Note on social choice allocation in exchange economies with many agents

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Abstract

In this paper we show that in pure exchange economies there exists no Pareto efficient and strategy-proof allocation mechanism which ensures positive consumptions to all agents. We also show that Pareto efficient, strategy-proof, and non-bossy allocation mechanism is dictatorial. We further show that if there exists three agents, then the allocation given by a Pareto efficient, strategy-proof, and non-dictatorial mechanism should depend only on one agent's preference who is always allocated zero consumption. That is, we prove Zhou's (1991) conjecture in three-agent economies and show that a Pareto efficient and strategy-proof social choice function in such an economy should be Satterthwaite and Sonnenschein's (1981) type.

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1 Introduction

Since the seminal work by Hurwicz (1972), the manipulability of an allocation mechanism in pure exchange economies has been intensively studied. Especially Zhou (1991) established that there exists no Pareto efficient, strategy-proof and non-dictatorial allocation mechanism in exchange economies with two agents having classical (i.e. continuous,

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strictly monotonic and strictly convex) preferences and conjectured that even in many-agent economies there would exist a agent (reverse dictator) who is always allocated zero consumption subject to a Pareto efficient and strategy-proof allocation mechanism. As shown by Satterthwaite and Sonnenchein (1981), in many-agent economies there exists a Pareto efficient, strategy and non-dictatorial allocation mechanism. In their example, some agent is allocated the whole endowment depending on the shape of the preference of a agent who is always allocated zero consumption.

The impossibility result in two-agent economies has been strengthened by proving that there exists no Pareto efficient, strategy-proof and non-dictatorial social choice function on more restricted domains of preferences (Schummer (1997), Ju (2003), Hashimoto (2008)). Nicolò (2004) however showed a Pareto efficient, strategy-proof and non-dictatorial social choice function on the domain of Leontief preferences.

Compared to these researches of two-agent economies, we would not still have a satisfactory understanding about allocation mechanism in economies with many agents. Serizawa (2002) established that there exists no Pareto efficient, strategy-proof, and individually rational social choice function in many-agent economy. The individual rationality requires the social choice function to allocate a consumption preferred to each agent's initial endowment. Serizawa and Weymark (2003) established that there exists no Pareto efficient, strategy-proof and minimum consumption guarantee social choice function in many-agent economy. The minimum consumption guarantee condition requires the social choice function to allocate a consumption whose distance from the origin is greater than some ϵ. These results are short to Zhou's original conjecture. On the other hand, a counter example to Zhou's conjecture was found by Kato and Ohseto (2002). They showed that in economies with agents more than or equal to four, there exists a Pareto efficient and strategy-proof mechanism by which all agents have opportunities of receiving non-zero consumption. In their example, however, the whole endowment is always allocated to some agent as in the case of Satterthwaite and Sonnenschein's example. To the best of our knowledge, many economists believe that Zhou's conjecture would be true with such a correction though no formal proof has been obtained.

In this paper, we establish three results. First, we prove that there exists no Pareto efficient, strategy-proof and positive consumption guarantee social choice function. The positive consumption guarantee condition requires that the social choice function always allocates non-zero consumptions to all agents, which is therefore a slight relaxation of the minimum consumption guarantee condition by Serizawa and Weymark (2003). Second, we prove that there exists no Pareto efficient, strategy-proof, non-bossy, and non-dictatorial social choice function. A social choice function is non-bossy if a change of preference of an agent does not affect the allocation as long as it does not affect the agent's own consumption. In two-agent economies, Pareto efficiency implies non-bossiness. It is believed that the difficulty of many-agent economies is in the lack of non-bossiness. Third, we characterize Pareto efficient and strategy-proof social choice functions in three-agent economies. We prove that in three-agent economies the allocation given by a Pareto efficient, strategy-proof and non-dictatorial social choice function depends only on one agent's preference who is always allocated zero consumption. That is, in three-agent economies, Zhou's conjecture is true and a Pareto efficient and strategy-proof social choice function should be dictatorial or Satterthwaite and Sonnenschein's type. We prove these results on the domain of classical, homothetic, and smooth preferences. It is also proved that the same results hold on larger domains of classical preferences.

The rest of this paper is organized as follows. Section 2 describes the model and shows the results. Sections 3 and 4 explain our two techniques we use in this paper, which might have a wide range of applications in the literature. Sections 5-8 are proofs of theorems.

2 The model and Results

We consider an economy with N agents indexed by $\mathbf{N} = \{1, \ldots, N\}$ where $N \geq 2$ and L goods indexed by $\mathbf{L} = \{1, \ldots, L\}$ where $L \geq 2$. The consumption set for each agent is R_+^L. A consumption bundle for agent $i \in \mathbf{N}$ is a vector $x^i = (x_1^i, \ldots, x_L^i) \in R_+^L$. The total endowment of goods for the economy is $\Omega = (\Omega_1, \ldots, \Omega_L) \in R_{++}^L$. An allocation is a vector $\mathbf{x} = (x^1, \ldots, x^N) \in R_+^{LN}$. The set of feasible allocation for the economy with N agents and L goods is thus

$$X = \left\{ \mathbf{x} \in R_+^{LN} \middle| \sum_{i \in \mathbf{N}} x^i \leq \Omega \right\}$$

A preference R is a complete, reflexive, and transitive binary relation on R_+^L. The corresponding strict preference P_R and indifference I_R are defined in the usual way. For any x and x' in R_+^L, xP_Rx' implies that $xRx'$and not $x'Rx$, and xI_Rx' implies that xRx' and $x'Rx$. Given a preference R and a consumption bundle $x \in R_+^L$, the upper contour set of R at x is $UC(R, x) = \{x' \in R_+^L | x'Rx\}$ and the lower contour set of R at x is $LC(R, x) = \{x' \in R_+^L | xRx'\}$. Further we let $I(x; R) = \{x' \in R_+^L | x'I_Rx\}$ denote the indifference set of R at x and $P(x; R) = \{x' \in R_+^L | x'P_Rx\}$ denote the strictly preferred set of R at x.

A preference R is continuous if $UC(R, x)$ and $LC(R, x)$ are both closed for any $x \in R_+^L$. A preference R is strictly convex on R_{++}^L if $UC(R, x)$ is strictly convex for any $x \in R_{++}^L$. A preference R is strictly monotonic on R_{++}^L if for any x and x' in R_{++}^L, $x > x'$ implies xP_Rx'. A preference R is homothetic if for any x and x' in R_+^L and any $t > 0$, xRx' implies $(tx)R(tx')$. A preference R is smooth if for any $x \in R_{++}^L$ there exists a unique vector $p \in S^{L-1} \equiv \{x \in R_+^L | \parallel x \parallel = 1\}$ such that p is the normal of a supporting hyperplane to $UC(R, x)$ at x. We call the vector p as gradient vector of R at x

We let $\mathcal{R}_{\mathcal{C}}$ denote the set of classical preferences R that is continuous on R^L_+, strictly convex and strictly monotonic on R^L_{++}. Further we let $\mathcal{R}$ denote the set of preferences R that is continuous on R^L_+, strictly convex and strictly monotonic on R^L_{++}, smooth, and homothetic. In this paper, the results are proved on the restricted domain $\mathcal{R}$, and then extended to $\mathcal{R}_{\mathcal{C}}$.

A preference profile is an N-tuple $\mathbf{R} = (R^1, \dots, R^N) \in \mathcal{R}^N$. The subprofile obtained by removing R^i from $\mathbf{R}$ is $\mathbf{R}^{-i} = (R^1, \dots, R^{i-1}, R^{i+1}, \dots, R^N)$. It is sometimes convenient to write the profile $(R^1, \dots, R^{i-1}, \bar{R}^i, R^{i+1}, \dots, R^N)$ as $(\bar{R}^i, \mathbf{R}^{-i})$.

A social choice function $f : \mathcal{R}^N \to X$ assigns a feasible allocation to each preference profile in $\mathcal{R}^N$. The set $\mathcal{R}^N$ is the domain of the social choice function. For a preference profile $\mathbf{R} \in \mathcal{R}^N$, the outcome chosen can be written as $f(\mathbf{R}) = (f^1(\mathbf{R}), \dots, f^N(\mathbf{R}))$ where $f^i(\mathbf{R})$ is the consumption bundle allocated to agent i by f.

Definition 1. A social choice function f is strategy-proof if $f^i(\mathbf{R}) R^i f^i(\bar{R}^i, \mathbf{R}^{-i})$ for any $i \in \mathbf{N}$, any $\mathbf{R} \in \mathcal{R}^N$, and any $\bar{R}^i \in \mathcal{R}$.

A feasible allocation is Pareto efficient if there is no other feasible allocation that would benefit someone without worsening anyone else. That is $\mathbf{x} \in X$ is Pareto efficient for the preference profile $\mathbf{R}$ if there exists no $\bar{\mathbf{x}} \in X$ such that $\bar{x}^i R^i x^i$ for any $i \in \mathbf{N}$ and $\bar{x}^j P_{R^j} x^j$ for some $j \in \mathbf{N}$. We say a social choice function is Pareto efficient when it always assigns Pareto efficient allocations.

Definition 2. A social choice function f is Pareto efficient if $f(\mathbf{R})$ is Pareto efficient for any $\mathbf{R} \in \mathcal{R}^N$.

We say a social choice function guarantee positive consumption if it always assigns non-zero consumptions to all agents. This is a weaker condition than the minimum consumption guarantee in Serizawa and Weymark (2003).

Definition 3. A social choice function f is positive consumption guarantee if $f^i(\mathbf{R}) \neq 0$ for any $i \in \mathbf{N}$ and any $\mathbf{R} \in \mathcal{R}^N$.

We say a social choice function is dictatorial if there exists an agent who is always allocated the total endowment.

Definition 4. A social choice function f is dictatorial if there exists $i \in \mathbf{N}$ such that $f^i(\mathbf{R}) = \Omega$ for any $\mathbf{R} \in \mathcal{R}^N$.

Following Satterthwaite and Sonnenschein (1981), we define SS mechanism, which includes a dictatorial social choice function as a special case, as follows.

Definition 5. A social choice function f is an SS mechanism if the following conditions are satisfied.

(i) There exists $i \in \mathbf{N}$ such that $f^i(\mathbf{R}) = 0$ for any $\mathbf{R} \in \mathcal{R}^N$.

(ii) For each $\mathbf{R} \in \mathcal{R}^N$, there exists some $j \in \mathbf{N}$ such that $f^j(\mathbf{R}) = \Omega$.

(iii) For any $j \neq i$, $f^j(R^i, \mathbf{R}^{-i}) = f^j(R^i, \bar{\mathbf{R}}^{-i})$ for any $\mathbf{R}^{-i}$ and $\bar{\mathbf{R}}^{-i}$ in $\mathcal{R}^{N-1}$, where i is the agent satisfying (i).

That is, a social choice function is an SS mechanism if the total endowment is allocated to some agent depending on agent i's preference who is always allocated zero consumption.

We say a social choice function is non-bossy, if any change of a agent's preference does not affect the allocation as long as his consumption is not changed.

Definition 6. A social choice function f is non-bossy if $f^i(R^i, \mathbf{R}^{-i}) = f^i(\bar{R}^i, \mathbf{R}^{-i})$ implies $f(R^i, \mathbf{R}^{-i}) = f(\bar{R}^i, \mathbf{R}^{-i})$ for any $i \in \mathbf{N}$, any $R^i, \bar{R}^i \in \mathcal{R}$ and any $\mathbf{R}^{-i} \in \mathcal{R}^{N-1}$.

The paper's main results follow.

Theorem 1. If a social choice function $f : \mathcal{R}^N \to X$ is Pareto efficient and strategy-proof, then it violates positive consumption guarantee.

Theorem 2. If a social choice function $f : \mathcal{R}^N \to X$ is Pareto efficient, strategy-proof, and non-bossy, then it is dictatorial.

Theorem 3. Suppose that $N = 3$. If a social choice function $f : \mathcal{R}^N \to X$ is Pareto efficient and strategy-proof, then it is an SS mechanism.

These results are proved on the preference domain $\mathcal{R}$. Let $\bar{\mathcal{R}}$ be a preference domain such that $\mathcal{R} \subset \bar{\mathcal{R}} \subset \mathcal{R}_{\mathcal{C}}$ and redefine Definitions 1-6 on $\bar{\mathcal{R}}$.

It is easy to observe that Theorems 1 and 2 respectively implies the same results on the larger domain $\bar{\mathcal{R}}^N$.[1] It would however need a proof that Pareto efficient and strategy-proof social choice function $f : \bar{\mathcal{R}}^N \to X$ should be an SS mechanism.

Corollary 1. Suppose $N = 3$ and $\mathcal{R} \subset \bar{\mathcal{R}} \subset \mathcal{R}_{\mathcal{C}}$. If a social choice function $f : \bar{\mathcal{R}}^N \to X$ is Pareto efficient and strategy-proof, then it is an SS mechanism.

[1] If $f : \bar{\mathcal{R}}^N \to X$ is Pareto efficient, strategy-proof and positive consumption guarantee, then it should be so on the restricted domain $\mathcal{R}^N$, which however contradicts to Theorem 1. If $f : \bar{\mathcal{R}}^N \to X$ is Pareto efficient, strategy-proof and non-bossy, then it should be so on the restricted domain $\mathcal{R}^N$; hence, by Theorem 2, f is dictatorial on $\mathcal{R}^N$ and there is a dictator i receiving the total endowment for any $\mathbf{R} \in \mathcal{R}^N$. Change each agent's preference to any $R^j \in \bar{\mathcal{R}}$ in turn. Under the strategy-proofness, this should not change the agent's own consumption because the agent receives the total endowment or zero consumption; hence this does not change the allocation because of the non-bossiness. Thus the agent i should be a dictator of $f : \bar{\mathcal{R}}^N \to X$.

3 Preliminary result I

In the following two sections we show two preliminary results which would be useful to investigate the allocations assigned by Pareto efficient and strategy-proof social choice functions. The first in this section is a slight generalization of a result proved by Hashimoto (2008) and generalized by Momi (2011). They proved that in a two-agent economy where the preferences are represented by Cobb-Douglas utility functions, if a social choice function is Pareto efficient and strategy-proof, then any change of a agent's preference should not affect the other's utility level.

Next proposition insists that if an agent's preference is changed while the other agents have the same preference $\tilde{R}$, then the sum of the new consumptions allocated to the others should be indifferent to the sum of the old consumptions with respect to the preference $\tilde{R}$.

Proposition 1. Suppose that $f : \mathcal{R}^N \to X$ is a Pareto efficient and strategy-proof social choice function. For any $i \in \mathbf{N}$, any $R^i, \bar{R}^i \in \mathcal{R}$ and any $\tilde{\mathbf{R}}^{-i} = (\tilde{R}, \dots, \tilde{R}) \in \mathcal{R}^{N-1}$, $(\sum_{j \neq i} f^j(R^i, \tilde{\mathbf{R}}^{-i})) I_{\tilde{R}} (\sum_{j \neq i} f^j(\bar{R}^i, \tilde{\mathbf{R}}^{-i}))$.

The proof is essentially same as that in Momi (2011). For simple exposition we let $\tilde{U} : R^L_+ \to R$ be a differentiable utility function representing the preference $\tilde{R}$. Following Kannai (1970), we introduce a topology into $\mathcal{R}$ and let $t \to R_t$ be a continuous map mapping a parameter t on any interval T to a preference $R_t \in \mathcal{R}$. For such a map, we let $f(t) = f(R_t, \tilde{\mathbf{R}}^{-i})$ denote the allocation given by f when agent i's preference is R_t and the others' are $\tilde{R}$. All we have to prove is that $\tilde{U}(\sum_{j \neq i} f^j(t')) = \tilde{U}(\sum_{j \neq i} f^j(t''))$ or equivalently that $\tilde{U}(\Omega - f^i(t')) = \tilde{U}(\Omega - f^i(t''))$. Note that $\mathcal{R}$ is connected, hence that for any $R^i \in \mathcal{R}$ and $\bar{R}^i \in \mathcal{R}$, we can pick a continuous mapping $t \mapsto R_t$ such that $R_{t'} = R^i$ and $R_{t''} = \bar{R}^i$ at some t' and t''.

Lemma 1. Let $t \mapsto R_t \in \mathcal{R}$ be a continuous map and let $\tilde{\mathbf{R}}^{-i} = (\tilde{R}, \dots, \tilde{R}) \in \mathcal{R}^{N-1}$. If f is a Pareto efficient and strategy-proof social choice function, then $f^i(t) = f^i(R_t, \tilde{\mathbf{R}}^{-i})$ is a continuous function of t.

Proof. As $t \to \bar{t}$, $f(t)$ converges because X is compact. We let $f(t) \to \bar{\mathbf{x}} = (\bar{x}^1, \dots, \bar{x}^N)$. All we have to show is that $\bar{x}^i = f^i(\bar{t})$.

We let $U^i(\cdot; t) : R^L_+ \to R$ be a differentiable utility function representing the preference R_t of agent i. Since f is strategy-proof, $U^i(f^i(t); t) \geq U^i(f^i(\bar{t}); t)$ holds for any t. Especially at the limit of $t \to \bar{t}$, $U^i(\bar{x}^i; \bar{t}) \geq U^i(f^i(\bar{t}); \bar{t})$ holds. If this equation holds with strict inequality, then the agent would manipulate f by reporting $R_{\tilde{t}}$ where $\tilde{t}$ is sufficiently close to $\bar{t}$ when his true preference is $R_{\bar{t}}$ because $f^i(\tilde{t})$ is close to $\bar{x}^i$, and hence $U^i(f^i(\tilde{t}); \bar{t})$ is close to $U^i(\bar{x}^i; \bar{t})$. This violate the strategy-proofness of f. Therefore the equation holds with equality: $U^i(\bar{x}^i; \bar{t}) = U^i(f^i(\bar{t}); \bar{t})$.

We next show that $\bar{\mathbf{x}}$ should be a Pareto efficient allocation in the economy where one agent's preference is $R_{\bar{t}}$ and others' are $\tilde{R}$. Suppose that $\bar{\mathbf{x}}$ is not Pareto optimal. Then in the economy with preferences $R_{\bar{t}}$ and $\tilde{R}$, which are both strictly convex, there exists $\hat{\mathbf{x}} = (\hat{x}^1, \ldots, \hat{x}^N) \in X$ such that $U^i(\hat{x}^i; \bar{t}) > U^i(\bar{x}^i; \bar{t})$ and $\tilde{U}(\hat{x}^j) > \tilde{U}(\bar{x}^j)$ for any $j \neq i$. When $\tilde{t}$ is sufficiently close to $\bar{t}$, $f(\tilde{t})$ is sufficiently close to $\bar{\mathbf{x}}$ and $R_{\tilde{t}}$ is sufficiently close to $R_{\bar{t}}$. Therefore $U^i(\hat{x}^i; \tilde{t}) > U^i(f^i(\tilde{t}); \tilde{t})$ and $\tilde{U}(\hat{x}^j) > \tilde{U}(f^j(\tilde{t}))$ hold. This violates the Pareto efficiency of f. Thus $\bar{\mathbf{x}}$ is a Pareto efficient allocation.

It is easy to observe that in the Edgeworth Box with consumer i's preferences R_t and the others' $\tilde{R} \in \mathcal{R}$, the set of Pareto efficient allocations intersects consumer i's one indifference surface only once. Therefore if $U^i(\bar{x}^i; \bar{t}) = U^i(f^i(\bar{t}); \bar{t})$, and $\bar{\mathbf{x}}$ and $f(\bar{t})$ are both Pareto efficient allocations, then $\bar{x}^i = f^i(\bar{t})$ holds. ■

Lemma 2. If f is a Pareto efficient and strategy-proof social choice function, then $\tilde{U}(\Omega - f^i(t')) = \tilde{U}(\Omega - f^i(t''))$ for any t' and t''.

Proof. We suppose that there exists t' and t'' such that $\tilde{U}(\Omega - f^i(t')) \neq \tilde{U}(\Omega - f^i(t''))$ and show a contradiction. Without loss of generality we assume $t' < t''$.

We first consider the case where $\tilde{U}(\Omega - f^i(t')) > \tilde{U}(\Omega - f^i(t''))$. Note that $\tilde{U}(\Omega - f^i(t))$ is a continuous function of t by Lemma 1 proved above. Then there exist $\bar{t} \in (t', t'')$ and a sequence $\{\epsilon_n\}$ which converges to 0 from the right hand side, $\epsilon_n > 0$ and $\epsilon_n \to 0$ as $n \to \infty$, such that

$$\lim_{n\to\infty} \frac{\tilde{U}(\Omega - f^i(\bar{t} + \epsilon_n)) - \tilde{U}(\Omega - f^i(\bar{t}))}{\epsilon_n} < 0.$$ [2]

Since the utility function $\tilde{U}$ is differentiable, the equation becomes

$$\sum_{l=1}^{L} \frac{\partial \tilde{U}(\Omega - f^i(\bar{t}))}{\partial x_l} \lim_{n\to\infty} \frac{-f_l^i(\bar{t} + \epsilon_n) + f_l^i(\bar{t})}{\epsilon_n} < 0.$$

Since f is Pareto efficient and $\tilde{U}$ represents the homothetic preference $\tilde{R}$ of all agents but i, $(\frac{\partial \tilde{U}(\Omega - f^i(t))}{\partial x_1}, \ldots, \frac{\partial \tilde{U}(\Omega - f^i(t))}{\partial x_L})$ is parallel to $(\frac{\partial U^i(f^i(t);t)}{\partial x_1}, \ldots, \frac{\partial U^i(f^i(t);t)}{\partial x_L})$. Therefore we have

$$\sum_{l=1}^{L} \frac{\partial U^i(f^i(\bar{t}); \bar{t})}{\partial x_l} \lim_{n\to\infty} \frac{f_l^i(\bar{t} + \epsilon_n) - f_l^i(\bar{t})}{\epsilon_n} > 0,$$

hence,

$$\lim_{n\to\infty} \frac{U^i(f^i(\bar{t} + \epsilon_n); \bar{t}) - U^i(f^i(\bar{t}); \bar{t})}{\epsilon_n} > 0,$$

[2]To the contrary, suppose that $\lim_{n\to\infty} \frac{\tilde{U}(\Omega - f^i(\bar{t}+\epsilon_n)) - \tilde{U}(\Omega - f^i(\bar{t}))}{\epsilon_n} \geq 0$ for any $\bar{t} \in (t', t'')$ and any sequence $\{\epsilon_n\}$ converging to 0 from right hand side. It clearly contradicts to that $\tilde{U}(\Omega - f^i(\cdot))$ is a continuous function and $\tilde{U}(\Omega - f^i(t')) > \tilde{U}(\Omega - f^i(t''))$.

This implies $U^i(f^i(\bar{t}+\epsilon_n);\bar{t}) > U^i(f^i(\bar{t});\bar{t})$ with sufficiently large n because $\epsilon_n > 0$. This violates the strategy-proofness of f because agent i whould manipulate f by reporting $R_{\bar{t}+\epsilon_n}$ when his preference is $R_{\bar{t}}$.

Next, we consider the case where $\tilde{U}(\Omega - f^i(t')) < \tilde{U}(\Omega - f^i(t''))$. Then there exist $\bar{t} \in (t', t'')$ and a sequence $\{\epsilon_n\}$ which converges to 0 from the left hand side, $\epsilon_n < 0$ and $\epsilon_n \to 0$ as $n \to \infty$, such that

$$\lim_{n\to\infty} \frac{\tilde{U}(\Omega - f^i(\bar{t}+\epsilon_n)) - \tilde{U}(\Omega - f^i(\bar{t}))}{\epsilon_n} > 0.$$

By the same discussion, we have

$$\lim_{n\to\infty} \frac{U^i(f^i(\bar{t}+\epsilon_n);\bar{t}) - U^i(f^i(\bar{t});\bar{t})}{\epsilon_n} < 0.$$

This implies $U^i(f^i(\bar{t}+\epsilon_n);\bar{t}) > U^i(f^i(\bar{t});\bar{t})$ with sufficiently large n because $\epsilon_n < 0$. This again violates the strategy-proofness of f. ■

4 Preliminary result II

In this section we show an application of Maskin monotonic transformation. Consider a preference $R \in \mathcal{R}$ and a consumption bundle $x \in R^L_+$. A preference $\bar{R}$ is called Maskin monotonic transformation of R at x if $\bar{x} \in UC(\bar{R}, x)$ and $\bar{x} \neq x$ implies $\bar{x} P_R x$. If an agent receives the commodity bundle x at a profile $\mathbf{R}$, strategy-proofness implies that this agent receives the same commodity bundle when his preference is subject to a Maskin monotonic transformation at x.

Lemma 3. Suppose that $f : \mathcal{R}^N \to X$ is a strategy-proof social choice function. For any $\mathbf{R} \in \mathcal{R}^N$ and any $i \in \mathbf{N}$, if $\bar{R}^i \in \mathcal{R}$ is a Maskin monotonic transformation of R^i at $f^i(R)$, then $f^i(\bar{R}^i, \mathbf{R}^{-i}) = f^i(\mathbf{R})$.

In addition to the preference R and the consumption bundle x, consider another preference $\tilde{R}$ and another consumption bundle $\tilde{x}$. If these are as in Figure 1, it would be possible to image a preference which is a Maskin monotonic transformation of R at x and also a Maskin monotonic transformation of $\tilde{R}$ at $\tilde{x}$. Next proposition shows when such a transformation exists. For $x \in R^L_+ \setminus 0$, we let $[x]$ denote the ray in the consumption set R^L_+ starting from the origin and passing through x. Keep in mind two preliminary facts about homothetic preferences.

Lemma 4. For $R, \tilde{R} \in \mathcal{R}$, if $UC(x;R) = UC(x;\tilde{R})$ at a consumption bundle $x \in R^L_{++}$, then R and $\tilde{R}$ are the same preference.

Lemma 5. If $\tilde{R} \in \mathcal{R}$ is a Maskin monotonic transformation of $R \in \mathcal{R}$ at $x \in R^L_{++}$, then $\tilde{R}$ is a Maskin monotonic transformation of R at any non-zero consumption $x' \in [x]$.

Proposition 2. For any $R, \tilde{R} \in \mathcal{R}$ and any $x, \tilde{x} \in R^L_{++}$, if $x \in P(I(x;R) \bigcap [\tilde{x}]; \tilde{R})$, then there exists a preference $\bar{R} \in \mathcal{R}$ that is a Maskin monotonic transformation of R at x and of $\tilde{R}$ at $\tilde{x}$.

Proof. Figure 2 (i) describes an example of R, $\tilde{R}$, x and $\tilde{x}$ satisfying the condition in the proposition. We first consider a special case where $\tilde{x} \in P(x;R)$ and $x \in P(\tilde{x};\tilde{R})$, and observe that there exists a preference $\bar{R}$ which is a Maskin monotonic transformation of R at x and of $\tilde{R}$ at $\tilde{x}$. Figure 2 (ii) draws this situation. It is easy to image a desired Maskin monotonic transformation. A rigorous discussion follows.

Suppose $\tilde{x} \in P(x;R)$ and $x \in P(\tilde{x};\tilde{R})$. We pick a preference $R' \in \mathcal{R}$ that is a Maskin monotonic transformation of R at x and $\tilde{x} \in P(x;R')$. We also pick a preference $\tilde{R}' \in \mathcal{R}$ that is a Maskin monotonic transformation of $\tilde{R}$ at $\tilde{x}$ and $x \in P(\tilde{x};\tilde{R}')$. We then construct a strictly convex closed set $Y \subset R^L_+$ such that (i) $Y \subset UC(x;R') \bigcap UC(\tilde{x};\tilde{R}')$; (ii) for any $x \in Y$, $x + R^L_+ \subset Y$; (iii) the boundary of Y, ∂Y, is smooth; (iv) $x \in \partial Y$, and $UC(x;R')$ and Y have the same hyperplane at x; and (v) $\tilde{x} \in \partial Y$, and $UC(\tilde{x};\tilde{R}')$ and Y have the same hyperplane at $\tilde{x}$. To obtain such a set Y, for example, let $B_\epsilon(y)$ be a closed ball with center y and radius ϵ. Fix sufficiently small ϵ and let Y be a sum of $B_\epsilon(y)$ over y's such that $B_\epsilon(y) \subset UC(x;R') \bigcap UC(\tilde{x};\tilde{R}')$. That is, $Y = \{x \in R^L_+ | x \in B_\epsilon(y)$ for some y such that $B_\epsilon(y) \subset UC(x;R') \bigcap UC(\tilde{x};\tilde{R}')\}$. If the ϵ is sufficiently small, Y is a desirable set satisfying (i)–(v). We let $\bar{R} \in \mathcal{R}$ be the preference such that Y is the upper contour set of $\bar{R}$ at x (and $\tilde{x}$). As in Lemma 4, $\bar{R}$ is determined uniquely. It is clear from the construction that $\bar{R}$ is a Maskin monotonic transformation of R at x and of $\tilde{R}$ at $\tilde{x}$.

We next consider a general case. Suppose $R, \tilde{R} \in \mathcal{R}$ and $x, \tilde{x} \in R^L_{++}$ satisfy the condition in the proposition. We pick $\hat{x} \in [\tilde{x}]$ such that $\hat{x} \in P(x;R)$ and $x \in P(\hat{x};\tilde{R})$. For example, pick $\hat{x} \in [\tilde{x}]$ which is preferred to x with respect to R and sufficiently close to $I(x;R) \bigcap [\tilde{x}]$. This $\hat{x}$ clearly satisfies $\hat{x} \in P(x;R)$ and $x \in P(\hat{x};\tilde{R})$. From the above discussion there exists a preference $\bar{R} \in \mathcal{R}$ that is a Maskin monotonic transformation of R at x and of $\tilde{R}$ at $\hat{x}$. Then, as in Lemma 5, $\bar{R}$ is also a Maskin monotonic transformation of $\tilde{R}$ at $\tilde{x}$ because $\tilde{x}$ and $\hat{x}$ are on the same ray. ■

5 Proof of Theorem 1

Suppose that f is a Pareto efficient and strategy-proof social choice function which guarantees positive consumptions. When all agents have the same preference, all agents should be allocated positive portions of Ω: $f^i(\mathbf{R}) = \lambda^i \Omega$ with some $0 < \lambda^i < 1$ for $\mathbf{R} = (R, \dots, R)$. Pick two different preferences R and $\tilde{R}$, and consider the allocations given by f at $\mathbf{R} = (R, \dots, R)$ and $\tilde{\mathbf{R}} = (\tilde{R}, \dots, \tilde{R})$.

We let $A(x;R)$ denote the set of consumption bundle x' such that $\Omega - x'$ is indifferent to $\Omega - x$ with respect to R

$$A(x;R) = \{x' \in R_+^L | (\Omega - x') I_R (\Omega - x)\}$$

and let $A^+(x;R) = \{x' \in R_+^L | (\Omega - x) P_R (\Omega - x')\}$, which is the upper right part of the consumption set partitioned by $A(x;R)$ and let $A^-(x;R) = \{x' \in R_+^L | (\Omega - x') P_R (\Omega - x)\}$, which is the lower left part.

Without loss of generality we assume $f^1(\mathbf{R}) \geq f^1(\tilde{\mathbf{R}})$. See the Edgeworth Box described in Figure 3, where the consumption of agent 1 is measured from the lower left vertex and sum of consumptions of the other agents is measured from the upper right vertex. We pick $\bar{x}^1 \in A(f^1(\tilde{\mathbf{R}}); \tilde{R})$ in the neighborhood of $f^1(\tilde{\mathbf{R}})$ so that $\bar{x}^1$ is in $A(f^1(\mathbf{R}); R)^-$ and $\bar{x}^1$ is not parallel to Ω. Next, let x' be the intersection of $A(f^1(\mathbf{R}); R)$ and the segment $[\bar{x}^1, \Omega]$ and pick $\hat{x}^1 \in A(f^1(\mathbf{R}); R)$ in the neighborhood of x' so that $\hat{x}^1 \in A(x'; \tilde{R})^-$.

As we observed in Proposition 1, agent 1's consumption should be on $A(f^1(\mathbf{R}); R)$ (resp. $A(f^1(\tilde{\mathbf{R}}); \tilde{R})$) when other agents' preference is R (resp. $\tilde{R}$) and agent 1's preference is changed. Let $\bar{R}$ and $\hat{R}$ be agent 1's preferences such that $f^1(\bar{R}, \mathbf{R}^{-1}) = \bar{x}^1$ and $f^1(\hat{R}, \tilde{\mathbf{R}}^{-1}) = \hat{x}^1$.

We let $\check{R}$ be a preference which is a Maskin monotonic transformation of R at $\Omega - \hat{x}^1$ and of $\tilde{R}$ at $\Omega - \bar{x}^1$. Observe that our choice of $\bar{x}^1$ and $\hat{x}^1$ ensures the condition in Proposition 2: $\Omega - \bar{x}^1 \in P(I(\Omega - \hat{x}^1; R) \bigcap [\Omega - \bar{x}^1]; \tilde{R})$ and supports the existence of such a Maskin monotonic transformation.

We observe that the consumption allocated to agent 1 should not be changed when the preferences of agents other than agent 1 are changed to $\check{R}$ from the profile $(\bar{R}, \mathbf{R}^{-1})$ or from $(\hat{R}, \tilde{\mathbf{R}}^{-1})$

Since f is Pareto efficient and positive consumption guarantee, at the profile $(\bar{R}, \mathbf{R}^{-1})$, agent 1 receives $\bar{x}^1$ and each of the other agents $i = 2, \ldots, N$, receives a positive portion of $\Omega - \bar{x}^1$: $\bar{\lambda}^i(\Omega - \bar{x}^1)$, $i = 2, \ldots, N$, where $0 < \bar{\lambda}^i < 1$ and $\sum_{i=2}^N \bar{\lambda}^i = 1$. Note that since we have chosen $\bar{x}^1$ not parallel to Ω, the vectors $\bar{x}^1$ and $\Omega - \bar{x}^1$ are independent. Now, let us change agent 2's preference to $\check{R}$ from R. Write the new profile as $(\bar{R}, \check{R}, \mathbf{R}_{-2})$ where agent 1's preference is $\bar{R}$, agent 2's $\check{R}$ and the others' R.

Since $\check{R}$ is a Maskin monotonic transformation of R at $\Omega - \bar{x}^1$, it is so at agent 2's consumption as observed in Lemma 5. Therefore agent 2's consumption should not be changed and her gradient vector at the consumption should not be changed. Because of the Pareto efficiency, all agents' gradient vectors at their consumptions should be the same. Hence, all agents have the same gradient vector at the both profiles $(\bar{R}, \mathbf{R}^{-1})$ and $(\bar{R}, \check{R}, \mathbf{R}_{-2})$. Since the preferences are homothetic, the equality of the gradient vectors implies that each agent's consumptions at the both profiles should be parallel. That is, at the new profile, agent 1's consumption is parallel to $\bar{x}^1$ and the other agents' consumptions are parallel to $\Omega - \bar{x}^1$. Because of the Pareto efficiency, the consumptions at the new profile should sum up to the total endowment Ω. Then agent 1's consumption should be still $\bar{x}^1$.

Next we further change agent 3's preference to $\check{R}$ from R. Discussions are the same. Because $\tilde{R}$ is a Maskin monotonic transformation of R at agent 3's consumption, agent 3's consumption at the new profile $(\bar{R}, \check{R}, \check{R}, \mathbf{R}_{-3})$, where agent 1's preference is $\bar{R}$, agent 2's and 3's $\check{R}$ and the others' R is same as the consumption at the old profile $(\bar{R}, \check{R}, \mathbf{R}_{-2})$. Hence, the gradient vectors at the consumptions are also the same at the both profiles. Then, all agents have the same gradient vector at their consumptions, and hence their consumptions are parallel at the both profiles $(\bar{R}, \check{R}, \check{R}, \mathbf{R}_{-3})$ and $(\bar{R}, \check{R}, \mathbf{R}_{-2})$. That is, at the new profile, the consumption of agent 1 is again parallel to $\bar{x}^1$ and the others' are parallel to $\Omega - \bar{x}^1$. Then agent 1's consumption at the profile $(\bar{R}, \check{R}, \check{R}, \mathbf{R}_{-3})$ should be still $\bar{x}^1$.

By applying the discussions repeatedly until all preferences but agent 1's are changed to $\check{R}$, we finally obtain that $f^1(\bar{R}, \check{\mathbf{R}}^{-1}) = \bar{x}^1$ where $\check{\mathbf{R}}^{-1} = (\check{R}, \dots, \check{R}) \in \mathcal{R}^{N-1}$. Discussions are the same for the profile $(\hat{R}, \tilde{\mathbf{R}}^{-1})$ and we obtain $f^1(\hat{R}, \check{\mathbf{R}}^{-1}) = \hat{x}^1$.

Finally, remember our choice of $\hat{x}^1$ and $\bar{x}^1$. From the construction, x' is strictly preferred to $\bar{x}$ with respect to any preference and $\hat{x}^1$ can be chosen arbitrarily close to x'. Therefore $\hat{x}^1$ could have been chosen to be preferred to $\bar{x}^1$ with respect to agent 1's preference $\bar{R}$. This violates the strategy-proofness of f. This ends the proof of Theorem 1.

6 Proof of Theorem 2

We let f be a Pareto efficient, strategy-proof and non-bossy social choice function.

We first pick a preference R and prove that there exists some agent i who is allocated the total endowment when all agents have the same preference R: $f^i(\mathbf{R}) = \Omega$ where $\mathbf{R} = (R, \dots, R)$. To the contrary, suppose that there are at least two agents who receive non-zero consumptions at the profile $\mathbf{R} = (R, \dots, R)$. Without loss of generality we assume agent 1 is one of them: $f^1(\mathbf{R}) \neq \{0, \Omega\}$. When all agents have the same preference, the consumption of any agent should be parallel to Ω including the case of zero consumption because of Pareto efficiency of f. Therefore $f^i(\mathbf{R}) = \lambda^i \Omega$ where $0 \leq \lambda^i < 1$ for any $i \in \mathbf{N}$ and $0 < \lambda^1 < 1$ especially.

We let $\tilde{R}$ be a Maskin monotonic transformation of R at $\lambda^1 \Omega$. Note that $\tilde{R}$ is then a Maskin Monotonic transformation of R at any consumption parallel to Ω as observed in Lemma 5. Replace the preference R to $\tilde{R}$ for all agents but agent 1. Because of non-bossiness, this replacement should not affect the allocation: Whether agent j receives positive or zero consumption, the replacement of his preference to $\tilde{R}$ from R should not change his consumption, and hence the allocation should not be affected under the non-bossiness. Thus we have $f(\mathbf{R}) = f(R, \tilde{R}, \dots, \tilde{R})$.

The following discussion is almost the same as that of Theorem 1. In fact it is much easier by the non-bossiness. See the Edgeworth Box drawn in Figure 4, where the consumption of agent 1 is measured from the lower left vertex and the sum of consumptions of the other agents is measured from the upper right vertex. As we did in the proof of Theorem 1, we pick $\bar{x}^1 \in A(f^1(\mathbf{R}); \tilde{R})$ in the neighborhood of $f^1(\mathbf{R})$ so that $\bar{x}^1$ is in $A(f^1(\mathbf{R}); R)^-$ and $\bar{x}^1$ is not parallel to Ω. Next, let x' be the intersection of $A(f^1(\mathbf{R}); R)$ and the segment $[\bar{x}^1, \Omega]$ and pick $\hat{x}^1 \in A(f^1(\mathbf{R}); R)$ in the neighborhood of x' so that $\hat{x}^1 \in A(x'; \tilde{R})^-$.

As proved in Proposition 1, any change of agent 1's preference from the preference profile $(R, R, \ldots, R)$ (or $(R, \tilde{R}, \ldots, \tilde{R})$) does not affect the utility level of the sum of others' consumptions with respect to the preference R (or $\tilde{R}$). Let $\bar{R}$ and $\hat{R}$ be preferences of agent 1 such that $f^1(\bar{R}, \mathbf{R}^{-1}) = \bar{x}^1$ and $f^1(\hat{R}, \tilde{\mathbf{R}}^{-1}) = \hat{x}^1$.

We let $\check{R}$ be a preference which is a Maskin monotonic transformation of R at $\Omega - \hat{x}^1$ and of $\tilde{R}$ at $\Omega - \bar{x}^1$. Our choice of $\bar{x}^1$ and $\hat{x}^1$ ensures the condition in Proposition 2: $\Omega - \bar{x}^1 \in P(I(\Omega - \hat{x}^1; R) \bigcap [\Omega - \bar{x}^1]; \tilde{R})$ and supports the existence of such a Maskin monotonic transformation.

Because of the non-bossiness, the allocation should not be affected by replacing the preferences of agents other than agent 1 to $\check{R}$ from the profile $(\bar{R}, \mathbf{R}^{-1})$ or from $(\hat{R}, \tilde{\mathbf{R}}^{-1})$. Therefore we have $f^1(\bar{R}, \check{\mathbf{R}}^{-1}) = \bar{x}^1$ and $f^1(\hat{R}, \check{\mathbf{R}}^{-1}) = \hat{x}^1$ where $\check{\mathbf{R}}^{-1} = (\check{R}, \ldots, \check{R}) \in \mathcal{R}^{N-1}$.

From the construction, x' is strictly preferred to $\bar{x}$ with respect to any preference and $\hat{x}^1$ can be chosen arbitrarily close to x'. Therefore $\hat{x}^1$ could have been chosen to be preferred to $\bar{x}^1$ with respect to agent 1's preference $\bar{R}$. This violates the strategy-proofness of f. Thus there exists some agent $i \in \mathbf{N}$ who is allocated the total endowment at the profile $\mathbf{R} = (R, \ldots, R)$.

We have proved that $f^i(\mathbf{R}) = \Omega$ for some i and $f^j(\mathbf{R}) = 0$ for $j \neq i$ at the profile $\mathbf{R} = (R, \ldots, R)$. This agent i should be a dictator. Change any agent's preference arbitrarily. This change does not affect the agent's consumption because of the strategy-proofness of f, and hence this change does not affect the allocation because of the non-bossiness. Therefore we have $f^i(\mathbf{R}') = \Omega$ for any preference profile $\mathbf{R}' \in \mathcal{R}^N$. That is, f is dictatorial. This ends the proof of Theorem 2.

7 Proof of Theorem 3

We first prove a lemma.

Lemma 6. Let $N = 3$ and f be a Pareto efficient and strategy-proof social choice function. Let $\mathbf{R} = (R, R, R)$ and $\bar{\mathbf{R}} = (\bar{R}, \bar{R}, \bar{R})$ be preference profiles where all agents have the same preferences R and $\bar{R}$ respectively. If one agent is given the total endowment

Ω by f at $\mathbf{R}$ and another agent is given Ω at $\bar{\mathbf{R}}$, then there exists no preference $\tilde{R}$ such that the other agent is given Ω at $\tilde{\mathbf{R}} = (\tilde{R}, \tilde{R}, \tilde{R})$.

Proof. Without loss of generality we assume $f(\mathbf{R}) = (\Omega, 0, 0)$ and $f(\bar{\mathbf{R}}) = (0, \Omega, 0)$ and prove $f(\tilde{\mathbf{R}}) \neq (0, 0, \Omega)$. To the contrary, we suppose $f(\tilde{\mathbf{R}}) = (0, 0, \Omega)$.

$f(\mathbf{R}) = (\Omega, 0, 0)$ implies $f(\bar{R}, R, R) = (\Omega, 0, 0)$ because of the strategy-proofness of f. This, again by the strategy-proofness of f, implies $f^2(\bar{R}, \bar{R}, R) = 0$. On the other hand, $f(\bar{\mathbf{R}}) = (0, \Omega, 0)$ implies $f^3(\bar{R}, \bar{R}, R) = 0$. Therefore we obtain $f(\bar{R}, \bar{R}, R) = (\Omega, 0, 0)$, hence $f(\tilde{R}, \bar{R}, R) = (\Omega, 0, 0)$.

By the same discussions, $f(\tilde{\mathbf{R}}) = (0, 0, \Omega)$ implies $f(\tilde{R}, \tilde{R}, \bar{R}) = (0, 0, \Omega)$, which implies $f^2(\tilde{R}, \bar{R}, \bar{R}) = 0$. $f(\bar{\mathbf{R}}) = (0, \Omega, 0)$ implies $f^1(\tilde{R}, \bar{R}, \bar{R}) = 0$. Therefore $f(\tilde{R}, \bar{R}, \bar{R}) = (0, 0, \Omega)$, hence $f(\tilde{R}, \bar{R}, R) = (0, 0, \Omega)$. This is a contradiction. ■

We next prove a technical lemma. Consider a two-agent economy with agent i, the other agent j, and the total endowment Ω. Let $X_2 = \{(x^i, x^j) \in R_+^{2L} | x_i + x_j \leq \Omega\}$ denote the set of feasible allocation of the economy. Let $PO^i(R^i, R^j)$ denotes the set of Pareto efficient allocations $(x^i, x^j) = (x^i, \Omega - x^i) \in X_2$ with respect to agent i's preference R^i and agent j's R^j. For $(\bar{x}^i, \Omega - \bar{x}^i) \in PO^i(R^i, R^j)$, define

$$B^i(R^i, R^j, \bar{x}^i) = \left\{ x \in R_+^L \;\middle|\; \begin{array}{l} x \in I(\bar{x}^i, R^i), \\ (sx, \Omega - sx) \in PO^i(R^i, R^j) \text{ for some } s \geq 1 \end{array} \right\}.$$

See Figure 5. $B^i(R^i, R^j, \bar{x}^i)$ is the set of agent i's consumption x which is indifferent to $\bar{x}^i$ with respect to R^i and $(sx, \Omega - sx)$ with some $s \geq 1$ is a Pareto efficient allocation with respect to R^i and R^j.

Further we let $C \subset R_+^L$ be a closed set such that $C \bigcap B^i(R^i, R^j, \bar{x}^i) = \bar{x}^i$ and let $\mathcal{R}(R^i, R^j, \bar{x}^i, C)$ be a set of agent i's preference R such that the set of consumption which is preferred to $\bar{x}^i$ with respect to R and not strictly preferred to $\bar{x}^i$ with respect to R^i is included in the closed set C.

Lemma 7. Let $N = 3$ and f be a Pareto efficient and strategy-proof social choice function. Suppose that $f^1(R', \tilde{R}, \tilde{R}) \notin \{0, \Omega\}$, $f^2(R', \tilde{R}, \tilde{R}) \notin \{0, \Omega\}$ and $f^3(R', \tilde{R}, \tilde{R}) = 0$. Let $t \mapsto R_t$ be a continuous map such that $R_{\bar{t}} = \tilde{R}$ and $R_t \in \mathcal{R}(\tilde{R}, R', f^2(R', \tilde{R}, \tilde{R}), C)$ for $t \neq \bar{t}$ where $C \subset R_+^L$ is a closed set satisfying $C \bigcap B^2(\tilde{R}, R'; f^2(R', \tilde{R}, \tilde{R})) = f^2(R', \tilde{R}, \tilde{R})$. Then $f^2(R', R_t, \tilde{R}) \to f^2(R', \tilde{R}, \tilde{R})$ as $t \to \bar{t}$.

Note the difference between this lemma and Lemma 1. If $R' = \tilde{R}$, then because of Lemma 1, $f^2(R', R_t, \tilde{R})$ is a continuous function of t with any continuous map $t \mapsto R_t$. In this lemma, where R' and $\tilde{R}$ might be different, we claim the continuity of $f^2(R', R_t, \tilde{R})$ only at $t = \bar{t}$ satisfying $R_{\bar{t}} = \tilde{R}$ under the condition that $f^3(R', \tilde{R}, \tilde{R}) = 0$ and the technical choice of the map $t \to R_t \in \mathcal{R}(\tilde{R}, R', f^2(R', \tilde{R}, \tilde{R}), C)$.

Proof. Letting $f(R', R_t, \tilde{R}) \to \tilde{x} = (\tilde{x}^1, \tilde{x}^2, \tilde{x}^3)$ as $t \to \bar{t}$, all we have to show is $\tilde{x}^2 = f^2(R', \tilde{R}, \tilde{R})$. The discussions are the same as those in the proof of Lemma 1 to prove that $\tilde{x}^2$ is indifferent to $f^2(R', \tilde{R}, \tilde{R})$ with respect to the preference $\tilde{R}$: $\tilde{x}^2 \in I(f^2(R', \tilde{R}, \tilde{R}); \tilde{R})$, and that $\tilde{x}$ is a Pareto efficient allocation for the preference profile $(R', \tilde{R}, \tilde{R})$.

Note that when both agents 2 and 3 have the same preference $\tilde{R}$, their consumptions $\tilde{x}^2$ and $\tilde{x}^3$ should be parallel (including the case where $\tilde{x}^3 = 0$), and $(\tilde{x}^2 + \tilde{x}^3, \tilde{x}^1)$ should be a Pareto efficient allocation in the two-agent economy with agent 2 of preference $\tilde{R}$ and the other (agent 1) of preference R': $(\tilde{x}^2 + \tilde{x}^3, \tilde{x}^1) = (s\tilde{x}^2, \Omega - s\tilde{x}^2) \in PO^2(\tilde{R}, R')$ with some $s \geq 1$.

On the other hand, since $f^3(R', \tilde{R}, \tilde{R}) = 0$ by assumption, $(f^2(R', \tilde{R}, \tilde{R}), f^1(R', \tilde{R}, \tilde{R}))$ is also a Pareto efficient allocation in the two-agent economy: $(f^2(R', \tilde{R}, \tilde{R}), f^1(R', \tilde{R}, \tilde{R})) = (f^2(R', \tilde{R}, \tilde{R}), \Omega - f^2(R', \tilde{R}, \tilde{R})) \in PO^2(\tilde{R}, R')$.

From these discussions, we have $\tilde{x}^2 \in B^2(\tilde{R}, R', f^2(R', \tilde{R}, \tilde{R}))$. Now remember our choice of $t \mapsto R_t \in \mathcal{R}(\tilde{R}, R', f^2(R', \tilde{R}, \tilde{R}), C)$ with a closed set C which has no intersection with $B^2(\tilde{R}, R', f^2(R', \tilde{R}, \tilde{R}))$ but $f^2(R', \tilde{R}, \tilde{R})$. Because of the strategy-proofness of f, $f^2(R', R_t, \tilde{R})$ should be preferred to $f^2(R', \tilde{R}, \tilde{R})$ with respect to R_t and should not be strictly preferred to $f^2(R', \tilde{R}, \tilde{R})$ with respect to $\tilde{R}$. Therefore $f^2(R', \tilde{R}, \tilde{R})$ should be in the closed set C. Hence at the limit of $t \to \bar{t}$, $\tilde{x}^2$ should be still in C. Thus we obtain $\tilde{x}^2 = f^2(R', \tilde{R}, \tilde{R})$. ■

We divide the proof of Theorem 3 into two parts

Part 1. In this part, we prove that if f is a Pareto efficient and strategy-proof social choice function, then some agent should be allocated the total endowment Ω at each profile (R, R, R) where all agents have the same preference. This part consists of 5 steps.

(1) We pick a preference R such that at least two agents receive non-zero consumptions at the profile (R, R, R).[3] In this step, we show that there exists another preference $\tilde{R}$ such that at least two agents receive non-zero consumptions at the profile $(\tilde{R}, \tilde{R}, \tilde{R})$.

We suppose that there exists no such $\tilde{R}$ different from R that at least two agents receive non-zero consumptions at $\tilde{\mathbf{R}} = (\tilde{R}, \tilde{R}, \tilde{R})$. That is, we suppose that some agent receives the total endowment Ω at each $\tilde{\mathbf{R}} = (\tilde{R}, \tilde{R}, \tilde{R})$, $\tilde{R} \neq R$.

As observed in Lemma 6, all agents 1,2 and 3 cannot be the receivers of the total endowment when all agents have same preferences. Without loss of generality, we assume that agent 3 receives zero consumption at any profiles $\tilde{\mathbf{R}} = (\tilde{R}, \tilde{R}, \tilde{R})$, $\tilde{R} \neq R$; agent 1 or 2 receives the total endowment at each profiles $\tilde{\mathbf{R}} = (\tilde{R}, \tilde{R}, \tilde{R})$, $\tilde{R} \neq R$; and agent 1 receives the total endowment at at least one such profile. We consider the following two cases separately and show contradictions: (i) $f^3(R, R, R) = 0$ and (ii) $f^3(R, R, R) \notin \{0, \Omega\}$.

[3] If there exists no such preference, then some agent receives the total endowment at each profile where all agents have the same preference.

(i) We conider the case $f^3(R,R,R) = 0$. Let R' be a preference such that $f(R',R',R') = (\Omega,0,0)$. Then $f(R'',R',R') = (\Omega,0,0)$ for any R'', and hence $f^2(R'',R'',R') = 0$ because of the strategy-proofness of f. On the other hand, if $R'' \neq R$, then $f^3(R'',R'',R'') = 0$ by assumption, and hence $f^3(R'',R'',R') = 0$ because of the strategy-proofness of f. Therefore, if $R'' \neq R$, then $f(R'',R'',R') = (\Omega,0,0)$. Then, $f(\bar{R},R'',R') = (\Omega,0,0)$ for any $\bar{R}$ and any $R'' \neq R$, and hence $f^3(\bar{R},R'',R) = 0$ because of the strategy-proofness of f. Remember that at the profile (R,R,R) agent 3 receives zero consumption, and agents 1 and 2 receive non-zero consumptions. This implies that $f(\cdot,\cdot,R)$ defined on the preference profile set $\mathcal{R} \times \mathcal{R}$ is a Pareto efficient, strategy-proof, and non-dictatorial social choice function in the two-agent economy with the agents 1 and 2. This is a contradiction.

(ii) We consider the case $f^3(R,R,R) \notin \{0,\Omega\}$. As in the above case, we let R' be a preference such that $f(R',R',R') = (\Omega,0,0)$ and obtain $f(\bar{R},R'',R') = (\Omega,0,0)$ for any $\bar{R}$ and any $R'' \neq R$. Note that this implies $f^2(\bar{R},R,R') = 0$ for any $\bar{R}$ because of the strategy-proofness of f, and hence $f^2(R,R,R') = 0$ especially. $f^3(R,R,R) \notin \{0,\Omega\}$ implies $f^3(R,R,R') \notin \{0,\Omega\}$, and this implies $f^1(R,R,R') \notin \{0,\Omega\}$ because consumer 2 receives zero consumption at the profile, and hence $f^1(\bar{R},R,R') \notin \{0,\Omega\}$ for any $\bar{R}$. Thus at the profile $(\bar{R},R,R')$ for any $\bar{R}$, agent 2 receives zero-consumption and agents 1 and 3 receive non-zero consumptions. Hence, $f^3(\bar{R},R,\hat{R}) \neq 0$ for any $\bar{R}$ and any $\hat{R}$.

Now suppose there exists $\hat{R}$ different from R' such that $f(\hat{R},\hat{R},\hat{R}) = (0,\Omega,0)$. Then by the symmetric discussion, we have $f(R'',\bar{R},\hat{R}) = (0,\Omega,0)$ for any $\bar{R}$ and any $R'' \neq R$. Especially, $f(R'',R,\hat{R}) = (0,\Omega,0)$ for any $R'' \neq R$. This contradicts to the conclusion in the above paragraph.

From the discusssions in the above paragraphs, we have $f(R',R',R') = (\Omega,0,0)$ for any $R' \neq R$ and $f^2(\bar{R},R,R') = 0$, $f^1(\bar{R},R,R') \notin \{0,\Omega\}$ and $f^3(\bar{R},R,R') \notin \{0,\Omega\}$ for any $\bar{R}$. This implies that $f(\cdot,R,\cdot)$ defined on the profile set $\mathcal{R} \times (\mathcal{R} \setminus R)$ is a Pareto efficient, strategy-proof, and positive consumption guarantee social choice function in the two-agent economy with the agents 1 and 3. This is a contradiction.[4]

(2) Since there are three agents, there exists at least one agent who receives non-zero consumption at the both profile $\mathbf{R} = (R,R,R)$ and $\tilde{\mathbf{R}} = (\tilde{R},\tilde{R},\tilde{R})$. Without loss of generality we let agent 1 be the consumer: $f^1(\mathbf{R}) \notin \{0,\Omega\}$ and $f^1(\tilde{\mathbf{R}}) \notin \{0,\Omega\}$. Without loss of generality we assume $f^1(\mathbf{R}) \geq f^1(\tilde{\mathbf{R}})$.

We change the preference of agent 1 as we did in the proof of Theorem 1. Remember Figure 3. We pick $\bar{x}^1 \in A(f^1(\tilde{\mathbf{R}});\tilde{R})$ in the neighborhood of $f^1(\tilde{\mathbf{R}})$ so that $\bar{x}^1$ is in $A(f^1(\mathbf{R});R)^-$ and $\bar{x}^1$ is not parallel to Ω. Next, let x' be the intersection of $A(f^1(\mathbf{R});R)$ and the segment $[\bar{x}^1,\Omega]$ and pick $\hat{x}^1 \in A(f^1(\mathbf{R});R)$ in a neighborhood of x' so that

[4] This especially implies that $f(\cdot,R,\cdot)$ is a Pareto efficient, strategy-proof, and non-dictatorial social choice function on a local domain of Cobb-Douglas preferences, which contradicts to the result proved by Hashimoto (2008) or Momi (2011).

$\hat{x}^1 \in A(x'; \tilde{R})^-$. As observed in Proposition 1, there exist consumer 1's preferences $\bar{R}$ and $\hat{R}$ such that $f^1(\bar{R}, \mathbf{R}^{-1}) = f^1(\bar{R}, R, R) = \bar{x}^1$ and $f^1(\hat{R}, \tilde{\mathbf{R}}^{-1}) = f^1(\hat{R}, \tilde{R}, \tilde{R}) = \hat{x}^1$.

(3) We focus on the profile $(\bar{R}, R, R)$ achieved in the previous step. Since $f^1(\bar{R}, R, R) = \bar{x}^1 \notin \{0, \Omega\}$, agent 2 or agent 3 receives non-zero consumption at the profile. In this step, we observe that even if one of them receives zero consumption at the profile, there exists a new profile in the neighborhood of $(\bar{R}, R, R)$ such that all agents receive non-zero consumptions. We also observe that agent 1's new consumption is then close to $\bar{x}^1$ and not parallel to the others' new consumptions

(i) We first consider new profiles (R', R, R) where agent 1's preference R' is a Maskin monotonic transformation of $\bar{R}$ at $\bar{x}^1$ and is close to $\bar{R}$. If agents 2 and 3 receive non-zero consumptions at such a profile, we achieve the desired result.

In this situation, since R' is a Maskin monotonic transformation of $\bar{R}$ at $\bar{x}^1$, $f^1(R', R, R) = \bar{x}^1$, and that $f^2(R', R, R)$ and $f^3(R', R, R)$ are both parallel to $\Omega - \bar{x}^1$ which is not parallel to $\bar{x}^1$.

(ii) We suppose that there exists no such profile (R', R, R) with the Maskin monotonic transformation R' in the neighborhood of $\bar{R}$. Without loss of generality, we let R' and R'' be preferences such that (a) R' and R'' are Maskin monotonic transformations of $\bar{R}$ at $\bar{x}^1$ and are close to $\bar{R}$, (b) R'' is a Maskin monotonic transformation of R' at $\bar{x}$, and (c) $f^3(R', R, R) = 0$ and $f^3(R'', R, R) = 0$. Note that then $f^1(R', R, R) = f^1(R'', R, R) = \bar{x}^1$ and $f^2(R', R, R) = f^2(R'', R, R) = \Omega - \bar{x}^1$.

Remembering Lemma 7, we let $t \mapsto R_t$ be a continuous map such that $R_{\bar{t}} = R$, $R_t \in \mathcal{R}(R, R', f^2(R', R, R), C)$ for $t \neq \bar{t}$ where C is a closed set satisfying $C \bigcap B^2(R, R', f^2(R', R, R)) = f^2(R', R, R)$, and the gradient vector of R_t, $t \neq \bar{t}$ at $f^2(R', R, R)$ is different from that of R. We consider the profiles (R', R_t, R) and (R'', R_t, R). Suppose there exists no t in the neighborhood of $\bar{t}$ such that $f^3(R', R_t, R) \neq 0$ or $f^3(R'', R_t, R) \neq 0$. Then, as shown in Proposition 1, $f^1(R', R_t, R)$ (resp. $f^1(R'', R_t, R)$) is indifferent to $\bar{x}^1$ with respect to the preference R' (resp. R'') for t in the neighborhood of $\bar{t}$ because the total endowment Ω should be allocated over the two agents 1 and 2. This, however, implies that $f^1(R'', R_t, R)$ is strictly preferred to $f^1(R', R_t, R)$ with respect to the preference R' because R'' is a Maskin monotonic transformation of R' at $\bar{x}^1$. This contradicts to the strategy-proofness of f. Therefore there should exists $\hat{t}$ in any neighborhood of $\bar{t}$ such that $f^3(R', R_{\hat{t}}, R) \neq 0$ or $f^3(R'', R_{\hat{t}}, R) \neq 0$.

We focus on the case $f^3(R', R_{\hat{t}}, R) \neq 0$ for $\hat{t}$ sufficiently close to $\bar{t}$ and show that $f^1(R', R_{\hat{t}}, R)$ is close to $\bar{x}^1$ and parallel to neither $f^2(R', R_{\hat{t}}, R)$ nor $f^3(R', R_{\hat{t}}, R)$. Because of Lemma 7, $f^2(R', R_{\hat{t}}, R)$ is close to $f^2(R', R, R) = \Omega - \bar{x}^1$ and the gradient vectors at the consumptions are also close to each other. The closeness of the gradient vectors implies that $f^1(R', R_{\hat{t}}, R)$ is on a ray close to $[\bar{x}^1]$ because of the homoceticity of R'. Further, the closeness between $R_{\hat{t}}$ and R implies that $f^3(R', R_{\hat{t}}, R)$ is on a ray close to $[f^2(R', R_{\hat{t}}, R)]$ hence is on a ray close to $[\Omega - \bar{x}^1]$. These implies that $f^1(R', R_{\hat{t}}, R)$ is close to $\bar{x}^1$ because $f^1(R', R_{\hat{t}}, R) + f^2(R', R_{\hat{t}}, R) + f^3(R', R_{\hat{t}}, R) = \Omega$, where $f^1(R', R_{\hat{t}}, R)$ is on the ray close to $[\bar{x}^1]$, $f^2(R', R_{\hat{t}}, R)$ and $f^3(R', R_{\hat{t}}, R)$ are on the rays close to $[\Omega - \bar{x}^1]$.

The conclusion of this step follows. There exist preferences $\bar{R}^1$, R^2 and R^3 respectively in the neighborhoods of $\bar{R}$, R, and R such that all agents are allocated non-zero consumptions at the profile $(\bar{R}^1, R^2, R^3)$ and $f^1(\bar{R}^1, R^2, R^3)$ is close to $f^1(\bar{R}, R, R) = \bar{x}^1$ and is parallel to neither $f^2(\bar{R}^1, R^2, R^3)$ nor $f^3(\bar{R}^1, R^2, R^3)$.

(4) The discussion is the same for the profile $(\hat{R}, \tilde{R}, \tilde{R})$ achieved in the second step. There exist preferences $\hat{R}^1$, $\tilde{R}^2$ and $\tilde{R}^3$ respectively in the neighborhoods of $\hat{R}$, $\tilde{R}$, and $\tilde{R}$ such that all consumers are allocated non-zero consumptions at the profile $(\hat{R}^1, \tilde{R}^2, \tilde{R}^3)$ and $f^1(\hat{R}^1, \tilde{R}^2, \tilde{R}^3)$ is close to $f^1(\hat{R}, \tilde{R}, \tilde{R}) = \hat{x}^1$ and is parallel to neither $f^2(\hat{R}^1, \tilde{R}^2, \tilde{R}^3)$ nor $f^3(\hat{R}^1, \tilde{R}^2, \tilde{R}^3)$.

(5) Let $\check{R}^2$ be a Maskin monotonic transformation of R^2 at $f^2(\bar{R}^1, R^2, R^3)$ and of $\tilde{R}^2$ at $f^2(\hat{R}^1, \tilde{R}^2, \tilde{R}^3)$. Note that there exists a Maskin monotonic transformation of R at $\Omega - \bar{x}^1$ and of $\tilde{R}$ at $\Omega - \hat{x}^1$ as we observed in the proof of Theorem 1. Then there exists a desired transformation $\check{R}^2$ for the preferences R^2 and $\tilde{R}^2$ close to $\tilde{R}$ and R respectively and the consumptions $f^2(\bar{R}^1, R^2, R^3)$ and $f^2(\hat{R}^1, \tilde{R}^2, \tilde{R}^3)$ on the rays close to $[\Omega - \bar{x}^1]$ and $[\Omega - \hat{x}^1]$ respectively.

By the discussion similar to that in the proof of Theorem 1, this transformation does not change consumptions of any agents. At the profile $(\bar{R}^1, R^2, R^3)$ for example, the transformation of agent 2's preference R^2 to $\check{R}^2$ does not change her consumption and her gradient vector at the consumption. Therefore $f^1(\bar{R}^1, \check{R}^2, R^3)$ and $f^3(\bar{R}^1, \check{R}^2, R^3)$ are parallel to $f^1(\bar{R}^1, R^2, R^3)$ and $f^3(\bar{R}^1, R^2, R^3)$ respectively and $f^1(\bar{R}^1, \check{R}^2, R^3) + f^3(\bar{R}^1, \check{R}^2, R^3) = f^1(\bar{R}^1, R^2, R^3) + f^3(\bar{R}^1, R^2, R^3)$ holds. Since $f^1(\bar{R}^1, R^2, R^3)$ and $f^3(\bar{R}^1, R^2, R^3)$ are independent vectors, this is satisfied only when $f^1(\bar{R}^1, \check{R}^2, R^3) = f^1(\bar{R}^1, R^2, R^3)$ and $f^3(\bar{R}^1, \check{R}^2, R^3) = f^3(\bar{R}^1, R^2, R^3)$.

Next, let $\check{R}^3$ be a Maskin monotonic transformation of R^3 at $f^3(\bar{R}^1, \check{R}^2, R^3)$ and of $\tilde{R}^3$ at $f^3(\hat{R}^1, \check{R}^2, \tilde{R}^3)$. The existence of this transformation is also supported by the fact that R^3 and $\tilde{R}^3$ are close to R and $\tilde{R}$ respectively and the consumptions $f^3(\bar{R}^1, \check{R}^2, R^3) = f^3(\bar{R}^1, R^2, R^3)$ and $f^3(\hat{R}^1, \check{R}^2, \tilde{R}^3) = f^3(\hat{R}^1, \tilde{R}^2, \tilde{R}^3)$ are respectively on the rays close to $[\Omega - \bar{x}^1]$ and $[\Omega - \hat{x}^1]$. Again this transformation does not change consumptions of any agents.

Thus we have that $f^1(\bar{R}^1, \check{R}^2, \check{R}^3) = f^1(\bar{R}^1, R^2, R^3)$, which is close to $\bar{x}^1$ and $f^1(\hat{R}^1, \check{R}^2, \check{R}^3) = f^1(\hat{R}^1, \tilde{R}^2, \tilde{R}^3)$, which is close to $\hat{x}^1$. This contradicts to the strategy-proofness of f because $\hat{x}^1$ could have been chosen to be preferred to $\bar{x}^1$ with respect to the preference $\bar{R}$ in the second step, and $\bar{R}^1$ could have been chosen to be sufficiently close to $\bar{R}$ in the third step. This ends the proof of Part 1.

Part 2. We have proved that one agent receives the total endowments at each preference profile $\mathbf{R} = (R, R, R)$ where all agents have the same preference. Because of Lemma 6, there exists an agent who receives zero consumption at any such profile $\mathbf{R} = (R, R, R)$. In this part, we prove that the allocation given by f should depend only on the preference of this agent.

Without loss of generality we assume that agent 3 is the agent receiving zero consumptions at such profiles: $f^3(R, R, R) = 0$ for any R.

Pick a preference R. We know $f(R, R, R)$ is $(\Omega, 0, 0)$ or $(0, \Omega, 0)$. We prove that if $f(R, R, R) = (\Omega, 0, 0)$, then $f(\tilde{R}, \bar{R}, R) = (\Omega, 0, 0)$ for any $\bar{R}, \tilde{R} \in \mathcal{R}$, and symmetrically if $f(R, R, R) = (0, \Omega, 0)$, then $f(\tilde{R}, \bar{R}, R) = (0, \Omega, 0)$ for any $\bar{R}, \tilde{R} \in \mathcal{R}$. That is, consumer 1 or 2 is allocated the total endowment depending on agent 3's preference.

We consider the case $f(R, R, R) = (\Omega, 0, 0)$. This implies $f(\bar{R}, R, R) = (\Omega, 0, 0)$. Hence $f^2(\bar{R}, \bar{R}, R) = 0$ for any $\bar{R}$. On the other hand, $f^3(\bar{R}, \bar{R}, \bar{R}) = 0$ implies $f^3(\bar{R}, \bar{R}, R) = 0$. Thus we have $f(\bar{R}, \bar{R}, R) = (\Omega, 0, 0)$. This implies $f(\tilde{R}, \bar{R}, R) = (\Omega, 0, 0)$ for any $\tilde{R}$.

The discussion is symmetric for the case $f(R, R, R) = (0, \Omega, 0)$. This ends the proof of Theorem 3.

8 Proof of Corollary 1

We suppose $f : \bar{\mathcal{R}}^3 \to X$ is a Pareto efficient and strategy-proof social choice function. Then it should be Pareto efficient and strategy-proof on the restricted domain $\mathcal{R}^3$. In Theorem 3, we proved that it should be an SS mechanism. Without loss of generality we assume that agent 1 or agent 2 receives the total endowment depending on the shape of agent 3's preference. Thus agent 3's preference domain $\mathcal{R}$ is divided into $\mathcal{R}_1$ and $\mathcal{R}_2$, and for any $R^1, R^2, R^3 \in \mathcal{R}$,

$$f(R^1, R^2, R^3) = (\Omega, 0, 0) \text{ if } R^3 \in \mathcal{R}_1,$$
$$f(R^1, R^2, R^3) = (0, \Omega, 0) \text{ if } R^3 \in \mathcal{R}_2.$$

Because of the strategy-proofness with respect to agent 2, $f^2(R^1, R^2, R^3) = 0$ for any $R^1 \in \mathcal{R}$, $R^2 \in \bar{\mathcal{R}}$, $R^3 \in \mathcal{R}_1$.

On the other hand, again because of the strategy-proofness with respect to agent 2, $f(R^1, R^2, R^3) = (0, \Omega, 0)$ for any $R^1 \in \mathcal{R}$, $R^2 \in \bar{\mathcal{R}}$, $R^3 \in \mathcal{R}_2$. Then, because of the strategy-proofness with respect to agent 3, $f^3(R^1, R^2, R^3) = 0$ for any $R^1 \in \mathcal{R}$, $R^2 \in \bar{\mathcal{R}}$, $R^3 \in \mathcal{R}_1$.

From these results we have $f(R^1, R^2, R^3) = (\Omega, 0, 0)$ for any $R^1 \in \mathcal{R}$, $R^2 \in \bar{\mathcal{R}}$, $R^3 \in \mathcal{R}_1$, and, because of the strategy-proofness of agent 1, $f(R^1, R^2, R^3) = (\Omega, 0, 0)$ for any $R^1, R^2 \in \bar{\mathcal{R}}$ and $R^3 \in \mathcal{R}_1$.

This implies that $f^3(R^1, R^2, R^3) = 0$ for any $R^1, R^2, R^3 \in \bar{\mathcal{R}}$ because of the strategy proofness with respect to agent 3. Therefore for any fixed $R^3 \in \bar{\mathcal{R}}$, $f(\cdot, \cdot, R^3)$ should be a Pareto efficient and strategy-proof social choice function for the two-agent economy with agents 1 and 2. Then it should be a dictatorial mechanism. That is, depending on agent 3's preference R^3, agent 1 or 2 should be allocated the total endowment Ω.

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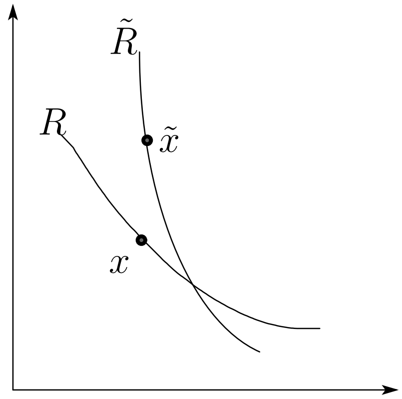

Figure 1

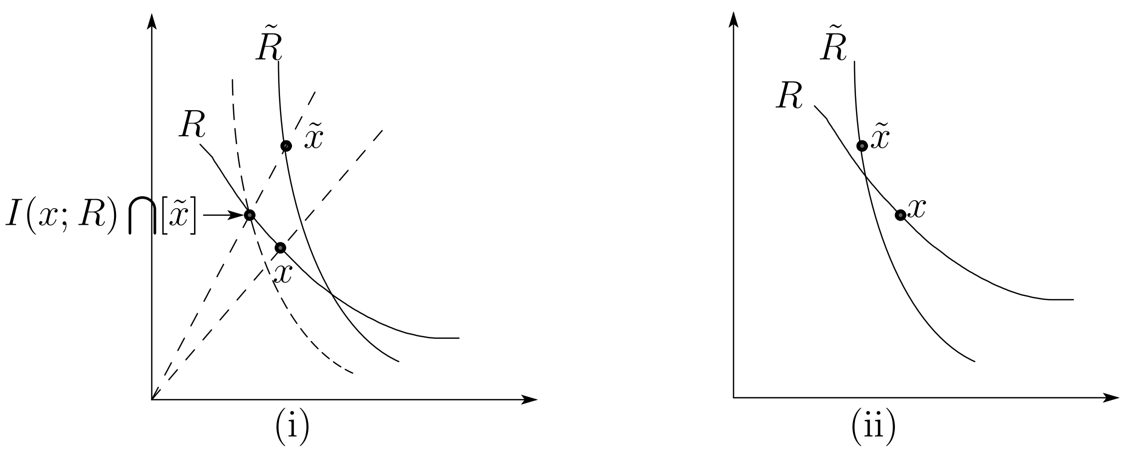

Figure 2

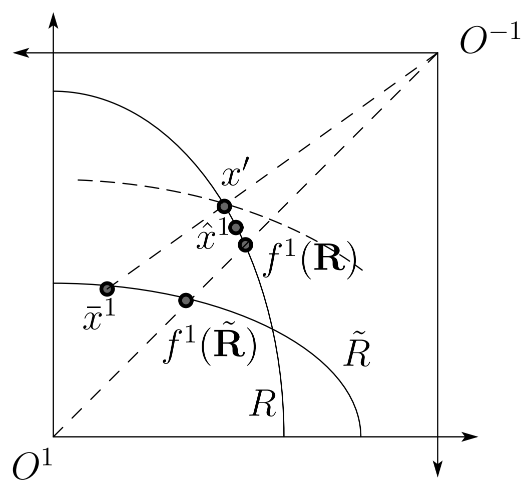

Figure 3

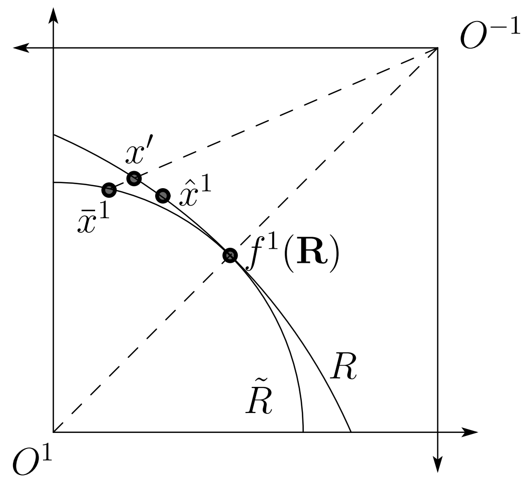

Figure 4

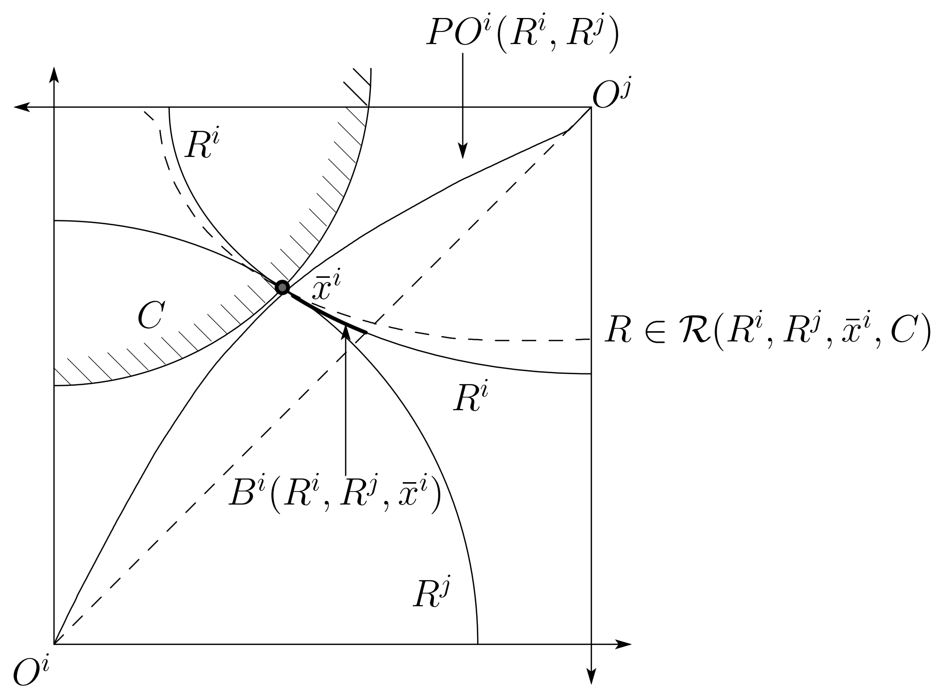

Figure 5