

# Why bubbles are accelerated? - Value of bankers' opportunistic behaviour behind the sub-prime problem<sup>★</sup>

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**Abstract.** This paper studies the relationship between bankers' opportunistic behaviour and bubble economies. We derive the value of bankers' opportunistic behaviour and show that these values are too large to overcome by the bankers' compensation profiles. When a fraction of bankers behave opportunistically, some loans are made without checking the applicants, and bubble economies are caused in the macroeconomy by the multiplier effect. We also derive the distribution of opportunistic behaviors by using Pareto-Lèvy distribution and thus we could know that it is more effective to monitor bankers and therefore reduce the likelihood of opportunistic behaviour.

**keywords.** Bellman equation, monitoring, and opportunistic behaviour.

**J.E.L. classification.** D39, E51, G01.

## 1 Introduction

This paper studies the relationship between bankers' opportunistic behaviour and bubble economies. Even though we assume that agents are all rational, there are always those who work opportunistically and those who do not. Opportunistic behaviour include shirk. Work that are done by opportunistic behaviour might become frauds or deceptions for predatory lendings.<sup>1</sup> It might also become embezzlement. If bankers (agents) lend money without checking the loan applicants, i.e. if they shirk, pretending that they have worked appropriately, the results are that the money supply caused by these loans are larger than its optimal amount. And thus the bubble begins.

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<sup>1</sup> Refer Engel and McCoy (2007) for the definition of predatory lending.

We present a paper that describes these lendings by opportunistic behaviour cause bubble economy and its burst.<sup>2</sup> There is historical evidence on the cumulating deterioration in the quality of credit during the period of prosperity that precedes severe depression (Moore, p.288, 1956).

Agents behave opportunistically whenever they are not monitored. Opportunity costs of monitoring become higher when the economies are in their booms for there are many areas that are profitable for banks. The opportunity costs of monitoring become higher at the boom for those who are monitoring could be applied to those sections that are profitable. Higher monitoring costs make less monitoring and therefore, increase opportunistic behaviour. This paper theoretically prove that the value of opportunistic behaviour are so large that it is infeasible to overcome them by any compensation profile. Actually we study that the value of opportunistic behaviour are increasing function of any compensation. The value of opportunistic behaviour are shown numerically in this paper. Hereafter we refer to the agents employed at banks or non-banks that are able to lend money as bankers. And without loss of generality, we assume that it is same for the bankers to deceive the loan applicants for predatory lending and the applicants to bribe the bankers to get the loan. This is because the money is received by bankers and the loan is lent do not change in both cases. Thus, henceforth we refer to the fraud done by bankers as the bribe taken by them.<sup>3</sup> We thereby generalize these incentive problems to take them into our model.

Even though in a complete labour market, we only know the possibilities of opportunistic behaviour of the agents and do not know when they would behave opportunistically. One can not punish those who have not yet behaved opportunistically even though one know that they might in the future. Thus, one can not reduce the expected cost of opportunistic behaviour beforehand from those agents. This makes the value of payment higher for those who behave opportunistically than those who do not, even though they receive the same wage for their same competence. Complete contracts that would avoid any of these incentive problems are impossible to make and it is up to the lender whether to take the bribe or not (Iacobucci and Winter, 2005).

The loans made by these employees would make the money supply at the macroeconomy larger than its optimal amount through the multiplier effect. Enlarged money supply would make bubble economy and its burst. After its burst, negative bubbles would also be brought by the opportunistic (shirking) behaviour of bankers insisting that it is in recession so it is risky to lend money without checking the loan applicants.

Theoretically, employees would be opportunistic whenever they perceive that the marginal benefits of opportunistic behaviour would exceed the marginal costs. (See Staten and Umbeck (1982) for empirical study of opportunistic behaviour.) Firms have a proportion of employees who will find the short-term gains from opportunistic behaviour quite irresistible (Nagin *et al.*, 2002). Authors who linked these agency problems with bubbles are Allen and Gale (2000) and Allen and Gorton (1993). Allen and Gale show that when price asset is financed by bank loans, it is priced highly than the price asset that is self financed, since at the time of bankruptcy, the cost of insolvency

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<sup>2</sup> See LeRoy (2004) for the excellent review of the economic literature on bubbles.

<sup>3</sup> Kiyotaki and Moore (1997) also connect bribe offered by the farmer to creditors to booms and recessions.

is covered by the bank for those who financed by bank loans. And this makes the price asset higher than it is self financed. Incentive fees that have been permitted since 1985 made the bad portfolio managers who are unable to identify undervalued firms to take risky position and thus exacerbated the bubble (Allen and Gorton, 1993). Kocherlakota (2008) study the story of bubble by perturbation of the stochastic solvency constraint. Zeira (1999) studies bubbles in asset prices, production function, and entry in an industry by overshooting of a bayesian inference. In each of these, bubble happens only once. Akerlof and Katz (1989) analyze that if the ratio of opportunistic behaviour' value to the probability of being caught opportunistic is at least paid as the compensation profile to the worker, there is never any opportunistic behaviour. But we study that this opportunistic behaviour value is too large to be paid by any firm when there is uncertainty regarding their opportunistic behaviour. Besides we study in this paper that value of opportunistic behaviour is increasing function of any compensation. We use Bellman equation for this calculation.

In most cases, investors are principals (employers) and bankers are agents (employees). We define principals as those who monitor agents and agents as those who are monitored whether they are working appropriately or not. Though in all of these cases we do neither refer to any production function nor utility function, if we know the rate of opportunistic behaviour, we know how large the bubble inflates. It is only assumed that utility function is monotonously increasing function. In our model, we give no role to corporate earnings. This is justified by the fact that though stock price rose sharply in the late 1990s in the U.S., after-tax corporate earnings as a production of GDP, shows that while earnings rose in the middle and late 1990s, even at their peak they were a smaller production of GDP than during most of the postwar period (LeRoy, 2004).

In Section 2 we present our model and simulate it. Section 3 provides some remarks about our conclusion.

## 2 The model and its simulations

In our model  $P$  is the payment earned by a banker and it includes his/her labour income and payment that a banker receives when a banker is behaving opportunistically such as taking bribe. It is expressed as

$$dP^i = \alpha_P P dt + \sigma_P^i P dz_P, \quad (1)$$

where  $\alpha_P$  is the trend rate of  $P$ ,  $\sigma_P$  is the uncertainty about whether a banker takes bribes or not, and  $dz_P$  is the increment of Weiner process of  $P$ . In this paper we only consider the simplest model and assume that only  $\sigma_P^i$  differs by the each agent.

To show that it is infeasible to prevent opportunistic behaviour by any compensation profile when the economy is at its boom, we apply the model of Dixit and Pindyck (chap. 6, 1994) that uses Bellman equation.

In our model,  $I$  is the fixed cost of monitoring that is incurred by bankers. Bank has monitoring costs to monitor their own employees and its fixed cost  $I$  is brought on the each banker equally from the time it is spent. That is, the incidence of  $I$  is all bankers, whether they behave opportunistically or not. Thus  $I/n$  is deprived from their wage, where  $n$  is the number of agents.  $I$  is assumed to be given because, if  $I$  is enlarged too

much it would deter those who do not behave opportunistically to work appropriately. In the competitive labour market  $I = 0$ , because utility earned by the agents who are working properly would be already 0 by the utility from their wage and the disutility of their labour. It would deprive the incentive to work if  $I > 0$  in the competitive labour market. Next we define the flow cost of opportunistic behaviour.  $C$  is the flow cost of opportunistic behaviour, that is the penalty multiplied by the probability that monitoring might catch someone who behaves opportunistically. Thus,  $C$  is increasing function of monitoring, where  $C$  is expressed as

$$dC = \alpha_C C dt + \sigma_C C dz_C, \quad (2)$$

where  $\alpha_C$  is the trend rate of  $C$ ,  $\sigma_C$  is the uncertainty of  $C$ , and  $dz_C$  is the increment of Weiner process of  $C$ .  $C$  is imposed on the bankers and includes the cost of being fired from the bank. When the economy is at its boom, banks reduce monitoring because their opportunity costs (flow costs of monitoring) become large. This is because the employees allocated to monitoring could be applied to sectors where it is profitable when the economy is in its good shape. When the economy is in a recession, the opportunity cost of monitoring becomes small and banks monitor their employees. This makes  $\sigma_P$  lower because monitoring reduces uncertainty regarding employees' opportunistic behaviour.  $V(P, C)$  is the value of opportunistic behaviour by the bankers, where  $V(P, C)$  is<sup>4</sup>

$$V_t = E \int_{t=0}^{\infty} \pi_s e^{-rs} ds = \pi_t / r. \quad (3)$$

Here  $\pi = \max[P - C, 0]$ , and  $r$  is the discount rate per unit time. We assume that discount rate and risk-free interest rate have the same values. Without loss of generality, we assume that opportunities to behave opportunistically in one's occupied carrier is infinite. The reason for this is that, as long as one can behave opportunistically we could take  $dt$  sufficiently small as the time to behave opportunistically, and opportunities to behave opportunistically could be an immense amount even in a working day. Thus, the aggregate opportunities in whole working life would be numerous and near infinity. Further we have

$$E[(dz_P)^2] = E[(dz_C)^2] = dt \quad (4)$$

and

$$E[(dz_P)(dz_C)] = \rho dt, \quad (5)$$

where we set  $\rho = -1$  for  $dz$  and  $dz_C$  move oppositely. Bellman equation for this model is

<sup>4</sup> It is easy to extend this model to finite horizon as,

$$V_t = E \int_{t=0}^Q \pi_s e^{-rs} ds = \pi_t [1 - e^{-rQ}] / r.$$

where  $Q$  is the time when employee quits the firm. The numerical results are almost same with  $Q < \infty$  and  $Q = \infty$ . See Majd and Pindyck (1987) and Dixit and Pindyck (1994, chap.10.2).

$$rV(\pi)dt = \pi dt + E[dV(\pi)] . \quad (6)$$

And because the disutility of marginal labour equals marginal utility of revenue in the competitive sector, the disutility of working appropriately is always equal to the utility of additional wage.<sup>5</sup> Thus, for those who do not behave opportunistically,  $V(P, C) = 0$  and  $\sigma = 0$ . When

$$V(\pi) > I , \quad (7)$$

agents behave opportunistically. Otherwise we assume that they work without behaving opportunistically or shirkingly. As we have noted earlier, note that only  $\sigma$  differs by the each agent. There are those who work opportunistically and those who do not. Those who work opportunistically are expressed in this model as  $\sigma > 0$  and those who do not as  $\sigma = 0$ . And because it is impossible to have  $\sigma = \infty$  for anyone, we assume that  $\sigma$  is distributed on  $[0, x]$ . In the calculation of  $V(\pi)$ , we use the technique used in Dixit and Pindyck (1994, p.210). As the value of the opportunistic behaviour is homogeneous of degree one  $V(P, C)$ , we could reduce the argument to one, i.e.  $p = P/C$ . We follow Dixit and Pindyck (1994, pp.187-189) for the derivation of  $V(\pi)$ . When agents are assumed to be risk neutral, we get,  $R_p = r - \alpha_p$  and  $R_C = r - \alpha_C$  (McDonald and Siegel, pp.334-335, 1985).  $V(\pi)$  would be derived by solving Cauchy-Euler equation. See the Appendix.

## 2.1 Bubbles

Bankers (agents) try to maximize  $\pi$ , while the employers (principals) try to minimize it by controlling labour incentives. Bankers might get some payment not only from their banks but also from their loan applicants. Applicants might try to bribe the bankers so as to get the loan. Bankers might receive the bribe or might not receive the bribe. Thus, if we set  $\sigma = 0.2$ ,  $V(P)$  would be more than 100 when  $P = 10$ ,  $R_p = R_C = r = 0.04$ ,  $\alpha_p = \alpha_C = 0$ ,  $\rho = -1$  and  $C = 10$ . See Fig.1. We have calculated here  $V(P) = cv(p)$ , where  $v(p) = V(\frac{P}{C})$ .

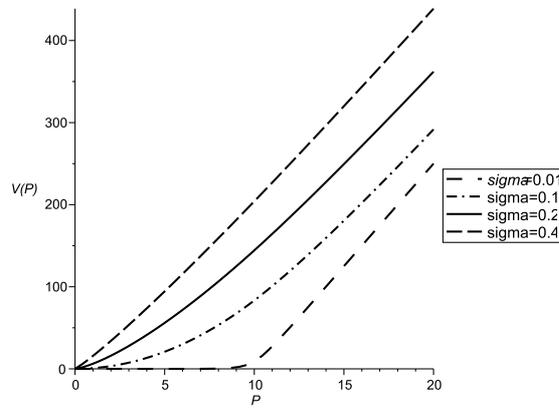
When  $C$  does not move as equation (2) and is stable through time, the figure would be as Fig.2 that is exactly the Dixit and Pindyck model (chap.5, 1994).

Thus  $V(P)$  easily exceeds  $I$ . But  $I$  could not be enlarged because it might deprive the will from those who are working properly. But increase of monitoring would make  $V(P)$  around 0 by making  $\sigma$  smaller as in the Fig.1. Fig.1 shows  $V(P)$  when  $C = 10$  and  $\sigma$  is set to 0.4, 0.2, 0.1, and 0.01. Further monitoring lowers the likelihood of large  $\sigma$ . We study about this later in section 2.2. We are able to see from these two figures that value of opportunistic behaviour  $V(P)$  is an increasing function of  $P$ .

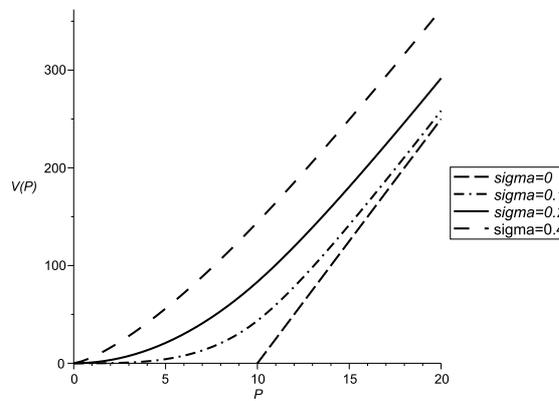
There are evidence that monitoring done by Office of Comptroller of the Currency (OCC) and new increased enforcement is significantly associated with decreased likelihood of subprime applications and rejections,<sup>6</sup> whereas the existence of a law itself has very small impact on the flow of subprime credit (Bostic *et al.* (2008)). The reason of this from our study is that strongest law is associated with large  $C$  and makes the value

<sup>5</sup> See Dixon (1994) for example.

<sup>6</sup> See Bostic *et al.* (2008) especially Tables 7 and Tables 8.

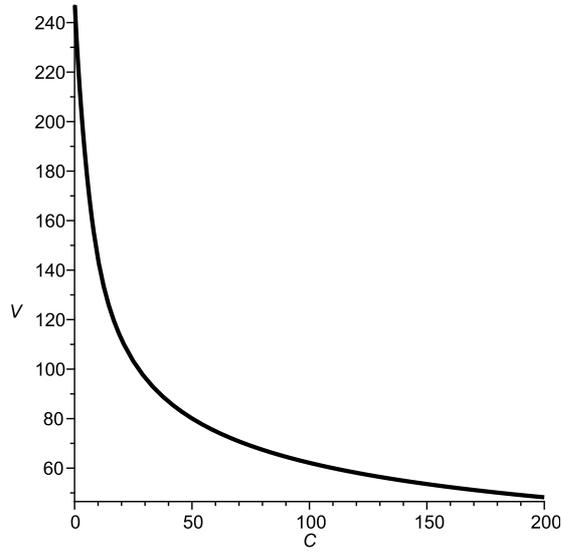


**Fig. 1.** Relationship between  $V(P)$  and  $\sigma$ .



**Fig. 2.** Relationship between  $V(P)$  and  $\sigma$  when  $C$  is constant.

of opportunistic behaviour lower whereas monitoring lowers the probability density function (pdf) of  $\sigma$ . Large  $C$  does not lowers pdf of opportunistic behaviour but only lowers  $V$ . See Fig.3. In this figure, however large  $C$  is made it does not make  $V \leq 0$ . For this reason  $V$  always stays  $V \geq I$  when  $C$  is only used to lower  $V$ . Therefore  $C$  is not effective in decreasing the likelihood of opportunistic behaviour. But uncertainty of  $\sigma$  is effective in decreasing the likelihood of opportunistic behaviour. We further explain about this in detail in section 2.2 when we study about  $\delta$  that is uncertainty of  $\sigma$ , and investigate about this particularly. Ho and Pennington-Cross (2006) study strength of law and restriction and reveal that strength of law has no effect on origination of subprime origination but strongest restriction reduce subprime applications by 50 percent.



**Fig. 3.** Relationship between  $V(P)$  and  $C$ .

## 2.2 Increased money stock during bubble economies

If we assume here that a fraction of bankers  $f$  behave opportunistically in every banks, then the resulting money supply  $M_s$  would be

$$M_s = d + f(1 - D)d + f^2(1 - D)^2d + \dots \quad (8)$$

$$f(1 - D)M_s = f(1 - D)d + f^2(1 - D)^2d + f^3(1 - D)^3d + \dots, \quad (9)$$

and subtracting (9) from (8) and arranging it we have the equation as below,

$$M_s = \frac{d}{1 - f + fD} \quad (10)$$

where  $d$  is the optimal money supply minus aggregate quantity of cash and  $D$  is reserve deposite requirement rate. When  $f = 0.3$  and  $D = 0.05$ ,  $M_s = 1.39d$ . This fraction  $f$  depends on the likelihood of opportunistic behaviour of bankers and we assume here that bankers imitate other colleagues who are getting benefit by being opportunistic. However, because every opportunistic agents who are behaving opportunistically try to hide their behaviour from others, other colleagues can not imitate perfectly. And we assume that agents are only able to imitate those, especially foregoers, whose  $\sigma$  are close to their own. That is, their  $\sigma$  needs to be close to those of foregoers. They can not imitate those who has much higher value of  $\sigma$  than their own. If agents are permutated from the largest  $\sigma$ , we assume that this  $\sigma$  has Pareto-Lèvy (P-L) distributions.<sup>7</sup> As in Mandelbrot (p.188, 1961)  $\log \sigma(t)$  has a random walk and  $\sigma$  is expressed as

$$\sigma(t+1) = T\sigma(t) + L, \quad T < 1, \quad (11)$$

where  $T$  is a constant and  $L$  is 0 or has P-L distribution. In our study,  $T$  is expressing the portion that one can imitate from others. We further modify this equation as below,

$$L = \begin{cases} 0 & \text{when } L \leq T\sigma(t) \\ \int_{t+2}^{\infty} T^s \sigma(s) ds & \text{when } L > T\sigma(t). \end{cases} \quad \text{Then assume } T\sigma(t) = 0. \quad (12)$$

The second equation of (12) expresses that when ones' foregoer colleagues' values of  $\sigma$  is not large compared to those values of inferiors' (who have smaller  $\sigma$  than one have) aggregated value of  $\sigma$ , one would refer the aggregated opportunistic behaviour of inferiors. Otherwise one would imitate the  $\sigma$  of foregoer colleague. At referring one's inferiors, one can calculate all the values of  $\sigma$  for one has observed the sequence of foregoers'  $\sigma$  and is able to sum them up to use them for one's growth rate of  $\sigma$ . If we make the above equations continuous,

$$\begin{aligned} \log \sigma(t+1) &= \log(T\sigma(t)) \\ &= \log T + \log \sigma(t) \\ \log \sigma(t+1) - \log \sigma(t) &= \log T, \quad T < 1 \end{aligned} \quad (13)$$

and because  $\log \sigma(t)$  has a random walk,

$$\begin{aligned} \log \sigma(t+1) - \log \sigma(t) &= d \log \sigma \\ d \log \sigma(t) &= \log T dt + \delta dw(t) \\ d\sigma_1 &= \sigma_1 \log T dt + \delta \sigma_1 dw(t), \end{aligned} \quad (14)$$

where  $\delta$  is the uncertainty of  $\sigma$ . And for when  $T\sigma(t) = 0$ ,

<sup>7</sup> See Mandelbrot (1960) and Mandelbrot (1961).

$$\begin{aligned}
\log \sigma(t+1) &= \log L \\
&= \log\left(-\frac{T^{(t+2)}}{\log T}\right)\sigma(t+2) \\
\log \sigma(t+2) - \log \sigma(t+1) &= -\log\left(-\frac{T^{(t+2)}}{\log T}\right) \\
d \log \sigma &= -\log\left(-\frac{T^{(t+2)}}{\log T}\right) + \delta dw(t) \\
d\sigma_2 &= -\sigma_2 \log\left(-\frac{T^{(t+2)}}{\log T}\right) + \delta\sigma_2 dw(t), \tag{15}
\end{aligned}$$

where we have calculated  $L$  when  $L \neq 0$ , as

$$\begin{aligned}
L &= \int_{t+2}^{\infty} T^s \sigma(s) ds \\
&= \left[ \frac{T^s}{\log T} \sigma(s) \right]_{t+2}^{\infty} \\
&= \frac{T^{\infty}}{\log T} \sigma(\infty) - \frac{T^{t+2}}{\log T} \sigma(t+2) \\
&= -\frac{T^{(t+2)}}{\log T} \sigma(t+2) > 0, \quad T < 1. \tag{16}
\end{aligned}$$

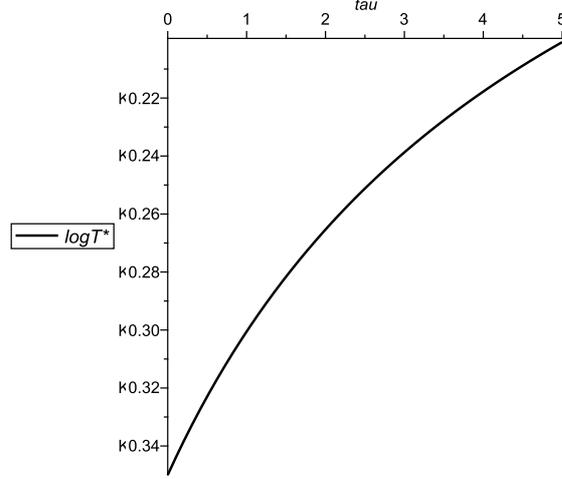
This is because agents could predict the succeeding process of  $\sigma$  after the long sequence of equation (11), and therefore agents could aggregate the process of  $\sigma$  to  $t = \infty$ . See the appendix for  $L_1 = \int_{t+3}^{\infty} T^s \sigma(s) ds$  which is almost equal to none.

As above equations we use two stochastic differential equations to depict pdf,  $p(\sigma, t)$ , for cases when  $L = 0$  or has P-L distribution. The reason of two is that agents imitate those foregoers when their  $\sigma$  is higher than ones'  $\sigma$  and do not when they are not and refer to all those who have lower  $\sigma$  than oneself. When (at time  $\tau$ ) foregoers'  $\sigma$  and aggregated inferiors'  $\sigma$  becomes the same value for those who are trying to refer others' behaviour and the two stochastic differential equations becomes the same. Thus<sup>8</sup>

$$\begin{aligned}
\log T &= -\log\left(-\frac{T^{(\tau+2)}}{\log T}\right) \\
T^{-1} &= -\frac{T^{(\tau+2)}}{\log T} \\
\log T &= -T^{(\tau+3)} \\
T^* &= \exp\left(-\frac{\text{Lambert}W(\tau+3)}{\tau+3}\right). \tag{17}
\end{aligned}$$

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<sup>8</sup> See Corless *et al.* (1996) for *LambertW* and use mathematical software to calculate this last equation.



**Fig. 4.** Relationship between  $\log T^* = -\frac{\text{LambertW}(\tau+3)}{\tau+3}$  and  $\tau$ .

Fig.4 depicts how  $\log T^*$  would evolve through  $\tau$ . It might have some time span when it is  $\sigma_1 = \sigma_2 = \sigma$ . It has the value of  $-0.35$  to  $-0.21$  when  $\tau \in [0, 5]$ . We also assume here that all agents' learning curves are the same so that every agents can imitate others at the same speed. To derive the pdf of  $\sigma$ , we substitute equations (14) and (15) into Kolmogorov forward equation:

$$\frac{\partial}{\partial t} X = -A(t, y) \frac{\partial}{\partial y} X + \frac{1}{2} B^2(t, y) \frac{\partial^2}{\partial y^2} X. \quad (18)$$

They become as follows,

$$\frac{\partial p(\sigma_1, t)}{\partial t} = -\sigma_1 \log T \frac{\partial p(\sigma_1, t)}{\partial \sigma_1} + \frac{1}{2} \delta^2 \sigma_1^2 \frac{\partial^2 p(\sigma_1, t)}{\partial \sigma_1^2}, \quad (19)$$

$$\frac{\partial p(\sigma_2, t)}{\partial t} = \sigma_2 \log\left(-\frac{T^{(\tau+2)}}{\log T}\right) \frac{\partial p(\sigma_2, t)}{\partial \sigma_2} + \frac{1}{2} \delta^2 \sigma_2^2 \frac{\partial^2 p(\sigma_2, t)}{\partial \sigma_2^2}. \quad (20)$$

And we get the pdf for the whole  $\sigma \in [0, x]$  as

$$p(\sigma, t) = \frac{2(\log T^* + \delta^2)}{\delta^2} x^{-\frac{2}{\delta^2} \log T^* - 2} \sigma^{\frac{2}{\delta^2} \log T^* + 1}. \quad (21)$$

See the appendix for the above proof. Probability density function (pdf),  $p(\sigma, t)$  becomes as Fig.5 when  $s = 1$ ,  $x = 2$ , and  $\tau = 2$ .  $s$  and  $\tau$  do not change  $p(\sigma, t)$  as much as  $\delta$ , where  $s$  is any constant that appears in Laplace transform and  $x$  is the maximum value that  $\sigma$  might take. Distribution function  $f(\sigma, t)$  becomes as Fig.6.  $\delta$  is the uncertainty of  $\sigma$ , and its smaller value makes smaller likelihood for large  $\sigma$ . This is because that monitoring makes each agents' opportunistic behaviour restrained, that is, they can not always behave opportunistically and thus makes the uncertainty ( $\delta$ ) of  $\sigma$  smaller and the

likelihood of opportunistic behaviour lower. Monitoring makes the value of  $\delta$  smaller and thus lowers  $p(\sigma, t)$  for large value of  $\sigma$ . Referring Fig.5 and Fig.6, see that one is able to make the likelihood of opportunistic behaviour smaller vehemently only by making  $\delta$  a little smaller from its former value. For example,  $\delta = 0.516$  would make  $f = 0.05$  for agents with  $\sigma = 0$  would occupy more than 95 percent of the all agents. Substituting this amount to (10), we have money supply almost  $M_s = 1.05d$ . This is an amount of money supply that might be allowable for central banks compared to  $M_s = 1.39d$ . If  $\delta$  is made smaller than  $\delta = 0.516$  the pdf of  $\sigma$  disappears, that is, any value of  $\sigma$  would have no probability.

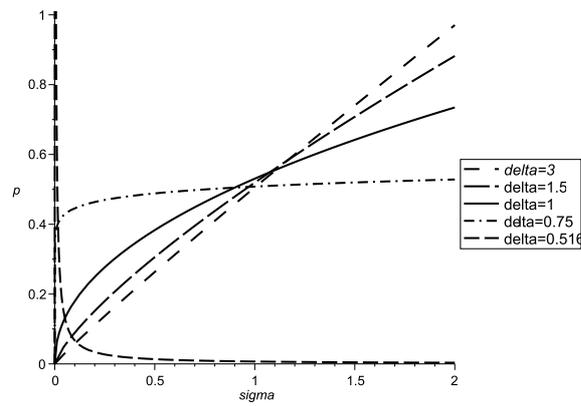


Fig. 5. Relationship between  $p$  and  $\sigma$ .

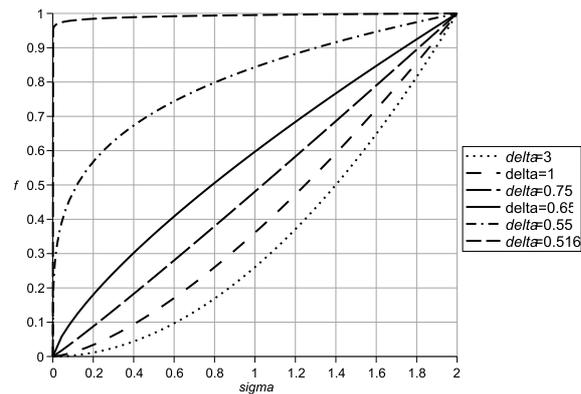


Fig. 6. Relationship between  $f$  and  $\sigma$ .

We can easily expand this study from bubble economy to negative bubble. When at negative bubble, agents at banks shirk by the excuses that it is because of recession that they do not lend any money or no inter-bank transactions. In this situation, they do not have to check loan applicants or other banks insisting that they do not know their assets. Notice here that there are loan applicants who have been rejected their loans without being properly checked because bankers value their shirking behaviour much higher than properly working when they are not monitored. The policy to cope with this situation is to make loan applicants or all banks' assets disclosed by enforcement law. If there is adequate competition in the market, disclosure makes inter-bank transactions arise because there are always those that are in better commercial conditions than other banks. And those that are in good situations can borrow the money from others. In this meaning, President Obama of United States made a good decision at the early stage of financial crisis. This policy could be easily extended to other industries. The policy that should be done by government is to disclose companies assets that belong to industries that government desires to grow. As in banking industries, there are always those that are in better commercial conditions than banks that lend the money. Consider that distribution of Fig.6 could easily be applied to revenue distribution of firms in an industry. Inter-bank transactions and ordinary loans are different but ordinary loan rates are much higher than inter-bank transactions rate. If inter-banking transactions rose at the financial market by the President Obama's policy of disclosing all assets of banks, then disclosing the companies' assets that belong to an industry would ignite the lendings by banks in that industry. Thus credit creation would be increased by the multiplier effect and the economy would reboot. From our study this disclosure of companies' assets means monitoring whether banks are adequately checking their loan applicants or not. Governments or central banks should not only check bank assets but also check their loan applicants in order to check whether banks are behaving opportunistically or not. Bankers shirk at the negative bubble and try to get money fraudulently or shirk at bubble economy because the value of opportunistic behaviour are much higher than their value of wages if they are not properly monitored. But because bankers only shirk at negative bubble, if they are properly monitored, that is checking whether bankers are shirking or not by checking their loan applicants at the same time, bankers would start behaving properly and credit creation would begin for their value of opportunistic behaviour are not as large as value of opportunistic behaviour at the time of bubble economy. This is because  $P$  does not include any bribe at recession for bankers do not lend any money.

Further because  $f(\sigma, t)$  could be easily affected by monitoring, central banks could use monitoring as a tool other than interest rate to control money supply. This is because, by making  $\delta$  smaller, one could make likelihood of large  $\sigma$  lower as in Fig.6. Also see Fig.7. Even though at the same interest rate, by making different  $\delta$  from one region from another through different monitoring extent, one could make one region monetary restraint and another region monetary relaxation. If one region needs monetary restraint, check banks and their loan applicants by severe standards, so that bankers are not able to lend money easily. One need not have to monitor all loan applicants but only some. The effect would spread through the banking industry by the imitation of agents that we have studied in this section. If one region needs monetary relaxation, one needs to be careful about what kind of situation one is in. If there is negative bubble and no

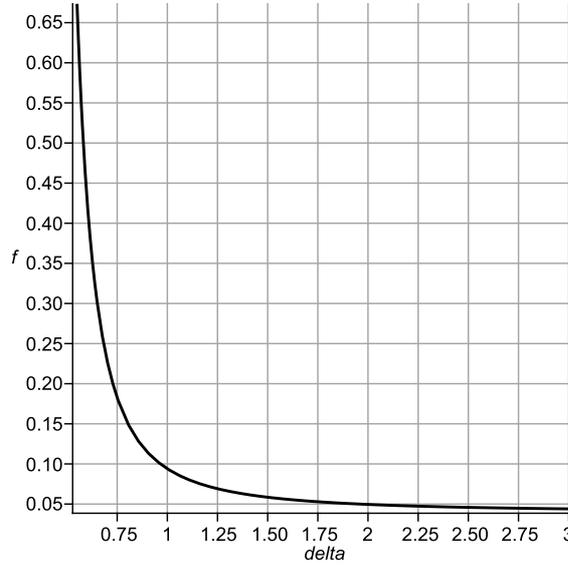


Fig. 7. Relationship between  $f$  and  $\delta$  when  $\sigma = 0.4$ .

credit creation, one needs to monitor bankers properly as is written above. If there is no negative bubble but still need to boost the economy, there is no need to monitor bankers by strict standards. No monitoring or monitor with low standards is the way to cope with this situation. But central banks should be the ones and not governments who should have the role to monitor banks, since governments are the ones who are likely to boost the economy though the economy is already in its bubble.

### 3 Concluding remarks

We study in this paper how large bankers' opportunistic behaviour values are and how it would affect the money supply in the macroeconomy. To analyze this carefully we have also derived the distribution of opportunistic behaviour by using Pareto-Lèvy distribution. The result is that by monitoring bankers and thereby making their opportunistic behaviour restrained, one is able to make the likelihood of opportunistic behaviour lower and thus recover the economy from any bubble.

## Appendix

### Derivation of $V(\pi)$ .

Bellman equation is

$$rVdt = \pi dt + E[dV], \quad (22)$$

where  $\pi = P - C$ . Using Ito's lemma

$$\begin{aligned} E[dV] &= E\left[\frac{\partial V}{\partial P}dP + \frac{\partial V}{\partial C}dC + \frac{1}{2}\left\{\frac{\partial^2 V}{\partial P^2}(dP)^2 + \frac{\partial^2 V}{\partial C^2}(dC)^2 + \frac{\partial^2 V}{\partial P\partial C}(dP)(dC)\right\}\right] \\ &= \frac{\partial V}{\partial P}\alpha_P P dt + \frac{\partial V}{\partial C}\alpha_C C dt + \frac{1}{2}\left\{\frac{\partial^2 V}{\partial P^2}\sigma_P^2 P^2 dt + \frac{\partial^2 V}{\partial C^2}\sigma_C^2 C^2 dt + 2\frac{\partial^2 V}{\partial P\partial C}\sigma_P\sigma_C\rho dt\right\} \end{aligned}$$

and because of the first and second derivatives of  $v(p) = V(P/C)$ ,<sup>9</sup>

$$E[dV] = v'(p)\alpha_P P dt + \{v(p) - v'(p)p\}\alpha_C C dt + \frac{1}{2}\left\{\frac{v''(p)}{C}\sigma_P^2 P^2 dt + \frac{v''(p)}{C}p^2\sigma_C^2 C^2 dt - 2\frac{pv''(p)}{C}\sigma_P\sigma_C\rho dt\right\}. \quad (23)$$

Thus, Bellman equation is

$$rV dt = (P - C)dt + v'(p)\alpha_P P dt + \{v(p) - v'(p)p\}\alpha_C C dt + \frac{1}{2}\left\{\frac{v''(p)}{C}\sigma_P^2 P^2 dt + \frac{v''(p)}{C}p^2\sigma_C^2 C^2 dt - 2\frac{pv''(p)}{C}\sigma_P\sigma_C\rho dt\right\}. \quad (24)$$

Dividing both sides by  $C$  and rearraging, we have

$$0 = (\alpha_C - r)v(p) + (p - 1)v'(p) + \frac{1}{2}\{v''(p)p^2\sigma_P^2 + v''(p)p^2\sigma_C^2 - 2p^2v''(p)\sigma_P\sigma_C\rho\}. \quad (25)$$

When  $\pi = 0$ , we substitute  $v(p) = K_1 p^{\beta_1}$ . Otherwise we substitute  $v(p) = B_2 p^{\beta_2} + p/R_P - 1/R_C$ . Thus, when  $\pi = 0$ , equation (25) becomes

$$\begin{aligned} 0 &= (\alpha_C - r)K_1 p^{\beta_1} + \beta_1 K_1 p^{\beta_1}(\alpha_P - \alpha_C) + \frac{1}{2}\beta_1(\beta_1 - 1)K_1 p^{\beta_1}\{\sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho\} \\ &= (\alpha_C - r) + \beta_1(\alpha_P - \alpha_C) + \frac{1}{2}\beta_1(\beta_1 - 1)\{\sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho\}. \end{aligned} \quad (26)$$

When  $\pi > 0$ , equation (25) becomes

$$\begin{aligned} 0 &= (\alpha_C - r)\left(B_2 p^{\beta_2} + \frac{p}{R_P} - \frac{1}{R_C}\right) + \beta_2 B_2 p^{\beta_2} + \frac{1}{R_P}p(\alpha_P - \alpha_C) \\ &\quad + \frac{1}{2}B_2\beta_2(\beta_2 - 2)p^{\beta_2-2}p^2\{\sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho\} + p - 1 \\ &= (\alpha_C - r)B_2 p^{\beta_2} + (\alpha_C - r)\left(\frac{p}{R_P} - \frac{1}{R_C}\right) + \beta_2 B_2 p^{\beta_2}(\alpha_P - \alpha_C) + \frac{p}{R_P}(\alpha_P - \alpha_C) \\ &\quad + \frac{1}{2}\beta_2 B_2(\beta_2 - 1)p^{\beta_2}\{\sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho\} + p - 1, \end{aligned} \quad (27)$$

where

<sup>9</sup> See p.210 of Dixit and Pindyck (1994).

$$(\alpha_C - r)\left(\frac{p}{R_P} - \frac{1}{R_C}\right) + \frac{p}{R_P}(\alpha_P - \alpha_C) + p - 1 = 0 \quad (28)$$

because of  $R_P = r - \alpha_P$  and  $R_C = r - \alpha_C$ . Thus equation (27) becomes,

$$\begin{aligned} 0 &= (\alpha_C - r)B_2p^{\beta_2} + \beta_2B_2p^{\beta_2}(\alpha_P - \alpha_C) + \frac{1}{2}B_2p^{\beta_2}\beta_2(\beta_2 - 1)\{\sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho\} \\ &= (\alpha_C - r) + \beta_2(\alpha_P - \alpha_C) + \frac{1}{2}\beta_2(\beta_2 - 1)\{\sigma_P^2 + \sigma_C^2 - 2\sigma_P\sigma_C\rho\}. \end{aligned} \quad (29)$$

We set the larger  $\beta$  as  $\beta_1$  in the above equations. We have  $K_1$  and  $B_2$  as,

$$K_1 = \frac{1}{\beta_1 - \beta_2} \left( \frac{1 - \beta_2}{R_P} + \frac{\beta_2}{R_C} \right) \quad (30)$$

and

$$B_2 = \frac{1}{\beta_1 - \beta_2} \left( \frac{1 - \beta_1}{R_P} + \frac{\beta_1}{R_C} \right). \quad (31)$$

This is because of Smooth-Pasting condition. We calculate here similarly as Dixit and Pindyck (pp.188-189, 1994).

### Proof that only

$$L = \int_{t+2}^{\infty} T^s \sigma(s) ds$$

matters.

First of all, we derive the relationship between  $\sigma(t)$  and  $\sigma(t + 2)$ . Because,

$$\sigma(t + 1) = T\sigma(t) + L, \quad T < 1, \quad (32)$$

$\sigma(t + 2)$  becomes,

$$\begin{aligned} \sigma(t + 2) &= T\sigma(t + 1) + L_1, \quad T < 1, \\ &= T(T\sigma(t) + L) + L_1 \\ &= T^2\sigma(t) + TL + L_1 \end{aligned} \quad (33)$$

where  $L_1$  is

$$L_1 = \begin{cases} 0 & \text{when } L_1 \leq T\sigma(t + 1) \\ \int_{t+3}^{\infty} T^s \sigma(s) ds & \text{when } L_1 > T\sigma(t + 1). \end{cases} \quad \text{Then we assume } T\sigma(t + 1) = 0. \quad (34)$$

And,

$$\begin{aligned}
L_1 &= \int_{t+3}^{\infty} T^s \sigma(s) ds \\
&= -\frac{T^{(t+3)}}{\log T} \sigma(t+3) \sim 0.
\end{aligned} \tag{35}$$

This is because, when  $T = 0.3$ ,  $\log T = -1.203$  and when  $T = 0.2$ ,  $\log T = -1.609$  and  $-\frac{T^{(t+3)}}{\log T}$  would be 0.006645 and 0.004971 for each value of  $T$  where substituting  $t = 0$ .

### Proof of

$$p(\sigma, t) = \frac{2(\log T + \delta^2)}{\delta^2} x^{-\frac{2}{\delta^2} \log T - 2} \sigma^{\frac{2}{\delta^2} \log + 1}.$$

Using Laplace transform to equations (19) and (20) we have,

$$sF(\sigma_1, s) - p(\sigma_1, 0) = -\sigma_1 \log T F_{\sigma} + \frac{1}{2} \delta^2 \sigma_1^2 F_{\sigma\sigma} \tag{36}$$

and

$$sF(\sigma_2, s) - p(\sigma_2, 0) = \sigma_2 \log\left(-\frac{T^{(t+2)}}{\log T}\right) F_{\sigma} + \frac{1}{2} \delta^2 \sigma_2^2 F_{\sigma\sigma}, \tag{37}$$

where  $F(\sigma, s)$  is

$$F(\sigma, s) = \mathcal{L}(p(\sigma, t)) = \int_0^{\infty} e^{-st} p(\sigma, t) dt. \tag{38}$$

In the above equation,  $\mathcal{L}$  is the Laplace transform and  $s$  is any real number that is constant. Solving above equations we have,

$$-\frac{1}{2} \delta^2 \sigma_1^2 F_{\sigma\sigma} + \sigma_1 F_{\sigma} \log T + sF(\sigma_1, s) = p(\sigma_1, 0) \tag{39}$$

and

$$-\frac{1}{2} \delta^2 \sigma_2^2 F_{\sigma\sigma} - \sigma_2 F_{\sigma} \log\left(-\frac{T^{(t+2)}}{\log T}\right) + sF(\sigma_2, s) = p(\sigma_2, 0). \tag{40}$$

Hereafter we denote  $p(\sigma_1, 0) = p_1$  and  $p(\sigma_2, 0) = p_2$ . We substitute  $K'_1 \sigma_1^{\beta_1} + \frac{p_1}{s}$  for the general solution of the equation (39) and  $B'_2 \sigma_2^{\beta_2} + \frac{p_2}{s}$  for the other.

$$\begin{aligned}
-\frac{1}{2} \delta^2 \sigma_1^2 \beta_1 (\beta_1 - 1) K'_1 \sigma_1^{\beta_1 - 2} + \sigma_1 \beta_1 K'_1 \sigma_1^{\beta_1 - 1} \log T + s K'_1 \sigma_1^{\beta_1} + p_1 &= p_1 \\
-\frac{1}{2} \delta^2 \beta_1^2 + \frac{1}{2} \delta^2 \beta_1 + \beta_1 \log T + s &= 0.
\end{aligned} \tag{41}$$

Calculating similarly as above for equation (40) we have,

$$-\frac{1}{2}\delta^2\beta_2^2 + \frac{1}{2}\delta^2\beta_2 - \beta_2 \log\left(-\frac{T^{(\tau+2)}}{\log T}\right) + s = 0. \quad (42)$$

If we assume that every agents have the same learning speed as we have mentioned above, agents who assess values of  $\sigma_1$  and  $\sigma_2$  the same would be appeared at the same time and its distribution would be given at some time  $\tau$ . And because those who have the same value of  $\sigma_1$  and  $\sigma_2$  turn their referring the behaviour of their foregoers to inferiors instantly, it would be continuous at the point  $\sigma_1 = \sigma_2 = \sigma$ . That is, we assume here that Smooth-Pasting condition (Dixit and Pindyck, 1994) is satisfied.<sup>10</sup> Therefore,  $T$  would be  $T^* = \exp\left(-\frac{\text{LambertW}(\tau+3)}{\tau+3}\right)$  and  $\mu = \log T^* = -\log\left(-\frac{T^{(\tau+2)}}{\log T^*}\right)$ . Substituting  $\mu$  into equation (41) and (42) we have the solution for  $\beta$  as,

$$\beta = \frac{\mu}{\delta^2} + \frac{1}{2} \mp \frac{1}{\delta^2} \sqrt{\left(\mu + \frac{\delta^2}{2}\right)^2 + 2\delta^2 s}. \quad (43)$$

We denote the larger one in above equation as  $\beta_1$  and smaller one as  $\beta_2$ . Further,  $K'_1\sigma^{\beta_1} + \frac{p_1}{s} = B'_2\sigma^{\beta_2} + \frac{p_2}{s}$  and also for their derivatives where  $p'_1 = \frac{\partial p_1}{\partial \sigma} = 0$  for  $p_1 = p(\sigma_1(0), 0) = 0$ . This is because of Smooth-Pasting condition. Thus,

$$\begin{aligned} \beta_1 K'_1 \sigma^{\beta_1-1} &= \beta_2 B'_2 \sigma^{\beta_2-1} + \frac{p'_2}{s} \\ \beta_1 (B'_2 \sigma^{\beta_2} + \frac{p_2 - p_1}{s}) &= \beta_2 B'_2 \sigma^{\beta_2} + \frac{p'_2 \sigma}{s} \\ \frac{\beta_1 (p_2 - p_1)}{s} &= B'_2 \sigma^{\beta_2} (\beta_2 - \beta_1) + \frac{p'_2 \sigma}{s} \\ B'_2 &= \frac{\beta_1 (p_2 - p_1) - p'_2 \sigma}{(\beta_2 - \beta_1) s \sigma^{\beta_2}}. \end{aligned} \quad (44)$$

Calculating similarly for  $K'_1$  we have,

$$K'_1 = \frac{\beta_2 (p_2 - p_1) - p'_2 \sigma}{(\beta_2 - \beta_1) s \sigma^{\beta_1}}. \quad (45)$$

Thus we have,

$$\begin{aligned} F(\sigma_1, s) &= K'_1 \sigma^{\beta_1} + \frac{p_1}{s} \\ &= \frac{p_2 \beta_2 - p_1 \beta_1 - p'_2 \sigma}{s(\beta_2 - \beta_1)}. \end{aligned} \quad (46)$$

and

<sup>10</sup> It could be easily proved that Smooth-Pasting condition is satisfied, that is, there is no kink using the discussion of Dixit and Pindyck (pp.130-132, 1994).

$$\begin{aligned}
F(\sigma_2, s) &= B'_2 \sigma^{\beta_2} + \frac{p_2}{s} \\
&= \frac{p_2 \beta_2 - p_1 \beta_1 - p'_2 \sigma}{s(\beta_2 - \beta_1)}.
\end{aligned} \tag{47}$$

Therefore inverting the Laplace transform, we get

$$\begin{aligned}
\mathcal{L}^{-1}(F(\sigma, s)) &= (p(\sigma, t)) \\
&= \frac{p_2 \beta_2 - p_1 \beta_1 - p'_2 \sigma}{(\beta_2 - \beta_1)}.
\end{aligned} \tag{48}$$

Because  $p(\sigma_1, 0) = p_1$  and  $p(\sigma_2, 0) = p_2$ , equation (19) and (20) becomes,

$$-\sigma \log T \frac{\partial p(\sigma_1, 0)}{\partial \sigma} + \frac{1}{2} \delta^2 \sigma^2 \frac{\partial^2 p(\sigma_1, 0)}{\partial \sigma^2} = 0 \tag{49}$$

and

$$\sigma \log\left(-\frac{T^{(t+2)}}{\log T}\right) \frac{\partial p(\sigma_2, 0)}{\partial \sigma} + \frac{1}{2} \delta^2 \sigma^2 \frac{\partial^2 p(\sigma_2, 0)}{\partial \sigma^2} = 0 \tag{50}$$

where we used differential of  $p$  when  $t = 0$  is 0. This is because,  $p(\sigma_1, 0)$  is  $p(\sigma_1(0), 0)$  thus,  $p(\sigma_1, 0) = 0$  for  $\sigma_1$  does not exist at  $t = 0$ . Though notice that  $p(\sigma_2, 0) = p(\sigma_1, \tau)$  for  $\tau$  is the time when  $\sigma_1 = \sigma_2$  and thus it is given. Therefore,  $\frac{\partial p(\sigma_1, \tau)}{\partial t} = 0$ . The reason of this is that even though  $\tau$  has its time span, because of  $\log T^*$  its distribution does not change along with the transition of  $\tau$ . Solving the above equations we have,

$$p_2 = C_1 \sigma^{\frac{2}{\delta^2} \log T^* + 1}. \tag{51}$$

The above equation is given by substituting  $p_2 = C_1 \sigma^m$  in equation (49) and (50) using  $\sigma_1 = \sigma_2 = \sigma$  and  $\log T^* = -\log\left(-\frac{T^{(t+2)}}{\log T}\right)$  and we have,

$$\begin{aligned}
-\sigma C_1 m \sigma^{m-1} \log T^* + \frac{1}{2} \delta^2 \sigma^2 C_1 m(m-1) \sigma^{m-2} &= 0 \\
-m \log T^* + \frac{1}{2} \delta^2 (m^2 - m) &= 0 \\
-\log T^* + \frac{1}{2} \delta^2 m - \frac{1}{2} \delta^2 &= 0 \\
m &= \frac{2}{\delta^2} \log T^* + 1.
\end{aligned} \tag{52}$$

And calculating for  $\int_0^x p(\sigma, t) d\sigma = 1$  and substituting  $p_1 = 0$  we have,

$$\begin{aligned}
\int_0^x \frac{p_2 \beta_2 - p_2' \sigma}{(\beta_2 - \beta_1)} d\sigma &= 1 \\
\beta_2 - \beta_1 &= \beta_2 \int_0^x C_1 \sigma^{\frac{2}{\delta^2} \log T^* + 1} d\sigma - \left( \frac{2}{\delta^2} \log T^* + 1 \right) C_1 \int_0^x \sigma^{\frac{2}{\delta^2} \log T^* + 1} d\sigma \\
&= \beta_2 C_1 \left[ \frac{\delta^2}{2 \log T^* + 2\delta^2} \sigma^{\frac{2}{\delta^2} \log T^* + 2} \right]_0^x - C_1 \left[ \frac{2 \log T^* + \delta^2}{2 \log T^* + 2\delta^2} \sigma^{\frac{2}{\delta^2} \log T^* + 2} \right]_0^x \\
&= \beta_2 C_1 \frac{\delta^2}{2 \log T^* + 2\delta^2} x^{\frac{2}{\delta^2} \log T^* + 2} - C_1 \frac{2 \log T^* + \delta^2}{2 \log T^* + 2\delta^2} x^{\frac{2}{\delta^2} \log T^* + 2} \\
&= C_1 \frac{\beta_2 \delta^2 - (2 \log T^* + \delta^2)}{2 \log T^* + 2\delta^2} x^{\frac{2}{\delta^2} \log T^* + 2} \\
C_1 &= \frac{(\beta_2 - \beta_1)(2 \log T^* + 2\delta^2)}{\beta_2 \delta^2 - 2 \log T^* - \delta^2} x^{-\frac{2}{\delta^2} \log T^* - 2}. \tag{53}
\end{aligned}$$

Thus,

$$p_2 = \frac{(\beta_2 - \beta_1)(2 \log T^* + 2\delta^2)}{\beta_2 \delta^2 - 2 \log T^* - \delta^2} x^{-\frac{2}{\delta^2} \log T^* - 2} \sigma^{\frac{2}{\delta^2} \log T^* + 1}. \tag{54}$$

Therefore,

$$\begin{aligned}
p(\sigma, t) &= \frac{p_2 \beta_2 - p_2' \sigma}{\beta_2 - \beta_1} \\
&= \frac{\beta_2(2 \log T^* + 2\delta^2)}{\beta_2 \delta^2 - 2 \log T^* - \delta^2} x^{-\frac{2}{\delta^2} \log T^* - 2} \sigma^{\frac{2}{\delta^2} \log T^* + 1} \\
&\quad - \frac{(2 \log T^* + \delta^2)(2 \log T^* + 2\delta^2)}{\delta^2(\beta_2 \delta^2 - 2 \log T^* - \delta^2)} x^{-\frac{2}{\delta^2} \log T^* - 2} \sigma^{\frac{2}{\delta^2} \log T^* + 1} \\
&= \frac{\delta^2 \beta_2(2 \log T^* + 2\delta^2) - (2 \log T^* + \delta^2)(2 \log T^* + 2\delta^2)}{\delta^2(\beta_2 \delta^2 - 2 \log T^* - \delta^2)} x^{-\frac{2}{\delta^2} \log T^* - 2} \sigma^{\frac{2}{\delta^2} \log T^* + 1} \\
&= \frac{(2 \log T^* + 2\delta^2)(\delta^2 \beta_2 - (2 \log T^* + \delta^2))}{\delta^2(\beta_2 \delta^2 - 2 \log T^* - \delta^2)} x^{-\frac{2}{\delta^2} \log T^* - 2} \sigma^{\frac{2}{\delta^2} \log T^* + 1} \\
&= \frac{2(\log T^* + \delta^2)}{\delta^2} x^{-\frac{2}{\delta^2} \log T^* - 2} \sigma^{\frac{2}{\delta^2} \log T^* + 1} \tag{55}
\end{aligned}$$

Calculating integral of this pdf from 0 to  $x$  we have,

$$\begin{aligned}
\int_0^x p(\sigma, t) d\sigma &= \frac{2(\log T^* + \delta^2)}{\delta^2} x^{-\frac{2}{\delta^2} \log T^* - 2} \int_0^x \sigma^{\frac{2}{\delta^2} \log T^* + 1} d\sigma \\
&= \frac{2(\log T^* + \delta^2)}{\delta^2} x^{-\frac{2}{\delta^2} \log T^* - 2} \left[ \frac{\delta^2}{2(\log T^* + \delta^2)} \sigma^{\frac{2}{\delta^2} \log T^* + 2} \right]_0^x \\
&= \frac{2(\log T^* + \delta^2)}{\delta^2} x^{-\frac{2}{\delta^2} \log T^* - 2} \left[ \frac{\delta^2}{2(\log T^* + \delta^2)} x^{\frac{2}{\delta^2} \log T^* + 2} \right] \\
&= 1. \tag{56}
\end{aligned}$$

Therefore, we could see that equation (55) is correct.  $\square$

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