Urbanization and political redistribution

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Abstract

Urbanization (population concentration in metropolitan areas) is a common phenomenon in developed countries. Some research claim that political factors, expressed in the form of political urban bias influence urbanization. Looking into Japan's data, however, there seems to be a rural bias because local governments in rural regions receive more transfers from the central government than their urban counterparts. The aim of the paper is to explain the seemingly inconsistent phenomena in Japan.

We construct a simple political economy model, from which we obtained three results. First, the equilibrium policy is characterized by markup pricing: urban wages are higher than rural wages by a constant markup rate. The persistent wage inequality represents a political urban bias.

Second, the urban population ratio is proportional to per capita capital, which implies that urbanization is an outcome of capital accumulation.

Finally, net public transfers from urban to rural regions are hump-shaped with respect to the urban population ratio. At an earlier stage of urbanization, public transfers flow from urban to rural regions. The direction is reversed later. In a matured economy, an additional political urban bias appears in the sense that urban residents become gainers of the political redistribution.

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1 Introduction

Urbanization (population concentration in metropolitan areas) is a common phenomenon in developed countries. A 2016 report published by the Organisation for Economic Co-operation and Development (OECD) outlines the regional population distribution of OECD countries in 2014, as well as their changes in regional share of the population between 2000 and 2014. On average, 46.4% of the whole population lives in urban regions which cover approximately 5.7% of the whole surface area. Between 2000 and 2014, the proportion of people living in urban areas increased by 0.3%, accompanied by 0.3% and 0.6% drops in rural regions and intermediate regions populations, respectively. Closely looking into each country, marked heterogeneity is observed. The proportion of people living in urban areas varies significantly across countries, from 11.4% in Slovakia to 73.9% in the UK. Interregional population dynamics are modest in the UK, Slovakia, Belgium, Greece, and Mexico. However, Estonia's urban population share has increased by 5.2%, accompanied by 3.4% and 1.8% drops in rural regions and intermediate regions populations, respectively. The speed of concentration is relatively high in Canada, Finland, Austria, and Japan.

While it has been recognized that interregional wage differentials encourage urbanization (Mas-Colell and Razin, 1973; Henderson, 1974), some research claim that political factors matter (Henderson, 2003; Davis and Henderson, 2003; Shifa, 2013; Mourmouras and Rangazas, 2013). Public policies that are seemingly in favor of urban residents are called "urban bias."

Figure 1 illustrates the relationship between regional income and fiscal dependency of local governments of Japan. The 47 points correspond to prefectures of Japan, which are classified into metropolitan areas (colored black) and rural areas (white).[1] The horizontal axis represents per capita income in the prefecture, and the vertical axis represents total transfers to the prefecture.[2] The negative relationship is statistically significant; a 1% increase in prefectural income reduces the fiscal dependency rate by 2.3%.[3]

Table 1 summarizes the between-group difference in the fiscal dependency rate. The two confidence intervals are not overlapped which implies that on average the dependency of rural prefectures on the central government is statistically higher than that of urban prefectures.

Although our data is simple and limited, the result may be sensible because public policies conducted by Japan's central government show a "rural" bias. We seek to explain the seemingly inconsistent phenomena in Japan.

[Figures 1 is here]

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We construct a simple model of a political economy in which both time and space considerations are incorporated (Mourmouras and Rangazas, 2013). The state of the economy consists of economic equilibrium and political equilibrium. In a two-period, two-region OLG model, individuals make decisions about migration and savings in young adulthood. Competitive firms produce a homogeneous good in either an urban or rural region. Urban technology is more capital-intensive than rural technology. In a closed economy, regional wage rates and interest rates are determined in the corresponding factor markets. In the economic equilibrium, each endogenous variable is expressed as a function of the economy's state variable, which is in our model "per capita capital."

In each period, two political parties of the national parliament compete over region-specific transfer policies, taking the existing tax rate as given. The political equilibrium is specified by the Markov perfect equilibrium in the probabilistic voting model (Persson and Tabellini, 2000; Hassler et al., 2003; Song et al., 2012; Ono, 2015; Lancia and Russo, 2016). In equilibrium, the transfer policy in a period is stipulated by the state variable in the period. This politico-economic equilibrium evolves with the law of motion of the state variable (per capita capital).

[1] We used the Super-Mega Region (SMR) as the metropolitan area. SMR is a brandnew concept of Japan's metropolitan area, emerging after the Chuo Shinkansen, a central line of bullet trains with superconducting maglev technology. SMR will connect Japan's two major cities, Tokyo and Osaka in 67 minutes by 2045.

[2] Total transfers is the sum of the distribution of local allocation tax and the national treasury grants-in-aid.

[3] A simple OLS estimate yields

$$\ln(\text{Fiscal dependency}) = \underset{(11.0)}{22.1} - \underset{(-9.25)}{2.33} * \ln(\text{Income}), \quad R^2 = 0.655$$

where the parentheses indicate t-values.

In this model, we obtain three results. First, the equilibrium policy is characterized by markup pricing, that is, the wage rate of urban labor is higher than the wage rate of rural labor by a constant markup rate. This implies that a political "urban bias" does exist in the sense that urban workers earn more than rural workers.

Second, the equilibrium urban population ratio is proportional to per capita capital. This implies that urbanization is an outcome of capital accumulation.

Finally, net public transfer from urban to rural regions is hump-shaped with respect to the urban population ratio. Suppose that the urban population ratio is low, and the migration speed is high in a country, our model predicts that the central government commits to a large-scale interregional redistribution from urban to rural regions. This scenario can be applied to Japan. In an extreme case, the net public transfer could be negative when the urban population is dominant. In this matured economy, an additional urban bias emerges in the sense that urban residents receive more public transfers than rural residents.

This paper is closely related to Mourmouras and Rangazas (2013), who discuss the mechanism of urban bias in a small open two-period two-region OLG economy. In their model, more public resources are allocated to urban regions in per capita terms, which implies that the optimal policy involves an urban bias. This result arises from the assumption of a small open economy as well as the government's consideration of the migration equilibrium. In general, the government has to estimate the extent to which the interest rate changes in response to a policy change because the welfare of retirement is influenced by the interest rate. This aspect can be eliminated because the interest rate is constant in a small open economy. On one hand, the urban wage rate is constant because capital inflow or outflow makes the capital-labor ratio constant. On the other hand, rural wage rate is negatively related to the labor demand because rural production inputs are labor and land, not capital. Since potential workers move freely, the migration equilibrium is characterized by a familiar condition that urban wage income equals rural wage income.

The government can indirectly control migration by controlling regional wage rates (via providing productive public goods) or by remittance of regional benefits (via providing consumption goods). Mourmouras and Rangazas (2013) show that the optimal allocation is associated with the maximization of aggregate wage income. Because the demand curve of urban labor is horizontal, and the demand curve of rural labor is downward-sloped, shifting the horizontal line upwards increases aggregate wage income, which implies that the optimal policy is biased toward urban residents.

Our model shares characteristics in common with Mourmouras and Rangazas (2013) in that the interest rate is constant over time, and that the equilibrium strategy of political parties is to maximize aggregate wage income. However, the mechanism is quite different from Mourmouras and Rangazas (2013) because our model economy is closed and the government is not purely benevolent. Our result arises from competitive political parties' expectations about future policies.

Further, this paper presents an additional insight on the static model of Wildasin (1991) and Lucas's (2004) dynamic migration model. We obtain a dynamic relationship among migration, capital accumulation, and region-specific transfers, which could capture the role of political parties in the process of urban development in a simple way.

The remainder of this paper is organized as follows. In Section 2, we introduce the basic model and derive the short-run equilibrium. In Section 3, we examine the dynamics of migration. In Section 4, we extend the basic model by introducing a migration cost and congestion externality of migration. In Section 5, we present a numerical example. The final section concludes the paper.

2 The model

2.1 Setup

We used a two-period, two-region overlapping generations model. In each period, a mass of individuals enters the economy, whose size is constant, $\bar{N} > 0$. Individuals who live as young adults in period t and old adults in period $t+1$ are called generation t. The birthplace of an individual in generation t is either urban region or rural region, depending on the parent's residence. However, they could choose their own residence at the beginning of period t. We assume migration is costless.[4] After choosing a residence, they work, have a child, and allocate their disposable income between consumption and savings. In the second period, they receive capital income to consume in the same region. Bequest motives are omitted.

[4] We discuss the effect of migration cost in Section 4.1.

The utility function of an individual in generation t is given by

$$u_t = (1-\beta)\ln c_{1t} + \beta \ln c_{2t+1} \tag{1}$$

where c_{1t} and c_{2t+1} represent young-age consumption and old-age consumption, respectively. $0 < \beta < 1$ is a constant preference parameter. Specifically, a private discount factor is given by $\beta/(1-\beta)$.

The budget constraints in the first and second periods are given by

$$y_t^i = c_{1t} + s_t \tag{2}$$

$$R_{t+1}s_t = c_{2t+1} \tag{3}$$

where s_t and R_{t+1} represent private savings and gross interest rates, respectively. y_t^i stands for disposable income in region i, which is given by

$$y_t^i = \begin{cases} (1-\tau)w_t(1-\varepsilon) + g_t & \text{in urban} \\ (1-\tau)w_t^o + g_t^o & \text{in rural} \end{cases} \tag{4}$$

where w_t and w_t^o are the urban and rural wage rates, respectively. $0 \le \tau < 1$ is the wage income tax rate, and g_t (g_t^o) is a lump-sum transfers in the urban (rural) region. Because our focus is interregional income redistribution, we assume that the tax rate is constant. $0 < \varepsilon < 1$ represents a relative cost of urban life to rural life such as the opportunity cost of commuting or housing costs.[5]

In each period, competitive firms produce a homogeneous good in either an urban or rural region. We assume that labor is immobile and capital and goods move freely between the two regions.

The production function in the urban region is given by

$$Y_t = F(K_t, L_t) = AK_t^{\alpha}L_t^{1-\alpha} \tag{5}$$

where Y_t, K_t, and L_t represent the urban output, capital, and urban labor, respectively. $0 < \alpha < 1$ is the output elasticity of capital, and $A > 0$ is total factor productivity in the urban region.

The production function in the rural region is given by

$$Y_t^o = BL_t^o \tag{6}$$

where Y_t^o and L_t^o represent rural output and rural labor, respectively. $B > 0$ is labor productivity in the rural region.

The working population in the urban (rural) region in period t is denoted by N_t (N_t^o). By assumption, $N_t + N_t^o = \bar{N}$.

The central government undertakes a redistribution policy by collecting taxes to provide region-specific transfers. The government budget constraint is given by

$$\tau(w_tL_t + w_t^oL_t^o) = N_tg_t + N_t^og_t^o \tag{7}$$

The market clearing conditions for urban labor, rural labor, and capital are respectively given by

$$L_t = N_t(1-\varepsilon) \tag{8}$$

$$L_t^o = N_t^o \tag{9}$$

$$K_{t+1} = N_ts_t + N_t^os_t^o \tag{10}$$

where s_t (s_t^o) represents the per capita savings of urban (rural) residents.[6]

2.2 Timing of decisions

The timing in period t is as follows:

(i) Per capita capital, $k_t = K_t/\bar{N}$ is realized.
(ii) Two political parties compete over the redistribution policy (g_t, g_t^o).
(iii) Individuals in generation t choose their residence.
(iv) Factor prices are determined in the competitive markets.
(v) Workers and firms behave optimally in both regions.

First, we derive the economic equilibrium in period t by solving backward (iii) - (v). Second, we derive a political equilibrium, which consists of a redistribution policy (g_t, g_t^o) as a function of the state variable, k_t. Finally, we can derive the long-run equilibrium by examining the law of motion of k_t.

[5] We discuss the cost arising from congestion externalities in Section 4.2.

[6] The goods market clearing condition,

$$Y_t + Y_t^o = N_tc_{1t} + N_t^oc_{1t}^o + N_{t-1}c_{2t} + N_{t-1}^oc_{2t}^o + K_{t+1}$$

can be derived by Walras' law.

2.3 Economic equilibrium

The household maximization problem is formulated as

$$\max_{c_{1t},\, c_{2t+1}} u_t = (1-\beta)\ln c_{1t} + \beta \ln c_{2t+1} \quad \text{subject to } y_t^i = c_{1t} + \frac{c_{2t+1}}{R_{t+1}}$$

Solving this, the saving function is given by

$$s_t^i = \beta y_t^i \tag{11}$$

and the indirect utility function is given by

$$V_t^i = \ln y_t^i + \beta \ln R_{t+1} + (1-\beta)\ln(1-\beta) + \beta \ln \beta \tag{12}$$

Solving the profit maximization problem yields

$$\begin{aligned} R_t &= \alpha \frac{Y_t}{K_t} \\ w_t &= (1-\alpha)\frac{Y_t}{L_t} \\ w_t^o &= B \end{aligned}$$

2.4 Political equilibrium

In this section, we derive the political equilibrium.

In each period, two political parties compete over the redistribution policy (g_t, g_t^o), taking per capita capital, $k_t = K_t/\bar{N}$, and tax rate τ, as given. Denote the population share in urban region in period t by $\rho_t = N_t/\bar{N}$. The wage rate in the urban region is given by

$$w_t = (1-\alpha)Ak_t^\alpha[(1-\varepsilon)\rho_t]^{-\alpha} \tag{13}$$

We focus on cases in which individuals live in both regions. Eq.(12) implies that the migration equilibrium yields $y_t = y_t^o$, that is,

$$(1-\tau)w_t(1-\varepsilon) + g_t = (1-\tau)B + g_t^o \tag{14}$$

From (7), the government budget constraint in per capita terms is given by

$$\tau\left[\rho_t w_t(1-\varepsilon) + (1-\rho_t)B\right] = \rho_t g_t + (1-\rho_t)g_t^o \tag{15}$$

Finally, substituting (11) into (10) and using (7), we obtain:

$$k_{t+1} = \beta\left[\rho_t w_t(1-\varepsilon) + (1-\rho_t)B\right] \tag{16}$$

Following Mourmouras and Rangazas (2013), the objective function of political parties in period t is given by

$$W_t = \theta_t V_t + (1-\theta_t)V_t^o \tag{17}$$

where V_t and V_t^o are given by Eq.(12), and $0 \le \theta_t \le 1$ represents a relative weight on the welfare of urban residents, which reflects political power such as voter turnout or the urban population share of the old generations, ρ_{t-1} (Song et al., 2012).

The maximization problem of a political party in period t is given by

$$\max_{\rho_t,\, g_t,\, g_t^o} W_t = \theta_t \ln[(1-\tau)w_t(1-\varepsilon) + g_t] + (1-\theta_t)\ln[(1-\tau)B + g_t^o] + \beta \ln R_{t+1}^*$$

subject to Eqs.(13), (14), (15), and

$$R_{t+1}^* = \alpha A k_{t+1}^{\alpha-1}[(1-\varepsilon)\rho_{t+1}^*]^{1-\alpha} \tag{18}$$

Hence, ρ_{t+1}^* represents the urban population ratio in period $t+1$ which the political party in period t expects to be realized. In fact, the urban population ratio in period $t+1$ is realized in the political equilibrium in period $t+1$. Therefore, political parties in period t have to form expectations about it, based on their own redistribution policies.

Further, R^*_{t+1} is affected by the per capita capital in period $t+1$, k_{t+1}, which can be partly controlled by the redistribution policy in period t. Suppose that the urban transfer, g_t, increases. Then, young individuals in the rural region migrate to the urban region, which causes an increase in savings because urban income is larger than rural income. Accordingly, capital per worker increases, which lowers the interest rate. This change worsens the welfare of urban residents as well as rural residents because the reduction of interest rate decreases old-age consumption. Political parties choose their redistribution policies by analyzing the relationship between current policy and the future interest rate.

The following proposition summarizes the outcome of the Markov perfect equilibrium.

Proposition 1 *In the Markov perfect equilibrium, the urban wage, the urban population ratio, and the redistribution policies are given by the following four equations:*

$$w^*_t = \frac{B}{(1-\alpha)(1-\varepsilon)} \tag{19}$$

$$\rho^*_t = \left[\frac{(1-\alpha)A}{w^*_t}\right]^{\frac{1}{\alpha}} \frac{k_t}{1-\varepsilon} \tag{20}$$

$$g^*_t = \frac{\tau - \alpha(1-\rho^*_t)}{1-\alpha} B \tag{21}$$

$$g^{o*}_t = \left(\tau + \frac{\alpha}{1-\alpha}\rho^*_t\right) B \tag{22}$$

Proof. See Appendix A. ■

Eq.(19) implies that the equilibrium is characterized by markup pricing. There exists an urban bias in the sense that urban workers earn more income than rural workers. Eq.(20) states that the urban population ratio is proportional to per capita capital. Eqs.(21) and (22) represents the policy functions in period t. The region-specific transfers are adjusted to satisfy both the migration equilibrium (14) and the government budget constraint (15). Both transfers are positively related to ρ^*_t. Given a constant tax rate, the revenue from the central government increases with the number of urban residents because the urban wage is higher than the rural wage. The increased revenue is transferred to both regions in order to control inter-regional migration.

It would be insightful to compare the Markov perfect equilibrium in Proposition 1 with the Pareto-efficient allocation. Appendix E shows that the Pareto-efficient allocation is characterized by the following equations:

$$MRS_t = MRS^o_t = f_{k,t+1} \tag{23}$$

$$f_{\rho,t} - c_{1t} - \frac{c_{2t+1}}{f_{k,t+1}} = B - c^o_{1t} - \frac{c^o_{2t+1}}{f_{k,t+1}} \tag{24}$$

where $MRS_t = (\partial u_t/\partial c_{1t})/(\partial u_t/\partial c_{2t+1})$ is the marginal rate of substitution between current and future consumption of urban residents, and MRS^o_t is the marginal rate of substitution between current and future consumption of rural residents. $f_{k,t+1}$ is the marginal product of capital in period $t+1$, and $f_{\rho,t}$ is the marginal product of urban labor in period t.

Eq.(23) implies that the marginal benefit of current investment is $f_{k,t+1}$ in terms of future consumption, which equals to the marginal cost in the urban region, MRS_t, and the marginal cost in the rural region, MRS^o_t. This condition is satisfied in the Markov perfect equilibrium because household optimization behavior and competitive capital market yield $MRS_t = MRS^o_t = R_{t+1} = f_{k,t+1}$.

Eq.(24) implies that the net marginal benefit of the urban population equals the net marginal benefit of the rural population. In each region, the net marginal benefit is defined as the marginal product of labor net the present value of consumption per capita. If the left-hand side of (24) is larger than the right-hand side, then relocating population from the rural region to the urban region is Pareto-improving.

We can see that Eq.(24) is not satisfied in the Markov perfect equilibrium. Using the household budget constraints and $f_{\rho,t} = w_t(1-\varepsilon)$, the net marginal product of the urban population is evaluated as

$$\tau w^*_t(1-\varepsilon) - g^*_t = \frac{\alpha}{1-\alpha} B(1-\rho^*_t) > 0$$

while the net marginal product of rural population is given by

$$\tau B - g_t^{o*} = -\frac{\alpha}{1-\alpha} B \rho_t^* < 0$$

Therefore, the winner of the political competition adopts a biased transfer policy in the sense that it prevents migration from rural to urban regions. This urban bias arises from the expectation about the next period interest rate. Substituting (19) and (20) into (18), we obtain

$$R_{t+1}^* = \alpha A \left[\frac{(1-\alpha)^2 (1-\varepsilon) A}{B} \right]^{\frac{1-\alpha}{\alpha}}$$

that is, the expected interest rate is constant. Clearly, this clear cut result depends on simplified specifications. However, the constancy of expected interest rate makes clear the characteristics of Markov perfect equilibrium. Since political parties take the interest rate as given, their common objective is to maximize aggregate wage income, $w_t L_t + w_t^o L_t^o = \bar{N} \left[w_t \rho_t (1-\varepsilon) + B(1-\rho_t) \right]$, considering the demand for urban labor, $w_t = w(\rho_t, k_t)$. Eqs.(19) and (20) are the exact solution of this problem.

[Figure 2 is here]

Figure 2 illustrates the temporary equilibrium in labor market. The origin of urban labor is located in the lower left, and the origin of rural labor is in the lower right. Because urban labor is $\rho_t(1-\varepsilon)$ in per capita terms, and rural labor is $1-\rho_t$, the distance between the two origins is $1-\varepsilon\rho_t$. The downward-sloping curve represents the demand for urban labor. First, suppose that the central government adopts a lump-sum transfer policy. In this case, the equilibrium is E_0. The urban wage rate is $B/(1-\varepsilon)$, which makes urban income equal to rural income. This equilibrium satisfies Eq.(24) in the same manner as Todaro (1969), and Harris and Todaro (1970).

In our model, however, E_0 is not the political equilibrium because political parties are allowed for region-specific transfers. The equilibrium is E^*, which maximizes aggregate wage income as mentioned above.

As for the interregional difference in transfers, we obtain the following proposition.

Proposition 2 *There exists a unique urban population ratio, $\tilde{\rho} \in (0,1)$, such that $N_t^o g_t^{o*} \gtreqless N_t g_t^*$ if and only if $\rho_t^* \lesseqgtr \tilde{\rho}$.*

Proof. From Eqs.(21) and (22), the interregional difference in transfers is given by

$$N_t^o g_t^{o*} - N_t g_t^* = \frac{B\bar{N}}{1-\alpha} \left\{ 2\alpha(1-\rho_t^*)\rho_t^* + \tau[(1-\alpha)(1-\rho_t^*) - \rho_t^*] \right\}$$

Denote a quadratic function by

$$f(\rho) = 2\alpha(1-\rho)\rho + \tau[(1-\alpha)(1-\rho) - \rho]$$

We know $f(0) = \tau(1-\alpha) > 0$, $f(1) = -\tau < 0$, and $f''(\rho) < 0$. Therefore, there exists a unique $\tilde{\rho} \in (0,1)$ such that $f(\rho) \gtreqless 0$ if $\rho \lesseqgtr \tilde{\rho}$. ■

3 Dynamics and the steady state

In this section, we examine the dynamics of migration. Our model complements the discussion of Lucas (2004) on the time paths of farm employment and cities employment. Eqs.(16), (21), and (22) determine the law of motion of per capita capital as

$$k_{t+1} = \beta B \left(1 + \frac{\alpha}{1-\alpha} \rho_t^* \right) = \beta B \left(1 + \frac{\alpha}{1-\alpha} \gamma k_t \right) \tag{25}$$

where

$$\gamma = \left[\frac{A}{B} (1-\alpha)^2 (1-\varepsilon)^{1-\alpha} \right]^{\frac{1}{\alpha}}$$

Because $\rho_t^* = \gamma k_t$, the interior condition $\rho_t^* < 1$ requires $k_t < 1/\gamma$. In the following, we assume[7]

$$\beta B\gamma < 1-\alpha \tag{26}$$

The dynamics of the urban population ratio is summarized in the following proposition.

Proposition 3 *We assme that Eq.(26) is satisfied. Then the urban population ratio in period t is given by*

$$\rho_t^* = \rho^* + (\rho_0 - \rho^*)\left(\frac{\alpha}{1-\alpha}\beta B\gamma\right)^t \tag{27}$$

where ρ_0 is the initial condition and ρ^* is the steady state urban population ratio, which is given by

$$\rho^* = \frac{\beta B\gamma}{1-\frac{\alpha}{1-\alpha}\beta B\gamma} \tag{28}$$

Proof. See Appendix B. ■

[Figure 3 is here]

Figure 3 illustrates the process of urbanization. If the initial urban population is small, that is, $\rho_0 < \rho^*$, then the urban population ratio increases monotonically and converges at the steady-state, ρ^*. The speed of convergence is high if the discount factor is large (β), urban productivity is high (A), and rural productivity is low (B), and/or the urban cost is small (ε).

4 Discussions

4.1 Migration cost

In this section, we extend the basic model by introducing the cost of migration. It can be seen that the result is the same as that of the basic model. Suppose that some individuals born in the rural region migrate to the urban region at a fixed cost and that individuals born in the urban region continue to live in urban region[8]. In this economy, individuals are classified into three groups according to their birth-place and the history of migration. We use a superscript m to identify individuals who migrate from rural to urban regions.

The objective function of a political party in period t is given by

$$W_t = \theta_t \ln y_t + \theta_t^m \ln y_t^m + (1-\theta_t-\theta_t^m)\ln y_t^o + \beta \ln R_{t+1}^*$$

where

$$\begin{aligned} y_t &= (1-\tau)w_t(1-\varepsilon) + g_t \\ y_t^m &= (1-\tau)w_t(1-\varepsilon) - Bz + g_t^m \\ y_t^o &= (1-\tau)B + g_t^o \end{aligned}$$

Hence, y_t^m represents migrants' income. The migration cost is measured by a constant share of rural income, $z > 0$. g_t^m represents the transfer migrants receive, and θ_t^m is the political power of migrants. The other variables are the same as the basic model.

The migration rate from rural to urban regions in period t is denoted by π_t. The urban population share in period t is $\rho_t = \rho_{t-1} + (1-\rho_{t-1})\pi_t$, and the rural share is $1-\rho_t = (1-\rho_{t-1})(1-\pi_t)$.

The government budget constraint is given by

$$\tau[\rho_t w_t(1-\varepsilon) + (1-\rho_t)B] = \rho_{t-1}g_t + (1-\rho_{t-1})\pi_t g_t^m + (1-\rho_t)g_t^o$$

Political parties choose g_t, g_t^m, g_t^o, and π_t to maximize the objective function subject to the government budget constraint and the migration equilibrium condition, $y_t = y_t^m = y_t^o$.[9] The condition $y_t = y_t^m$ yields $g_t^m = g_t + Bz$. Therefore, the political maximization problem is reformulated as follows:

$$\max_{\rho_t, g_t, g_t^o} W_t = (\theta_t + \theta_t^m)\ln[(1-\tau)w_t(1-\varepsilon) + g_t] + (1-\theta_t-\theta_t^m)\ln[(1-\tau)B + g_t^o] + \beta \ln R_{t+1}^*$$

[7] Appendix B shows that Eq.(26) ensures the interior condition.

[8] Allowing for migration from urban to rural regions does not change our result qualitatively.

[9] Individuals born at rural region are indifferent between staying there and moving to urban region if $y_t^m = y_t^o$. Theoretically, there are three cases: $y_t > y_t^m = y_t^o$, $y_t = y_t^m = y_t^o$, and $y_t < y_t^m = y_t^o$. We focus on the second case since the other cases generate ex-post incentives for migration.

subject to

$$\begin{aligned} \tau[\rho_t w_t(1-\varepsilon)+(1-\rho_t)B] &= \rho_t g_t+(1-\rho_t)g_t^o+(\rho_t-\rho_{t-1})Bz \\ (1-\tau)w_t(1-\varepsilon)+g_t &= (1-\tau)B+g_t^o \end{aligned}$$

The main difference from the basic model is an additional public expenditure component, $(\rho_t - \rho_{t-1})Bz$. This compensation for migration ensures that both regions exist.

The following proposition summarizes the result. The markup pricing rule is preserved in this extended model.

Proposition 4 *In the Markov perfect equilibrium with migration cost Bz, the urban wage, the urban population ratio, and the redistribution policies are given by the following five equations:*

$$\begin{aligned} w_t^* &= \frac{B(1+z)}{(1-\alpha)(1-\varepsilon)} \\ \rho_t^* &= \left[\frac{(1-\alpha)A}{w_t^*}\right]^{\frac{1}{\alpha}} \frac{k_t}{1-\varepsilon} \\ g_t^* &= B\left[\frac{\tau(1+z\rho_t^*)-\alpha(1-\rho_t^*)}{1-\alpha} - z(\rho_t^*-\rho_{t-1})\right] \\ g_t^{m*} &= g_t^* + Bz \\ g_t^{o*} &= B\left[\tau\left(1+\frac{z}{1-\alpha}\rho_t^*\right)+\frac{\alpha}{1-\alpha}\rho_t^* - z(\rho_t^*-\rho_{t-1})\right] \end{aligned}$$

Proof. See Appendix C. ■

4.2 Urban cost

In the basic model, we assume that urban cost is constant. This assumption might be restrictive because urban cost tends to be large when the population density is high (Sato, 2007). If the urban cost increases with population, then the political equilibrium would be in favor of rural residents to mitigate congestion externalities in the urban region.

Assume that the urban cost in period t, ε_t, is increasing in the urban population ratio in period t, ρ_t, that is, $\varepsilon_t = \varepsilon(\rho_t)$, $\varepsilon' > 0$.[10] The economic equilibrium is the same as the basic model because individuals and firms take ρ_t as given. The political equilibrium is somewhat different from the basic model because political parties try to assess the cost of congestion externalities.

Denote the urban labor income in period t by I_t,

$$I_t = w_t[1-\varepsilon(\rho_t)] = (1-\alpha)Ak_t^\alpha[1-\varepsilon(\rho_t)]^{1-\alpha}\rho_t^{-\alpha}$$

An increase in ρ_t decreases I_t because it decreases the wage rate and the labor supply. Therefore, political parties tend to discourage people from migrating into the urban region by decreasing transfers to urban residents, g_t.

One of the inevitable difficulties is to find a class of cost functions that are consistent with the Markov perfect equilibrium. For this purpose, we specify the cost function as follows:

$$\varepsilon(\rho_t) = \begin{cases} e - \frac{m}{\rho_t} & \text{if} \quad \frac{m}{e} \le \rho_t < 1 \\ 0 & \phantom{\text{if}} \quad 0 < \rho_t \le \frac{m}{e} \end{cases} \tag{29}$$

where $0 \le m < e \le 1+m$.

The results are summarized in the following proposition. Qualitatively, the result is the same as that of the basic model.

Proposition 5 *In the Markov perfect equilibrium with the urban cost specified by Eq.(29), the urban population ratio and the redistribution policies are given by the following equations:*

$$\begin{aligned} \rho_t^* &= \frac{1}{1-e}\left\{\left[\frac{A}{B}(1-\alpha)^2(1-e)\right]^{\frac{1}{\alpha}} k_t - m\right\} \\ g_t^* &= \tau I_t^* - (1-\rho_t^*)(I_t^*-B) \\ g_t^* &= \tau B + \rho_t^*(I_t^*-B) \end{aligned}$$

[10] More precisely, the urban cost would be $\varepsilon_t = \varepsilon(\rho_t + \rho_{t-1})$ because the urban population consists of working generations whose size is N_t, and retirement generations whose size is N_{t-1}. This general form is left for future research because it is a tough task to solve the Markov perfect equilibrium.

where I_t^* represents the equilibrium urban labor income,

$$I_t^* = \frac{B}{1-\alpha}\left[1 + \frac{m}{(1-e)\rho_t^*}\right]$$

Proof. See Appendix D. ■

5 Numerical example

In this section, we show a numerical example. We assume the output elasticity of capital is $\alpha = 0.33$, and the tax rate is $\tau = 0.2$.

[Figure 4 is here]

Figure 4 illustrates the interregional difference in transfers, $(N_t^o g_t^{o*} - N_t g_t^*)/(Y_t + Y_t^o)$ as a function of the urban population ratio, ρ_t^*. Proposition 2 shows that the curve intersects the horizontal axis at $\tilde{\rho}$, which is 0.76 in this example. Overall, the curve is hump-shaped.

[Figures 5 and 6 are here]

It would be more informative to separate the amount of regional transfers from regional outputs. In Figure 5, the solid line represents the urban output, Y_t, and the dashed line represents the rural output, Y_t^o (we assume $B = \bar{N} = 1$). The urban output is proportional to ρ_t^* because ρ_t^* is adjusted to be proportional to k_t. Because the solid line is steeper than the dashed line, an increase in ρ_t^* increases the aggregate output, which widens the tax base of income redistribution.

In Figure 6, the solid curve represents transfers to urban residents, $N_t g_t^*$, and the dashed curve represents transfers to rural residents, $N_t^o g_t^{o*}$. The figure is a mirror image. For urban residents, public transfers are negative when ρ_t^* is small (see Eq.(21)). There are two reasons. First, the central government compensates rural residents for not moving to the urban region. Second, the tax base is small because the aggregate output is small. As ρ_t^* increases, public transfers to urban residents increase not only because the tax base is widened but also because the urban population increases. The logic is the same for the rural residents.

6 Conclusions

In a simple model of a political economy, we examined the dynamic relationship between urbanization and political urban bias. The equilibrium is characterized by markup pricing: the wage rate in the urban region is set to be higher than the wage rate in the rural region by a constant markup rate. The persistent wage inequality represents a political urban bias. Further, the central government compensates rural residents for not moving to the urban region, because political parties are interested in the welfare of the rural voters. The dynamics of interregional transfers are not monotonic. At an earlier stage of urbanization, public transfers flow from urban to rural regions. The direction is reversed later.

Our model can be extended in several directions. It would be theoretically interesting to examine a decentralized economy where local governments compete over income redistribution and individuals choose their residence after observing local policies. Focusing on regional redistribution, we assume the tax rate is constant. It would be important in reality to include the size of revenue as a policy choice. We assume that individuals choose their residence once in a lifetime. It could be reasonable to allow for migration in later life because the length of active life has increased in developed countries (Gaigné and Thisse, 2009). Our model predicts that migration occurs in a single direction. However, one may observe a counterflow migration in the US (OECD, 2016), which has been induced by other public policies such as public childcare (Yakita, 2019). Further theoretical research will be necessary to explain the complicated dynamics of migration.

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Appendix A
[Proof of Proposition 1]
Suppose that the economy ends in period T. The procedure to solve for the Markov perfect equilibrium is as follows:

(i) Solve for the political party's problem in period T. The policy function (g_T^*, g_T^{o*}) is obtained.
(ii) Solve for the political party's problem in period $T-1$ by taking into account (g_T^*, g_T^{o*}). The policy function $(g_{T-1}^*, g_{T-1}^{o*})$ is obtained.
(iii) In the same way as (ii), solve for the political party's problem in period $t \le T-1$. The policy function (g_t^*, g_t^{o*}) is obtained.
(iv) Let $T \to \infty$. Then, we obtain the policy function (g_t^*, g_t^{o*}) in the infinite horizon OLG model.

(i) Period T
Individuals in generation T do not save. The welfare of urban and rural residents are given by

$$\begin{aligned} u_T &= \ln[(1-\tau)w_T(1-\varepsilon)+g_T] \\ u_T^o &= \ln[(1-\tau)B+g_T^o] \end{aligned}$$

respectively.
The political maximization problem is formulated as follows:

$$\max_{g_T, g_T^o, \rho_T} W_T = \theta_T \ln[(1-\tau)w_T(1-\varepsilon)+g_T] + (1-\theta_T)\ln[(1-\tau)B+g_T^o]$$

subject to

$$\tau\left[\rho_T w_T(1-\varepsilon)+(1-\rho_T)B\right] = \rho_T g_T + (1-\rho_T)g_T^o \tag{A1}$$
$$(1-\tau)w_T(1-\varepsilon)+g_T = (1-\tau)B+g_T^o \tag{A2}$$

where the wage rate is given by

$$w_T = (1-\alpha)Ak_T^\alpha[(1-\varepsilon)\rho_T]^{-\alpha} \tag{A3}$$

Note that per capita capital, $k_T = K_T/N$, is predetermined. Political parties understand that migration policy affects urban wages,

$$\frac{\partial w_T}{\partial \rho_T} = -\alpha\frac{w_T}{\rho_T} < 0 \tag{A4}$$

The Lagrangian function is given by

$$\begin{aligned} \Phi_T = {} & \theta_T \ln[(1-\tau)w_T(1-\varepsilon)+g_T] + (1-\theta_T)\ln[(1-\tau)B+g_T^o] \\ & + \lambda_T\left\{\tau[\rho_T w_T(1-\varepsilon)+(1-\rho_T)B] - \rho_T g_T - (1-\rho_T)g_T^o\right\} \\ & + \mu_T\left[(1-\tau)w_T(1-\varepsilon)+g_T-(1-\tau)B-g_T^o\right] \end{aligned}$$

where λ_T and μ_T are the multipliers attached to (A1) and (A2), respectively.
The first-order conditions require

$$\frac{\partial \Phi_T}{\partial g_T} = \frac{\theta_T}{y_T} - \lambda_T\rho_T + \mu_T = 0 \tag{A5}$$
$$\frac{\partial \Phi_T}{\partial g_T^o} = \frac{1-\theta_T}{y_T^o} - \lambda_T(1-\rho_T) - \mu_T = 0 \tag{A6}$$

and

$$\frac{\partial \Phi_T}{\partial \rho_T} = \frac{\theta_T}{y_T}(1-\tau)(1-\varepsilon)\frac{\partial w_T}{\partial \rho_T} + \lambda_T\left\{\tau[w_T(1-\varepsilon)-B] - (g_T-g_T^o) + \tau\rho_T(1-\varepsilon)\frac{\partial w_T}{\partial \rho_T}\right\} + \mu_T(1-\tau)(1-\varepsilon)\frac{\partial w_T}{\partial \rho_T} = 0 \tag{A7}$$

In equilibrium, $y_T = y_T^o$. Therefore, (A5) and (A6) yield $\lambda_T = y_T^{-1} > 0$.
Using (A5), the coefficient of $\partial w_T/\partial \rho_T$ in (A7) becomes

$$\frac{\theta_T}{y_T}(1-\tau)(1-\varepsilon) + \lambda_T\tau\rho_T(1-\varepsilon) + \mu_T(1-\tau)(1-\varepsilon) = \lambda_T\rho_T(1-\varepsilon)$$

Therefore, (A7) yields

$$g_T - g_T^o = \tau[w_T(1-\varepsilon) - B] + \rho_T(1-\varepsilon)\frac{\partial w_T}{\partial \rho_T} \tag{A8}$$

Combining (A2) and (A8), we obtain

$$(1-\varepsilon)\left(w_T + \rho_T \frac{\partial w_T}{\partial \rho_T}\right) = B \tag{A9}$$

The left-hand side represents the marginal benefit of migration from rural to urban regions, and the right-hand side represnets the marginal cost.

Substituting (A4) into (A9), we obtain

$$w_T^* = \frac{B}{(1-\alpha)(1-\varepsilon)} \tag{A10}$$

From (A3), the urban population ratio is given by

$$\rho_T^* = \left[\frac{(1-\alpha)A}{w_T^*}\right]^{\frac{1}{\alpha}} \frac{k_T}{1-\varepsilon} \tag{A11}$$

Note that ρ_T^* is proportional to the per capita capital, k_T. In our model, one of the political concerns is related to markup pricing (see Eq.(A10)). Therefore, political parties encourage urban migration to keep urban wages constant, when capital accumulation increases the marginal product of urban labor.

Eqs.(A1) and (A2) yield

$$\begin{aligned} g_T &= \tau w_T(1-\varepsilon) - (1-\rho_T)[w_T(1-\varepsilon) - B] & \text{(A12)} \\ g_T^o &= \tau B + \rho_T[w_T(1-\varepsilon) - B] & \text{(A13)} \end{aligned}$$

which implies that the interregional redistribution is based on the income difference, $w_T(1-\varepsilon) - B$, adjusted by the population ratio.

Substituting (A10) into (A12) and (A13), we obtain

$$g_T^* = \frac{B}{1-\alpha}\left[\tau - \alpha(1-\rho_T^*)\right] \tag{A14}$$

$$g_T^{o*} = B\left(\tau + \frac{\alpha}{1-\alpha}\rho_T^*\right) \tag{A15}$$

Because ρ_T^* increases with k_T, capital accumulation increases g_T^* and g_T^{o*}. In our model, political parties conduct a policy to keep the interregional income difference constant, $w_T^*(1-\varepsilon) - B = \alpha B/(1-\alpha)$. Rural residents benefit from an increase in ρ_T^* because the number of contributors to redistribution increases. Urban residents also benefit from an increase in ρ_T^*, because the number of recipients of redistribution decreases.

Finally, the interest rate in period T is given by

$$R_T = \alpha A k_T^{\alpha-1}[(1-\varepsilon)\rho_T^*]^{1-\alpha}$$

Using (A10) and (A11), we obtain

$$R_T^* = \alpha A\left[\frac{A}{B}(1-\alpha)^2(1-\varepsilon)\right]^{\frac{1-\alpha}{\alpha}} \tag{A16}$$

which implies that R_T^* is independent of k_T. On one hand, capital accumulation decreases marginal product of capital. On the other hand, political parties encourage urban migration in response to capital accumulation, which increases marginal product of capital. These two effects are entirely offset in our model.

(ii) Period $T-1$

The political maximization problem is formulated as follows:

$$\max_{g_{T-1}, g_{T-1}^o, \rho_{T-1}} W_{T-1} = \theta_{T-1}\ln[(1-\tau)w_{T-1}(1-\varepsilon) + g_{T-1}] + (1-\theta_{T-1})\ln[(1-\tau)B + g_{T-1}^o] + \beta \ln R_T^*$$

subject to

$$\tau[\rho_{T-1}w_{T-1}(1-\varepsilon)+(1-\rho_{T-1})B]=\rho_{T-1}g_{T-1}+(1-\rho_{T-1})g^o_{T-1}$$
$$(1-\tau)w_{T-1}(1-\varepsilon)+g_{T-1}=(1-\tau)B+g^o_{T-1}$$

where $w_{T-1}=(1-\alpha)Ak^{\alpha}_{T-1}[(1-\varepsilon)\rho_{T-1}]^{-\alpha}$, and R^*_T is given by Eq.(A15).

Political parties in period $T-1$ understand that if their policy increases aggregate savings, the interest rate in period T will decrease. Further, they understand that political parties in period T would increase urban migration in response to capital accumulation. Combining them, political parties in $T-1$ expect that the interest rate in period T would be independent of their own policy.

The political maximization problem in period $T-1$ is essentially the same as in period T. Therefore, the optimal policies consist of

$$w^*_{T-1}=\frac{B}{(1-\alpha)(1-\varepsilon)}$$
$$\rho^*_{T-1}=\left[\frac{(1-\alpha)A}{w^*_{T-1}}\right]^{\frac{1}{\alpha}}\frac{k_{T-1}}{1-\varepsilon}$$
$$g^*_{T-1}=\frac{\tau-\alpha(1-\rho^*_{T-1})}{1-\alpha}B$$
$$g^{o*}_{T-1}=\left(\tau+\frac{\alpha}{1-\alpha}\rho^*_{T-1}\right)B$$

(iii) Period $t\leq T-1$

The political optimization problem is formulated as follows:

$$\max_{g_t,\,g^o_t,\,\rho_t} W_t=\theta_t\ln[(1-\tau)w_t(1-\varepsilon)+g_t]+(1-\theta_t)\ln[(1-\tau)B+g^o_t]+\beta\ln R^*_{t+1}$$

subject to

$$\tau\left[\rho_t w_t(1-\varepsilon)+(1-\rho_t)B\right]=\rho_t g_t+(1-\rho_t)g^o_t$$
$$(1-\tau)w_t(1-\varepsilon)+g_t=(1-\tau)B+g^o_t$$

where $w_t=(1-\alpha)Ak^{\alpha}_t[(1-\varepsilon)\rho_t]^{-\alpha}$, and

$$R^*_{t+1}=\alpha A\left[\frac{(1-\alpha)^2(1-\varepsilon)A}{B}\right]^{\frac{1-\alpha}{\alpha}}$$

Using mathematical induction, we obtain

$$w^*_t=\frac{B}{(1-\alpha)(1-\varepsilon)} \tag{A16}$$
$$\rho^*_t=\left[\frac{(1-\alpha)A}{w^*_t}\right]^{\frac{1}{\alpha}}\frac{k_t}{1-\varepsilon} \tag{A17}$$
$$g^*_t=\frac{\tau-\alpha(1-\rho^*_t)}{1-\alpha}B \tag{A18}$$
$$g^{o*}_t=\left(\tau+\frac{\alpha}{1-\alpha}\rho^*_t\right)B \tag{A19}$$

(iv) Let $T\to\infty$. Policy functions in the Markov perfect equilibrium is given by Eqs.(A16) - (A19).

Appendix B

[Proof of Proposition 2]

The dynamics is formulated by a first-order difference equation of k_t,

$$k_{t+1}=\beta B\left(1+\frac{\alpha}{1-\alpha}\gamma k_t\right) \tag{B1}$$

The coefficient of k_t is smaller than one because $\beta B\gamma < 1-\alpha$. Therefore, the unique steady state is given by $k^* = \beta B/[1-\alpha\beta B\gamma/(1-\alpha)]$, which yields the steady state urban population ratio,

$$\rho^* = \frac{\beta B\gamma}{1-\frac{\alpha}{1-\alpha}\beta B\gamma}$$

Obviously, $\rho^* < 1$ because $\beta B\gamma < 1-\alpha$.

Appendix C

[Proof of Proposition 3]

The proof is the same as Appendix A.

(i) Period T

The Lagrangian function is given by

$$\begin{aligned}\Phi_T &= (\theta_T+\theta_T^m)\ln[(1-\tau)w_T(1-\varepsilon)+g_T] + (1-\theta_T-\theta_T^m)\ln[(1-\tau)B+g_T^o] \\ &\quad + \lambda_T\left\{\tau[\rho_T w_T(1-\varepsilon)+(1-\rho_T)B] - \rho_T g_T - (1-\rho_T)g_T^o - (\rho_T-\rho_{T-1})Bz\right\} \\ &\quad + \mu_T\left[(1-\tau)w_T(1-\varepsilon)+g_T-(1-\tau)B-g_T^o\right]\end{aligned}$$

The first-order conditions for g_T, g_T^o, and ρ_T require

$$\frac{\theta_T+\theta_T^m}{y_T} - \lambda_T\rho_T + \mu_T = 0 \tag{C1}$$

$$\frac{1-\theta_T-\theta_T^m}{y_T^o} - \lambda_T(1-\rho_T) - \mu_T = 0 \tag{C2}$$

and

$$\frac{\theta_T+\theta_T^m}{y_T}(1-\tau)(1-\varepsilon)\frac{\partial w_T}{\partial\rho_T} + \lambda_T\left\{\tau[w_T(1-\varepsilon)-B] - (g_T-g_T^o) - Bz + \tau\rho_T(1-\varepsilon)\frac{\partial w_T}{\partial\rho_T}\right\} + \mu_T(1-\tau)(1-\varepsilon)\frac{\partial w_T}{\partial\rho_T} = 0 \tag{C3}$$

In equilibrium, $y_T = y_T^o$. Therefore, (C1) and (C2) yield

$$\frac{1}{y_T} = \lambda_T \tag{C4}$$

Using (C1), the coefficient of $\partial w_T/\partial\rho_T$ in (C3) becomes

$$\frac{\theta_T+\theta_T^m}{y_T}(1-\tau)(1-\varepsilon) + \lambda_T\rho_T\tau(1-\varepsilon) + \mu_T(1-\tau)(1-\varepsilon) = \lambda_T\rho_T(1-\varepsilon)$$

Therefore, (C3) is reduced to

$$\lambda_T\left\{\tau[w_T(1-\varepsilon)-B] - (g_T-g_T^o) - Bz + \rho_T(1-\varepsilon)\frac{\partial w_T}{\partial\rho_T}\right\} = 0$$

Substituting $\partial w_T/\partial\rho_T = -\alpha w_T/\rho_T$ into this equation, we obtain

$$g_T - g_T^o = (\tau-\alpha)w_T(1-\varepsilon) - (\tau+z)B \tag{C5}$$

Substituting (C5) into $(1-\tau)w_T(1-\varepsilon)+g_T = (1-\tau)B = g_T^o$, we obtain

$$w_T^* = \frac{B(1+z)}{(1-\alpha)(1-\varepsilon)} \tag{C6}$$

and

$$g_T^* - g_T^{o*} = -\frac{(\alpha+z)B(1-\tau)}{1-\alpha} < 0 \tag{C7}$$

Since $w_T = (1-\alpha)Ak_T^\alpha[(1-\varepsilon)\rho_T]^{-\alpha}$, the urban population ratio is given by

$$\rho_T^* = \left[\frac{(1-\alpha)A}{w_T^*}\right]^{\frac{1}{\alpha}}\frac{k_T}{1-\varepsilon} \tag{C8}$$

Finally, (C7) and the government budget constraint yield

$$
\begin{aligned}
g_T^* &= B\left[\frac{\tau(1+z\rho_T^*)-\alpha(1-\rho_T^*)}{1-\alpha}-z(\rho_T^*-\rho_{T-1})\right] && \text{(C9)}\\
g_T^{o*} &= B\left[\tau\left(1+\frac{z\rho_T^*}{1-\alpha}\right)+\frac{\alpha\rho_T^*}{1-\alpha}-z(\rho_T^*-\rho_{T-1})\right] && \text{(C10)}
\end{aligned}
$$

Using (C8), the interest rate in period T is given by

$$
R_T^* = \alpha A k_T^{\alpha-1}[(1-\varepsilon)\rho_T^*]^{1-\alpha} = \alpha A\left[\frac{A(1-\alpha)^2(1-\varepsilon)}{B(1+z)}\right]^{\frac{1-\alpha}{\alpha}} \tag{C11}
$$

(ii) Period $T-1$

The political maximization problem is formulated as follows:

$$
\max_{g_{T-1},\, g_{T-1}^o,\, \rho_{T-1}} W_{T-1} = (\theta_{T-1}+\theta_{T-1}^m)\ln[(1-\tau)w_{T-1}(1-\varepsilon)+g_{T-1}]+(1-\theta_{T-1}-\theta_{T-1}^m)\ln[(1-\tau)B+g_{T-1}^o]+\beta\ln R_T^*
$$

subject to

$$
\begin{aligned}
\tau[\rho_{T-1}w_{T-1}(1-\varepsilon)+(1-\rho_{T-1})B] &= \rho_{T-1}g_{T-1}+(1-\rho_{T-1})g_{T-1}^o+(\rho_{T-1}-\rho_{T-2})Bz\\
(1-\tau)w_{T-1}(1-\varepsilon)+g_{T-1} &= (1-\tau)B+g_{T-1}^o
\end{aligned}
$$

Eq.(C11) implies that R_T^* is independent of the policy variables in period $T-1$. Therefore, the policy functions in period $T-1$ are given by Eqs.(C6), (C8), (C9), and (C10) by replacing the time script T with $T-1$.

(iii) Period $t \le T-1$

By mathematical induction, we obtain

$$
\begin{aligned}
w_t^* &= \frac{B(1+z)}{(1-\alpha)(1-\varepsilon)} && \text{(C12)}\\
\rho_t^* &= \left[\frac{(1-\alpha)A}{w_t^*}\right]^{\frac{1}{\alpha}}\frac{k_t}{1-\varepsilon} && \text{(C13)}\\
g_t^* &= B\left[\frac{\tau(1+z\rho_t^*)-\alpha(1-\rho_t^*)}{1-\alpha}-z(\rho_t^*-\rho_{t-1})\right] && \text{(C14)}\\
g_t^{o*} &= B\left[\tau\left(1+\frac{z\rho_t^*}{1-\alpha}\right)+\frac{\alpha\rho_t^*}{1-\alpha}-z(\rho_t^*-\rho_{t-1})\right] && \text{(C15)}
\end{aligned}
$$

(iv) Let $T\to\infty$. Policy functions in the Markov perfect equilibrium is given by Eqs.(C12) - (C15).

Appendix D

[Urban cost]

(i) Period T

The political maximization problem in period T is formulated as

$$
\max_{g_T,\, g_T^o,\, \rho_T} W_T = \theta_T\ln[(1-\tau)I_T+g_T]+(1-\theta_T)\ln[(1-\tau)B+g_T^o]
$$

subject to

$$
\begin{aligned}
\tau[\rho_T I_T+(1-\rho_T)B] &= \rho_T g_T+(1-\rho_T)g_T^o && \text{(D1)}\\
(1-\tau)I_T+g_T &= (1-\tau)B+g_T^o && \text{(D2)}
\end{aligned}
$$

where

$$
I_T = (1-\alpha)Ak_T^\alpha[1-\varepsilon(\rho_T)]^{1-\alpha}\rho_T^{-\alpha} \tag{D3}
$$

The Lagrangian function is given by

$$
\begin{aligned}
\Phi_T = {} & \theta_T\ln[(1-\tau)I_T+g_T]+(1-\theta_T)\ln[(1-\tau)B+g_T^o]\\
&+\lambda_T\{\tau[\rho_T I_T+(1-\rho_T)B]-\rho_T g_T-(1-\rho_T)g_T^o\}\\
&+\mu_T[(1-\tau)I_T+g_T-(1-\tau)B-g_T^o]
\end{aligned}
$$

The first-order conditions require

$$
\begin{aligned}
\frac{\theta_T}{(1-\tau)I_T + g_T} - \lambda_T \rho_T + \mu_T &= 0 && \text{(D4)}\\
\frac{1-\theta_T}{(1-\tau)B + g_T^o} - \lambda_T(1-\rho_T) - \mu_T &= 0 && \text{(D5)}
\end{aligned}
$$

and

$$
\frac{\theta_T(1-\tau)}{(1-\tau)I_T + g_T}\frac{\partial I_T}{\partial \rho_T} + \lambda_T\left\{\tau(I_T - B) - (g_T - g_T^o) + \tau\rho_T\frac{\partial I_T}{\partial \rho_T}\right\} + \mu_T(1-\tau)\frac{\partial I_T}{\partial \rho_T} = 0 \qquad \text{(D6)}
$$

Using (D2), (D4) and (D5) yield

$$
\frac{1}{(1-\tau)I_T + g_T} = \lambda_T
$$

Using (D4), the coefficient of $\partial I_T/\partial \rho_T$ is given by

$$
\frac{\theta_T(1-\tau)}{(1-\tau)I_T + g_T} + \lambda_T\tau\rho_T + \mu_T(1-\tau) = \lambda_T\rho_T
$$

Therefore, (D6) yields

$$
g_T - g_T^o = \tau(I_T - B) + \rho_T\frac{\partial I_T}{\partial \rho_T} \qquad \text{(D7)}
$$

Combining (D2) and (D7) yield

$$
I_T + \rho_T\frac{\partial I_T}{\partial \rho_T} = B \qquad \text{(D8)}
$$

Using (D3), we obtain

$$
\begin{aligned}
\frac{\partial I_T}{\partial \rho_T} &= -\alpha\frac{I_T}{\rho_T} - (1-\alpha)\frac{I_T}{1-\varepsilon_T}\frac{d\varepsilon_T}{d\rho_T}\\
&= -\frac{I_T}{\rho_T}\left[\alpha + (1-\alpha)\frac{\varepsilon_T}{1-\varepsilon_T}\sigma_T\right]
\end{aligned}
$$

where $\sigma_T = (\rho_T/\varepsilon_T)(d\varepsilon_T/d\rho_T)$ represents the elasticity of the urban cost with respect to the population ratio.

Substituting this into (D8), we obtain

$$
I_T = \frac{B}{(1-\alpha)\left(1 - \frac{\varepsilon_T}{1-\varepsilon_T}\sigma_T\right)} \qquad \text{(D10)}
$$

If $\varepsilon(\rho_T) = \varepsilon$, then Eq.(D10) is equal to Eq.(A10). Combining (D3) and (D10), we obtain the urban population ratio ρ_T^* as a function of k_T.

We derive a class of cost functions which have characteristics similar to the basic model. From (D10), the wage rate is given by

$$
w_T = \frac{B}{(1-\alpha)\left(1 - \varepsilon_T - \rho_T\frac{d\varepsilon_T}{d\rho_T}\right)}
$$

Suppose that the right-hand side is constant. Then, the capital-labor ratio in the urban region, $k_T/[(1-\varepsilon_T)\rho_T]$, is constant, which implies that the interest rate, R_T, is also constant. Then, we can obtain the Markov perfect equilibrium in the same way as the basic model.

Let $\varepsilon(\rho) + \rho\varepsilon'(\rho) = e$ $(0 < e < 1)$. Then, $\varepsilon'(\rho)/[\varepsilon(\rho) - e] = -\rho^{-1}$. Integrating both sides, we obtain $\ln|\varepsilon(\rho) - e| = -\ln\rho + M$, where M is a constant of integration. Denoting $m = e^M > 0$, the general solution is given by $|\varepsilon(\rho) - e| = m/\rho$. Focusing on increasing functions, we obtain

$$
\varepsilon(\rho) = e - \frac{m}{\rho}, \quad \left(\frac{m}{e} \leq \rho < 1\right) \qquad \text{(D11)}
$$

Under the cost function in (D11), the wage rate is given by

$$
w_T = \frac{B}{(1-\alpha)(1-e)}
$$

Using $[1-\varepsilon(\rho_T)]\rho_T = (1-c)\rho_T + m$, the urban population ratio is explicitly given by

$$\rho_T^* = \left[\frac{A}{B}(1-\alpha)^2(1-e)^{1-\alpha}\right]^{\frac{1}{\alpha}} k_T - \frac{m}{1-e} \tag{D12}$$

which shows that the interest rate is constant,

$$R_T^* = \alpha A k_T^\alpha [(1-\varepsilon(\rho_T^*))\rho_T^*]^{-\alpha} = \alpha A \left[\frac{A}{B}(1-\alpha)^2(1-e)\right]^{\frac{1-\alpha}{\alpha}} \tag{D13}$$

Finally, the redistribution policies in period T are given by

$$\begin{aligned} g_T^* &= \tau I_T^* - (1-\rho_T^*)(I_t^* - B) \\ g_T^{o*} &= \tau B + \rho_T^*(I_T^* - B) \end{aligned}$$

where

$$I_T^* = \frac{B[1-\varepsilon(\rho_T^*)]}{(1-\alpha)(1-e)} = \frac{B}{1-\alpha}\left[1 + \frac{m}{(1-e)\rho_T^*}\right]$$

If $m = 0$, g_T^* and g_T^{o*} are equal to Eqs.(A13) and (A14), respectively.

(ii) Period $t \leq T-1$

The political maximization problem in period $T-1$ is formulated as

$$\max_{g_{T-1},\, g_{T-1}^o,\, \rho_{T-1}} W_{T-1} = \theta_{T-1} \ln[(1-\tau)I_{T-1} + g_{T-1}] + (1-\theta_{T-1})\ln[(1-\tau)B + g_{T-1}^o] + \beta \ln R_T^*$$

subject to

$$\begin{aligned} \tau[\rho_{T-1}I_{T-1} + (1-\rho_{T-1})B] &= \rho_{T-1}g_{T-1} + (1-\rho_{T-1})g_{T-1}^o \\ (1-\tau)I_{T-1} + g_{T-1} &= (1-\tau)B + g_{T-1}^o \end{aligned}$$

where

$$I_{T-1} = (1-\alpha)Ak_{T-1}^\alpha[1-\varepsilon(\rho_{T-1})]^{1-\alpha}\rho_{T-1}^{-\alpha}$$

If the cost function is specified by Eq.(D11), then R_T^* is given by Eq.(D13). Therefore, the policy functions in period $T-1$ are of the same form as in period $T-1$. The same is true for all $t \leq T-1$.

Appendix E

[Pareto efficiency]

Following King and Ferguson (1993), and Wigger (1999), the Pareto efficient allocation in our OLG economy is characterized by the solution of the following problem:

$$\max_{c_{1t}, c_{2t+1}, c_{1t}^o, c_{2t+1}^o, N_t, N_t^o, K_{t+1}} u(c_{1t}, c_{2t+1})$$

subject to

$$\bar{u}_t = u^o(c_{1t}^o, c_{2t+1}^o) \tag{E1}$$

$$N_t + N_t^o = \bar{N} \tag{E2}$$

$$F(K_t, N_t(1-\varepsilon)) + BN_t^o = N_t c_{1t} + N_t^o c_{1t}^o + N_{t-1}c_{2t} + N_{t-1}^o c_{2t}^o + K_{t+1} \tag{E3}$$

$$F(K_{t+1}, N_{t+1}(1-\varepsilon)) + BN_{t+1}^o = N_{t+1}c_{1t+1} + N_{t+1}^o c_{1t+1}^o + N_t c_{2t+1} + N_t^o c_{2t+1}^o + K_{t+2} \tag{E4}$$

taking $\bar{u}_t$, K_t, N_{t-1}, N_{t-1}^o, c_{2t}, c_{2t}^o, N_{t+1}, N_{t+1}^o, c_{1t+1}, c_{1t+1}^o, and K_{t+2} as given.

Eq.(E1) is a utility constraint for rural residents and Eq.(E2) is the population constraint. Eqs.(E3) and (E4) are the resource constraints in period t and period $t+1$, respectively.

In the following, we use per capita variables. Denote the urban population share in period t by $\rho_t = N_t/\bar{N}$. The rural share is $\rho_t^o = 1-\rho_t$. We denote per capita capital and per capita urban output by $k_t = K_t/\bar{N}$ and $y_t = Y_t/\bar{N}$, respectively. Assuming that urban technology is constant-returns-to-scale, we obtain $y_t = F(k_t, \rho_t(1-\varepsilon)) \equiv f(k_t, \rho_t)$.

Let us set up the Lagrangian,

$$
\begin{aligned}
\Phi_t \;=\;& u(c_{1t}, c_{2t+1}) + \lambda_t[u^o(c^o_{1t}, c^o_{2t+1}) - \bar{u}_t] + \mu_t(1 - \rho_t - \rho^o_t) \\
& + \gamma_t \left[f(k_t, \rho_t) + B\rho^o_t - \rho_t c_{1t} - \rho^o_t c^o_{1t} - \rho_{t-1} c_{2t} - \rho^o_{t-1} c^o_{2t} - k_{t+1}\right] \\
& + \gamma_{t+1} \left[f(k_{t+1}, \rho_{t+1}) + B\rho^o_{t+1} - \rho_{t+1} c_{1t+1} - \rho^o_{t+1} c^o_{1t+1} - \rho_t c_{2t+1} - \rho^o_t c^o_{2t+1} - k_{t+2}\right]
\end{aligned}
$$

where λ_t and μ_t represent the multiplies attached to the utility constraint for rural residents, and to the population constraint, respectively. γ_t represents the multiplier attached to the resource constraint in period t.

The first-order conditions require

$$
\begin{aligned}
\frac{\partial \Phi_t}{\partial c_{1t}} &= \frac{\partial u_t}{\partial c_{1t}} - \gamma_t \rho_t = 0 \\
\frac{\partial \Phi_t}{\partial c_{2t+1}} &= \frac{\partial u_t}{\partial c_{2t+1}} - \gamma_{t+1} \rho_t = 0 \\
\frac{\partial \Phi_t}{\partial c^o_{1t}} &= \lambda_t \frac{\partial u^o_t}{\partial c^o_{1t}} - \gamma_t \rho^o_t = 0 \\
\frac{\partial \Phi_t}{\partial c^o_{2t+1}} &= \lambda_t \frac{\partial u^o_t}{\partial c^o_{2t+1}} - \gamma_{t+1} \rho^o_t = 0 \\
\frac{\partial \Phi_t}{\partial \rho_t} &= -\mu_t + \gamma_t \left(f_{\rho,t} - c_{1t}\right) - \gamma_{t+1} c_{2t+1} = 0 \\
\frac{\partial \Phi_t}{\partial \rho^o_t} &= -\mu_t + \gamma_t \left(B - c^o_{1t}\right) - \gamma_{t+1} c^o_{2t+1} = 0 \\
\frac{\partial \Phi_t}{\partial k_{t+1}} &= -\gamma_t + \gamma_{t+1} f_{k,t+1} = 0
\end{aligned}
$$

Let us denote the marginal rate of substitution between current and future consumption of urban residents by $MRS_t = (\partial u_t/\partial c_{1t})/(\partial u_t/\partial c_{2t+1})$, and that of rural residents by $MRS^o_t = (\partial u^o_t/\partial c^o_{1t})/(\partial u^o_t/\partial c^o_{2t+1})$. Then, the Pareto-efficient conditions are summarized by the following three equations,

$$MRS_t = f_{k,t+1} \tag{E5}$$

$$MRS^o_t = f_{k,t+1} \tag{E6}$$

$$f_{\rho,t} - c_{1t} - \frac{c_{2t+1}}{f_{k,t+1}} = B - c^o_{1t} - \frac{c^o_{2t+1}}{f_{k,t+1}} \tag{E7}$$

In a decentralized economy, the competitive capital market yields $R_{t+1} = f_{k,t+1}$, which implies that household optimization behavior results in Eqs.(E5) and (E6). The left-hand side of (E7) represents the net marginal product of the urban population. Adding one person into the urban region, they supply $(1 - \varepsilon)$ units of labor, which increases urban output by $F_{L,t}(1 - \varepsilon) = f_{\rho,t}$. The additional output is reduced by $c_{1t} + c_{2t+1}/R_{t+1}$ because they consume as an urban resident. Similarly, the right-hand side of (E7) represents the net marginal product of the rural population. If the left-hand side is larger than the right-hand side, relocating population from rural to the urban region is Pareto-improving. Eq.(E7) implies that the net marginal products in both regions are equalized to each other at the optimum.

Table 1. Regional difference in the fiscal dependency rate

	Super Mega-Region	Other prefectures
Sample	23	24
Mean	28.81	45.32
99% confidence interval	[22.96, 34.66]	[41.77, 48.87]

Figure 1. Fiscal dependency of local governments in Japan (2016)

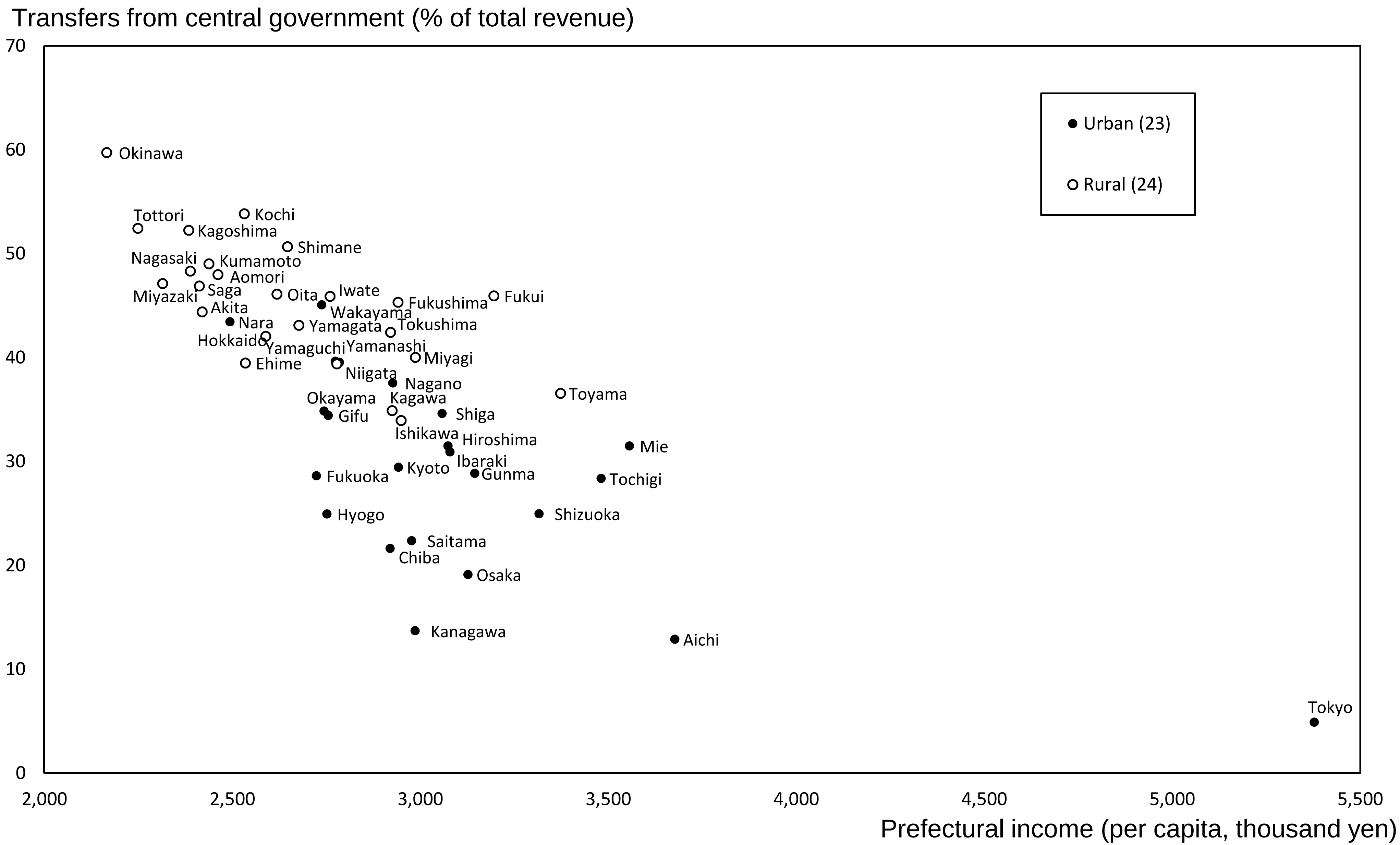


Figure 2. Temporary equilibrium

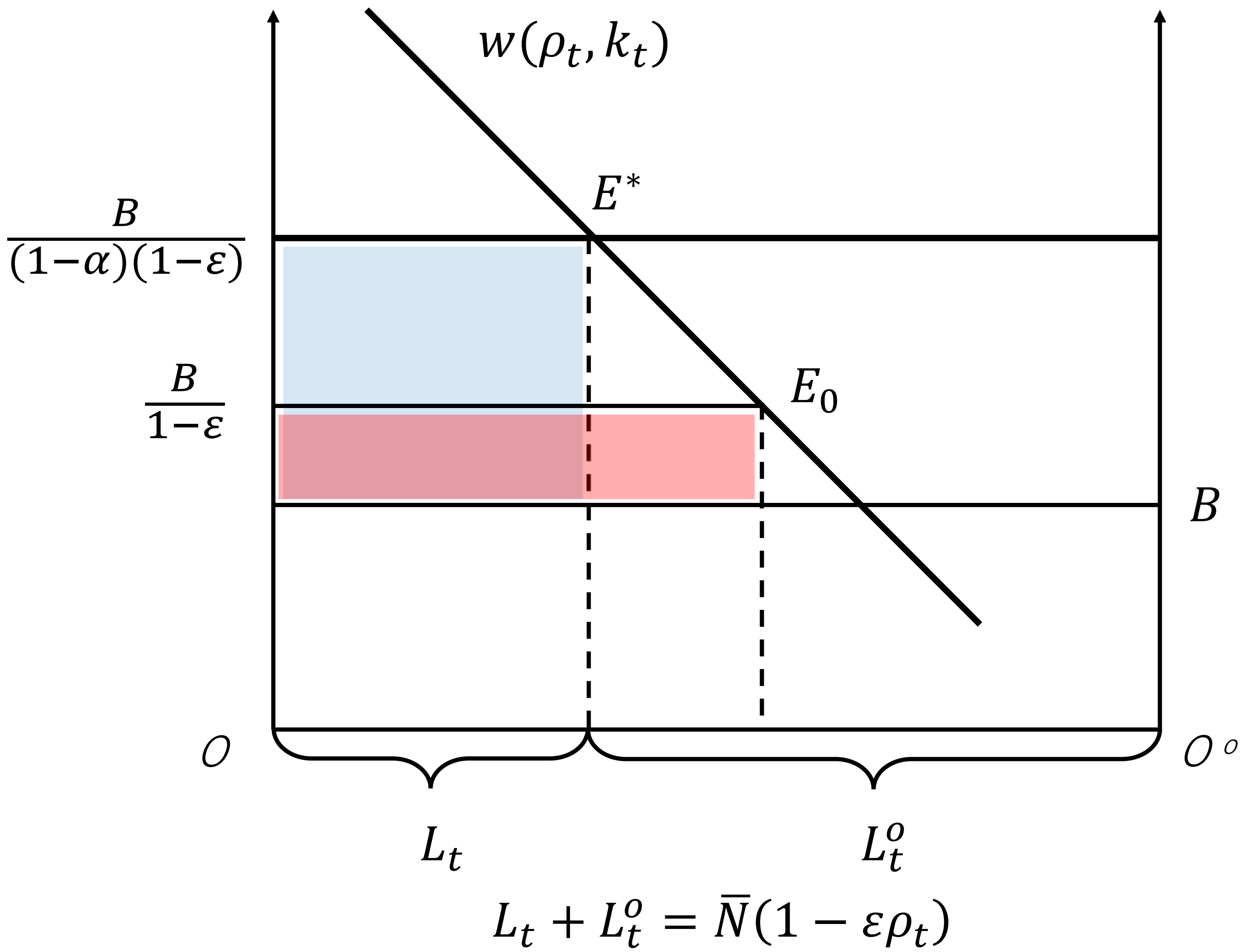


Figure 3. Dynamics of migration

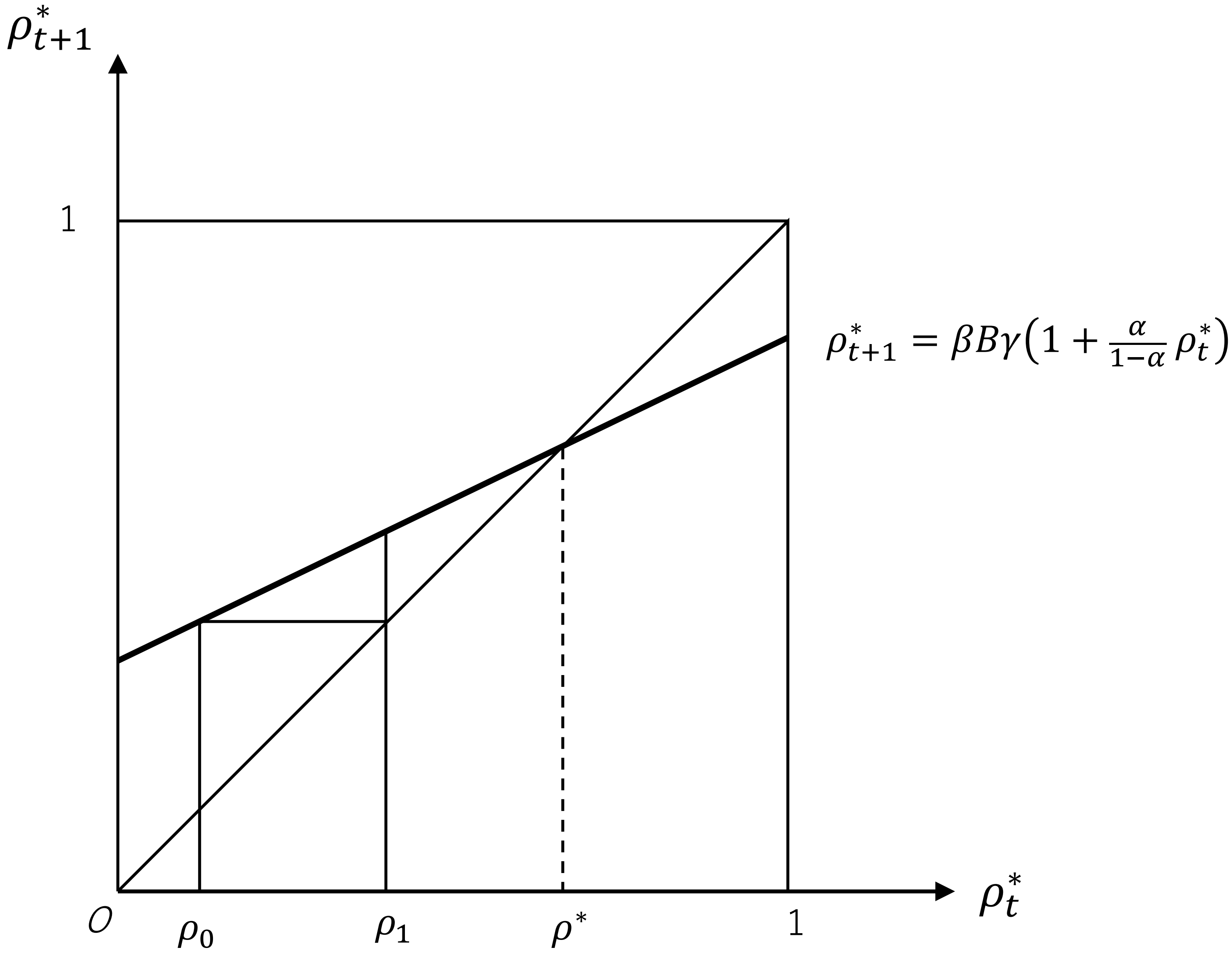
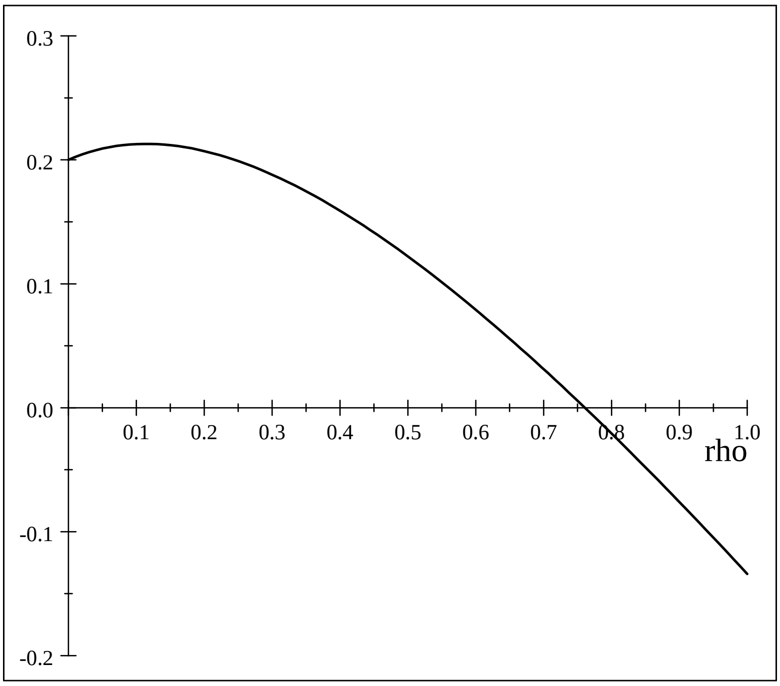
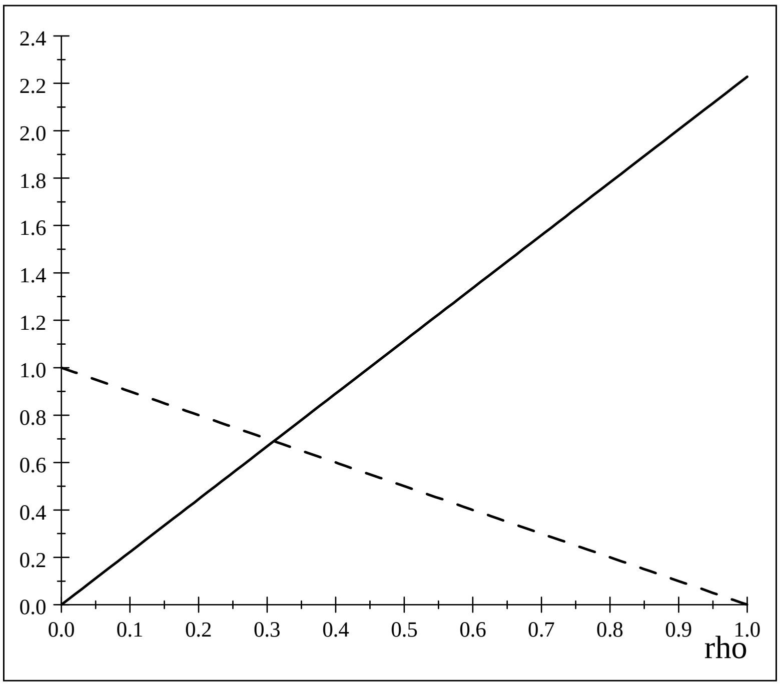


Figure 4. Interregional difference in transfers (% of GDP)



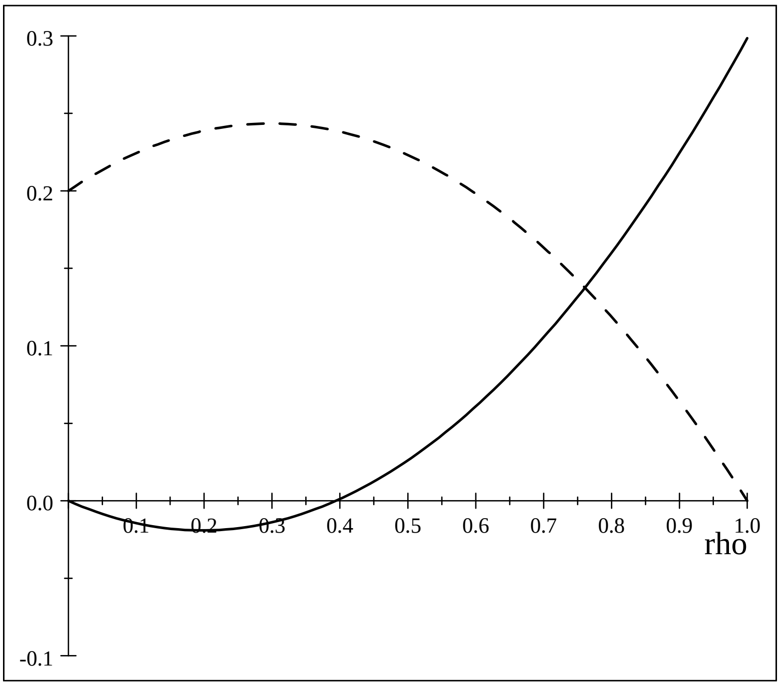
Note. The horizontal axis represents the urban population ratio ρ_t^*, and the vertical axis represents the ratio of interregional difference in transfers to GDP, $(N_t^o g_t^{o*} - N_t g_t^*)/(Y_t + Y_t^o)$. We assume the capital share is $\alpha = 0.33$, and the tax rate is $\tau = 0.2$.

Figure 5. Regional outputs



Note. The horizontal axis represents the urban population ratio ρ_t^*. The solid line represents the urban output, Y_t, and the dashed line the rural output, Y_t^o. We assume $\alpha = 0.33$, $\tau = 0.2$, $B = 1$, $\bar{N} = 1$.

Figure 6. Regional transfers



Note. The horizontal axis represents the urban population ratio ρ_t^*. The solid curve represents the transfers to urban residents, $N_t g_t^*$, and the dashed curve the one to rural residents, $N_t^o g_t^{o*}$. We assume $\alpha = 0.33$, $\tau = 0.2$, $B = 1$, $\bar{N} = 1$.