Development Trap Driven by both Expectations and Initial Conditions*

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Abstract

In a human capital-based growth model, it is demonstrated that the circumstance of development trap, where both initial conditions and expectations could be decisive for the eventual outcome, naturally occurs. Then, the relative importance of such is examined.

Keywords: development trap, initial conditions, expectations, education

JEL *Classifications*: J24, O11, O33

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1 Introduction

Development trap or multiple equilibria is possible when some economic variables are complementary related with non-convexity. If complementarities are intertemporal, the initial conditions determine the eventual outcome, while expectations are decisive if they are intergenerational, or precisely, characterized by strategic complementarities. Though many studies model development trap, in most of them, either the intertemporal or intergenerational aspect is focused on, and hence, the significance of initial conditions and expectations on the eventual outcome is argued in distinct contexts.

One main reason is that they deeply shed light on specific problems or circumstances originating development trap. However, if there are relevant channels connecting both intertemporal and intergenerational aspects, the interplay of initial conditions and expectations matters for the eventual outcome. Examining these configurations would be meaningful since the economic implications significantly differ depending on which matters.

One exceptional work emphasizing this point and explicitly verifying the relative importance of initial conditions and expectations is Krugman (1991). He starts with a simplified model in which workers' sectoral choice is associated by expectation-driven multiple equilibria due to pecuniary externality within generations. Then, the intertemporal aspect is introduced by incorporating adjustment costs on moving sectors, and dynamical system of sectral adjustment is analyzed. There he shows that, depending on the position of the economy, both initial conditions and expectations could be decisive for the particular equilibrium that the economy eventually converges in.

This paper presents an alternative example to consider the relative importance of initial conditions and expectations. I employ a simple human capital-based growth model with overlapping generations where the technological adaptability of education matters, and demonstrate the above circumstance without any extra assumptions. The idea that I accept is simple: the positive externality of human capital of old and young workers on productivity growth. The former is familiar in the studies modeling development trap driven by initial conditions, e.g., Azaliadis and Drazen (1990) and Galor and Tsiddon (1997).[1] The latter is adopted in those modeling expectation-driven

[1]This type of development trap is examined in studies incorporating endogenous fertility choice such as Becker, et al. (1990), capital-skill complementarity such as Acemoglu (1997), and capital market imperfection such as Galor

multiple equilibria such as Maki et al. (2005) and Redding (1996).[2]

Combining these elements presents a framework to study which is decisive for the eventual outcome. The analysis shows that, if initial conditions are sufficiently well or extremely bad, the initial condition itself determines the eventual outcome, whereas expectations are decisive if the economy is in an intermediate position. This is analogous to the result, the possibility of which is stressed in Krugman (1991), though suppositions of both models fairly differ.

2 The Model

2.1 Basic Setup

Consider an overlapping generations economy indexed by t. Each generation lives for two periods and consists of a continuum of homogenous individuals with total mass 1. In each period, each individual produces single consumption goods according to:

$$y_i^t = a_j h_{i,a_j}^t, \qquad i = t,\ t+1. \tag{1}$$

y_i^t is the amount of goods produced by an individual of generation t in period i. h_{i,a_j}^t is the efficiency unit of labor put into j-th technology, i.e., $h_{i,a_j}^t = (work\ time) \times (labor\ productivity\ for\ a_j)$. a_j is the leading-edge technology in period j, and $a_j < a_{j+1}$ is satisfied for all $j \geq 0$. As formally assumed below, generation t necessarily uses a_t when young, i.e., $j = t$, while the technology choice between a_t and a_{t+1} is required when old. Consumption goods produced in each period are consumed during that period, and the preferences of individuals are defined by:

$$U^t = \ln y_t^t + \rho \ln y_{t+1}^t, \qquad 0 < \rho < 1. \tag{2}$$

At birth, individuals of generation t are endowed with productivity 1 for the leading-edge technology of that period, a_t, though the validity is limited for the use of a_t. When young, they allocate ε^t of time endowment for education and the rest for production, whereas they specialize in production when old ($0 \leq \varepsilon^t \leq 1$). Thus, y_t^t is simply given by $a_t(1-\varepsilon^t)$.

Investment in education yields $\sigma(\varepsilon^t)$ units of skills, where $\sigma(0) = 0$, $\sigma'(.) > 0$, and $\sigma''(.) \leq 0$. The characteristic of these skills is technological adaptability, that is, they are valid for future

and Zeira (1993).

[2] Seminal contribution modeling this type of multiplicity in the macroeconomic field is Murphy, et al. (1989).

technology, as well as a_t. When old, individuals must decide whether to stay in the existing technology, a_t, or update to a_{t+1} which emerges at the beginning of period $t+1$. If they stay in a_t, their labor productivity rises to $1+\sigma(\varepsilon^t)$, while it falls to $\sigma(\varepsilon^t)$ if updating to a_{t+1}. Hence, which choice is preferable depends whether the gains from the advanced technology exceed the losses from erosion of labor productivity.

2.2 Individuals' Maximization

According to the backward decision making, the individuals' problem is represented by which choice attains higher utilities between remaining in a_t and updating to a_{t+1}, subject that, ε^t is set to the optimal level in both cases. Denoting indirect utilities corresponding to the former and the latter choices by $\underline{U}^t$ and $\hat{U}^t$, respectively, this is simply formulated as follows from (1) and (2):

$$\max\left[\hat{u}^t(g_t), \underline{u}^t\right] \ , \ \ where \tag{3}$$

$$\underline{u}^t = \ln(1-\underline{\varepsilon}^t) + \rho\ln[1+\sigma(\underline{\varepsilon}^t)], \tag{4}$$

$$\hat{u}^t(g_t) = \ln(1-\hat{\varepsilon}^t) + \rho\ln(g_t+1)\sigma(\hat{\varepsilon}^t) \ , \tag{5}$$

where $u^t = U^t - (1+\rho)\ln a_t$, and g_t is the rate of technological progress in period t, $g_t = (a_{t+1}-a_t)/a_t$. $\underline{\varepsilon}^t$ and $\hat{\varepsilon}^t$ are the values at which the RHS of (4) and (5) are maximized, respectively. From (4) and (5), these are defined by:

$$\underline{\varepsilon}: \ \ \frac{1}{1-\underline{\varepsilon}} = \rho\frac{\sigma'(\underline{\varepsilon})}{1+\underline{\varepsilon}}, \tag{6}$$

$$\hat{\varepsilon}: \ \ \frac{1}{1-\hat{\varepsilon}} = \rho\frac{\sigma'(\hat{\varepsilon})}{\hat{\varepsilon}}. \tag{7}$$

The LHS of (6) and (7) is the marginal disutility of education investment caused by increases in foregone income when young. The RHS of (6) and (7) is the marginal utility of it through increases in consumption when old. Since it is bigger in the case of updating to a_{t+1} from $\frac{\sigma'(\varepsilon^t)}{1+\varepsilon^t} < \frac{\sigma'(\varepsilon^t)}{\varepsilon^t}$, $\hat{\varepsilon} > \underline{\varepsilon}$.[3] (6) and (7) also indicate $0 < \hat{\varepsilon} < 1$ and $\underline{\varepsilon} < 1$. And $0 < \underline{\varepsilon}$ is assured if $\rho\sigma'(0) > 1$. As follows, the solution of (3) is determined by the value of g_t.

[3] In general, $\hat{\varepsilon}$ should be defined as the function of g_t. In this model, the constant $\hat{\varepsilon}$ rests on, first, logarithmic utility function, and second, the assumption that skills acquired by education are adaptable for a_{t+1} with no depreciation. Similarly in Galor and Moav (2000), if incorporating the depreciation factor, $\delta(g_t)$ ($\delta'(.) > 0$, $\delta(.) \geq 0$), and assuming that skills acquired from education fall to $\hat{\varepsilon} - \delta(g_t)$ when updating technology, the positive relation of $\hat{\varepsilon}$ to g_t is guaranteed as indicated in Galor and Moav, Redding (1996), and Gould et al. (2001).

Lemma There exists a unique value of g_t, $\bar{g}$, where $\underline{u}^t = \hat{u}^t(\bar{g})$ holds, and, staying in a_t becomes optimal if $0 \leq g_t < \bar{g}$, whereas updating to a_{t+1} is preferred if $\bar{g} < g_t$.

Proof. $\hat{u}^{t'}(g_t) > 0$ and $\hat{u}^t(0) < \ln(1-\hat{\varepsilon}) + \rho \ln[1+\sigma(\hat{\varepsilon})] < \underline{u}^t$ hold from (4) and (5). These indicate the unique existence of $\bar{g}$ satisfying $\underline{u}^t >$ (resp. $<$) $\hat{u}^t(g_t)$ for $g_t <$ (resp. $>$) $\bar{g}$. From (4) and (5), $\bar{g}$ is defined by:

$$\bar{g}: \quad \frac{1-\underline{\varepsilon}}{1-\hat{\varepsilon}} = \left[\frac{(\bar{g}+1)\sigma(\hat{\varepsilon})}{1+\sigma(\underline{\varepsilon})}\right]^{\rho},$$

and individuals' optimal investment in education, $\varepsilon^t(g_t)$, is classified by:

$$\varepsilon^t(g_t) = \begin{cases} \underline{\varepsilon} & \text{if} \quad 0 \leq g_t < \bar{g} \\ \hat{\varepsilon} & \text{if} \quad \bar{g} < g_t \end{cases}. \qquad \square \tag{8}$$

Intuition of this Lemma is simple. Gains of utility from updating technology increase with g_t, whereas losses of it are constant. $g_t > \bar{g}$ means g_t is large enough such that the former exceeds the latter, while net gains of updating technology are negative for $g_t < \bar{g}$.

3 General Equilibrium and Long-run Growth

3.1 Technological Progress

Among human capital-based growth studies, it is familiar that technological progress is positively related to the level of human capital. However, as emphasized in Gelb et al. (1991), Murphy et al. (1991), and Yotsuya (2002), the allocation of it also matters if the choice of occupation, technology, or sector is analyzed. In this model, as discussed in Yotsuya, it could be plausible to suppose that educated workers working in the leading-edge technology can contribute on the next technology. Hence, referring Z_t as the aggregation of time spent for education by individuals who work in a_t in period t, technological progress is assumed to externally occur according to:

$$g_t = \phi(Z_t), \tag{9}$$

where $\phi'(.) > 0$, $\phi''(.) \leq 0$, $\phi(0) = 0$, and Z_t is given by

$$Z_t = z^{t-1} + \int_0^1 \varepsilon^{t,k} dk, \quad z^{t-1} = \int_{k \in a_t} \varepsilon^{t-1,k} dk.$$

That is, g_t relies on both old and young individuals, though old individuals have an effect on g_t only if updating technology. The contribution of generation $t-1$ on technological progress in period t is denoted by z^{t-1}.[4]

As formally shown below, this is the key of the model. First, the contribution of old individuals yields intertemporal complementarities, and hence, multiple, locally stable steady states may arise. Second, the contribution of young individuals yields strategic complementarities among their investments in education. Thus, for some values of z^{t-1}, expectation-driven multiple equilibria may also arise in each period. Consequently, the eventual outcome of the economy could depend on both initial conditions and expectations. Another source of these multiplicity is discontinuity of $\varepsilon^t(g_t)$.

3.2 Rational Expectations Equilibrium and Long run Growth

Given (9), the rational expectations equilibrium in period t is defined by the situation that individuals' symmetric expectation for g_t is self-fulfilling, i.e., all individuals choose the same ε^t which satisfies (8) and (9), simultaneously. This situation is depicted in Figure 1, where the ε^t-curve and the g_t-curve respectively illustrate (8) and (9) subject to $\varepsilon^{t,k} = \varepsilon^t$ for all $i \in [0,1]$. As seen there, three situations are possible depending on the value of z^{t-1}.

Proposition 1 Suppose:

$$\phi(\hat{\varepsilon}) < \bar{g} < \phi(1 + \underline{\varepsilon}), \tag{10}$$

then, two threshold values of z^{t-1}, $\underline{z}$ and $\hat{z}$, defined below exist in $0 < \underline{z} < \hat{z} < 1$:

$$\phi(\underline{z} + \hat{\varepsilon}) = \phi(\hat{z} + \underline{\varepsilon}) = \bar{g}.$$

The rational expectations equilibrium is obtained uniquely at $\varepsilon^t = \underline{\varepsilon}$ if $0 \leq z^{t-1} \leq \underline{z}$ and $\varepsilon^t = \hat{\varepsilon}$ if $\hat{z} \leq z^{t-1} \leq 1$. However, if $\underline{z} < z^{t-1} < \hat{z}$, it contains both $\varepsilon^t = \underline{\varepsilon}$ and $\varepsilon^t = \hat{\varepsilon}$.

Proof. Obviously from (10) and $\phi'(.) > 0$, two curves cross only at once at $\varepsilon^t = \underline{\varepsilon}$ if $0 \leq z^{t-1} \leq \underline{z}$, and at $\varepsilon^t = \hat{\varepsilon}$ if $\hat{z} \leq z^{t-1} \leq 1$. If $\underline{z} < z^{t-1} < \hat{z}$, they cross once at $\varepsilon^t = \underline{\varepsilon}$ and at $\varepsilon^t = \hat{\varepsilon}$ again (See, Figure 1). □

[4]The contribution of young workers on g_t ought to be discounted since they are still in the course of learning. Or, supposing that skills acquired by education are available also when young, which is interpreted to mean that individuals join the labor force after finishing education, might be suitable. These modifications do not affect the main results of the model.

[Figure 1 around here]

This proposition states that, in each period, though the economy potentially faces a possibility of expectation-driven multiple equilibria, whether it realizes depends on the contribution of old workers on technology growth. Under a small contribution such that $z^{t-1} \leq \underline{z}$, the expectations of $g_t > \bar{g}$ cannot be self-fulfilling. Then, individuals of generation t choose to stay in a_t, and thus, technology growth remains at a low rate, $g_t = \phi(z^{t-1} + \underline{\varepsilon})$. In contrast, under a large contribution such that $z^{t-1} > \hat{z}$, individuals actively invest in education looking forward to updating technology since more than $\bar{g}$ rate of g_t is guaranteed irrespective of their choice. Consequently, a high pace of technology growth, $g_t = \phi(z^{t-1} + \hat{\varepsilon})$, is achieved via externality. And significantly, for an intermediate value such that $\underline{z} < z^{t-1} < \hat{z}$, both optimistic and pessimistic expectations for g_t can be self-fulfilling. So which equilibrium is realized is determined by individuals' expectations.

The dynamical system of the economy is governed by the transition of z^t. Notice here that, $z^t = 0$ if individuals of generation t choose to stay in a_t when old, while $z^t = \hat{\varepsilon}$ if updating technology. Then, from Proposition 1, it is given by:

$$z^t(z^{t-1}) = \begin{cases} 0 & \text{if } \ 0 \leq z^{t-1} \leq \underline{z} \\ 0 \ \text{ or } \ \hat{\varepsilon} & \text{if } \ \underline{z} < z^{t-1} < \hat{z} \\ \hat{\varepsilon} & \text{if } \ \hat{z} \leq z^{t-1} \leq 1 \end{cases} . \tag{11}$$

Figure 2 is an example of the phase diagram of (11). As depicted there, the dynamical path overlaps in $\underline{z} < z^{t-1} < \hat{z}$. In this area, which path the economy takes depends on the expectations of individuals, that is, not the initial conditions, but the expectations are crucial for the long-run outcome.

Proposition 2 Under (10) and $\hat{\varepsilon} > \hat{z}$, two locally stable steady states, z^*, $z^* = 0$ and $z^* = \hat{\varepsilon}$, coexist. If $z^{t-1} \leq \underline{z}$ or $z^{t-1} \geq \hat{z}$, the initial condition itself determines which steady state the economy converges in. Yet, it is determined by individuals' expectation if $\underline{z} < z^{t-1} < \hat{z}$.

Proof. As depicted in Figure 2, it is apparent that, if $\hat{\varepsilon} > \hat{z}$, $z^t(z^{t-1})$ cross the diagonal at $z^t = 0$ and $z^t = \hat{\varepsilon}$, and $z^* = 0$ and $z^* = \hat{\varepsilon}$ are stable in $0 \leq z^{t-1} \leq \underline{z}$ and $\hat{z} \leq z^{t-1} \leq 1$, respectively. In $\underline{z} < z^{t-1} < \hat{z}$, if either $\varepsilon^t = \underline{\varepsilon}$ or $\varepsilon^t = \hat{\varepsilon}$ is actually realized, then the economy immediately converges to $z^* = 0$ or $z^* = \hat{\varepsilon}$. □

[Figure 2 around here]

$z^* = \hat{\varepsilon}$ corresponds to the high-growth steady state, $g^* = \phi(2\hat{\varepsilon})$, where both young and old individuals contribute on technological progress. On the other hand, $z^* = 0$ corresponds to the low-growth steady state, $g^* = \phi(\underline{\varepsilon})$, where technological progress relies only on young individuals with a small investment in education, $\underline{\varepsilon}$. In both steady states, the growth rate of the aggregate output corresponds to that of technology since the production of goods is proportional to a_t. As shown in this Proposition, the situation in which both initial conditions and expectations can be decisive for the eventual outcome is able to naturally occur. The relative importance of both elements stated here is similar to the result pointed out in Krugman (1991).

4 Concluding Remarks

Combining two familiar ideas, intertemporal and intergenerational externality of human capital, which are separately examined in most cases, is the key for Proposition 2. In fact, if either factor is excluded from (9), the structure of the model is precisely reduced to that of focusing on only one factor. Probably, there also are other meaningful combinations in considering the interplay of initial conditions and expectations, and hence, should be jointly examined.

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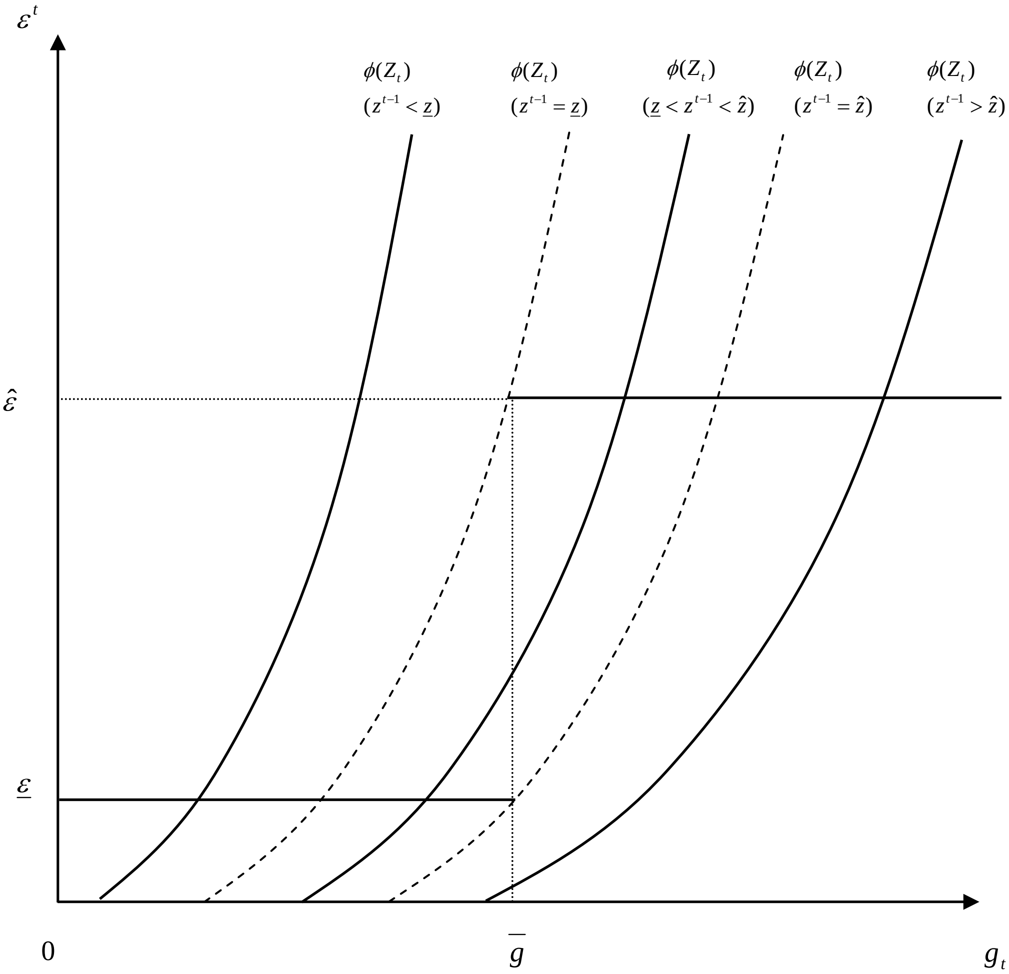

ε^t
$\phi(Z_t)$
$(z^{t-1} < \underline{z})$
$\phi(Z_t)$
$(z^{t-1} = \underline{z})$
$\phi(Z_t)$
$(\underline{z} < z^{t-1} < \hat{z})$
$\phi(Z_t)$
$(z^{t-1} = \hat{z})$
$\phi(Z_t)$
$(z^{t-1} > \hat{z})$
$\hat{\varepsilon}$
$\underline{\varepsilon}$
0
$\overline{g}$
g_t

Figure 1

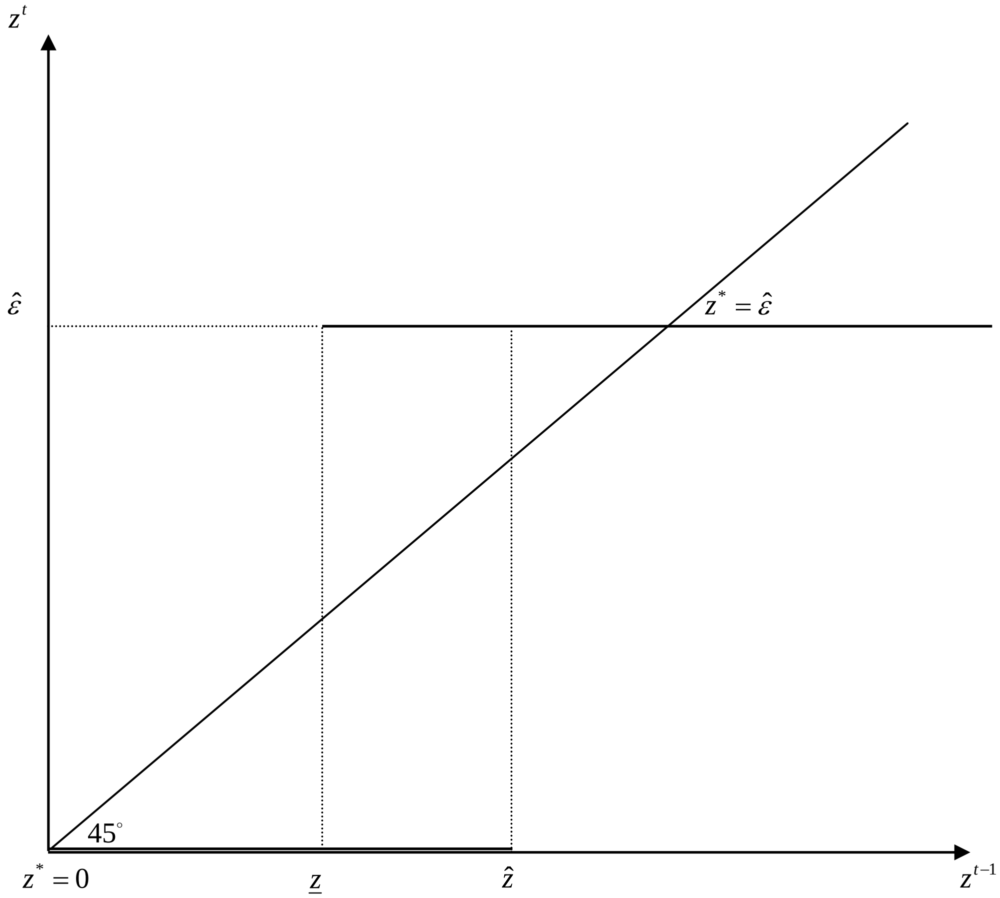
z^t
$\hat{\varepsilon}$
$z^* = \hat{\varepsilon}$
45°
$z^* = 0$
$\underline{z}$
$\hat{z}$
z^{t-1}

Figure 2