Higher education and the polarization of wages

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Abstract

In some countries, labor market polarization is a remarkable phenomenon driven by skill-biased technological change (SBTC) or globalization. The SBTC determines the occupational employment of the workers, which depends on their education and governs their wages. Hence, the income inequality arising from this polarization is attributed to education.

In this context, this study explains the mechanism of wage polarization based on a model comprising unskilled, routine-skilled, and non-routine-skilled laborers classified by their choice of higher education. Our result suggests that the polarization of wages could be attributed to a sizable reduction in the cost of college education relative to the cost of postgraduate education.

JEL Classification: I23, I24, J24, J42, O15

Keywords: Wage inequality, Higher education, Skill distribution, Skill-biased technological change

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1 Introduction

Polarization has been considered a remarkable phenomenon in the US and some European labor market since the 1990s (Autor, Katz, and Kearney 2006; Goldin and Katz 2007; Lemieux 2008; Goos, Manning, and Salomons 2009). Some research successfully explains the labor market polarization in the context of the skill-biased technological change (Autor and Dorn 2013; Cortes, Jaimovich, and Siu 2017; Cavenaile 2021) or in the context of globalization or offshoring (Beladi, Marjit, and Broll 2011; Cavenaile 2021).

In this context, this study explains the polarization of wages in the context of higher education, which has not been sufficiently analyzed. Autor (2014) points out that between 1980 and 2012, the real hourly earnings of full-time college-educated US males rose anywhere from 20% to 56%, with the greatest gains going to those with a postbaccalaureate degree. Hoffmann, Lee, and Lemieux (2020) calculate the contribution rates of education and other factors to the growth in the variance of total income. They conclude that education accounted for 56% of the growth in income inequality from the late 1970s to the late 2010s in the United States. In the United Kingdom and Germany, the contribution rate of education accounted for approximately 50%, though the sample periods for these countries have been shorter than those for in the United States. This empirical evidence calls for further research in education, especially in higher education, to explain the dynamics of wage distribution.

Hence, this study examines wage polarization in the context of education. To this end, it constructs a model comprising three types of labor (unskilled, routine-skilled, and non-routine-skilled). They are classified based on their choice of higher education—high-school graduates employed as unskilled labor in the services sector, general graduate degree holders employed as routine-skilled labor in the final consumption goods sector, and postgraduate degree holders employed as non-routine-skilled labor in the intermediate goods sector. In this model economy, we investigate the factors that influence the differential in wages among the unskilled, routine-skilled, and non-routine-skilled workers.

We obtain two results. First, a decrease in the production share of the non-routine-skilled labor leads to a respective increase and decrease in the top-end and low-end wage inequalities. This result is consistent with the literature attributing the labor market polarization to a skill-biased technological change (Autor and Dorn 2013). Second, and more importantly, a decrease in the cost of college education leads to a respective increase and decrease in the top-end and low-end wage inequalities. The top-end wage inequality increases because the increased relative supply of routine-skilled labor reduces the relative wage rate (Johnson 1997). The low-end wage inequality decreases because of an increased service demand from high-income individuals, which drives up the wage rate of the unskilled labor (Manning 2004; Mazzolari and Ragusa 2013). Our result highlights the important role of higher education in shaping the wage distribution, as pointed out by Autor (2014) and Hoffmann, Lee, and Lemieux (2020).

The reminder of this paper is organized as follows. Section 2 introduces the basic model. Section 3 presents the derivation of a general formula explicitly indicating the relationship between the top-end and low-end wage inequalities. Section 4 provides a numerical example of a case where skills are distributed uniformly. Section 5 concludes the paper.

2 The model

The economy consists of three production sectors (services, final consumption goods, and intermediate goods) (See Table 1). Individuals are heterogeneous with respect to their inherent effective labor or human capital, which determines the choice of higher education. Labor markets are perfectly competitive and assumed to be segmented by educational status. Profits in the intermediate goods sector are allocated among individuals as dividends. Since this study emphasizes wage inequality, we assume that firms in the intermediate goods sector can access the international capital markets. This assumption implies that capital income inequality is beyond this study's scope.

[Table 1 here]

2.1 Individuals

Total population is constant and normalized to unity. Individuals are heterogenous in their effective labor, which is denoted by $h \in [0, 1]$. The cumulative distribution function is denoted by $F(h)$.

The utility function of an individual whose effective labor is h is given by

$$u(h) = (1 - \alpha) \ln c_g(h) + \alpha \ln c_s(h)$$

where $c_g(h)$ and $c_s(h)$ represent individual h's consumption of goods and services, respectively. $0 < \alpha < 1$ is a preference parameter attached to services.

The budget constraint is given by

$$y(h) + \omega(h)\Pi = c_g(h) + p_s c_s(h)$$

where $y(h)$ represents individual h's labor income, net of the cost of education (details are explained below). p_s and Π represent the price of services and the dividends accrued from the intermediate goods sector, respectively. $\omega(h)$ is the ownership ratio of individual h, which satisfies

$$\int_0^1 \omega(h) dF(h) = 1$$

Solving the utility maximization problem, the demand for goods and services for individual h are, respectively, given by

$$\begin{aligned} c_g(h) &= (1-\alpha)[y(h) + \omega(h)\Pi] \\ c_s(h) &= \alpha \frac{y(h) + \omega(h)\Pi}{p_s} \end{aligned} \tag{1}$$

The net labor income is defined by

$$y(h) = \begin{cases} w_L h & & \text{unskilled labor } (L) \\ w_M h - e_1 & \text{if employed as} & \text{routine-skilled labor } (M) \\ w_H h - e_1 - e_2 & & \text{non-routine-skilled labor } (H) \end{cases} \tag{2}$$

where w_L, w_M, and w_H represent the wage rates of the unskilled, routine-skilled, and non-routine-skilled laborers, respectively. We assume, in equilibrium, $w_H > w_M > w_L$. e_1 and e_2 represent the costs of college and postgraduate education, respectively; they are specified by

$$e_1 = \theta_1(w_M - w_L), \quad e_2 = \theta_2(w_H - w_M), \quad 0 < \theta_1 < \theta_2 < 1 \tag{3}$$

Solving the income maximization problem with Eqs. (2) and (3), we obtain

$$y(h) = \begin{cases} y_L(h) = w_L h & & 0 < h \leq \theta_1 \\ y_M(h) = w_M h - e_1 & \text{if} & \theta_1 \leq h \leq \theta_2 \\ y_H(h) = w_H h - e_1 - e_2 & & \theta_2 \leq h \leq 1 \end{cases} \tag{4}$$

[Figure 1 here]

Figure 1 illustrates the optimal choice of education. Low-skill individuals ($h < \theta_1$) do not go to college and work as unskilled workers. Individuals in the intermediate range ($\theta_1 < h < \theta_2$) go to college and work as routine-skilled workers. High-skill individuals ($\theta_2 < h$) obtain a postgraduate professional degree and work as non-routine-skilled workers.

The cost structure in Eq. (3) is not specific. From Eqs. (3) and (4), we obtain the net rates of return on college and postgraduate education, which are given by

$$\begin{aligned} \frac{y_M(h) - y_L(h)}{w_L h} &= \left(\frac{w_M}{w_L} - 1\right)\left(1 - \frac{\theta_1}{h}\right) \\ \frac{y_H(h) - y_M(h)}{w_M h} &= \left(\frac{w_H}{w_M} - 1\right)\left(1 - \frac{\theta_2}{h}\right) \end{aligned}$$

respectively. The net rate of return on college education is positive for individual $h > \theta_1$ and increasing in h. The net rate of return on postgraduate education is positive for individual $h > \theta_2$ and increasing in h. Therefore, Eq. (3) enables us to link the differences in effective labor (human capital) directly to the differences in the net rate of return on higher education among individuals.

From Eq. (4), the supply of the unskilled, routine-skilled, and non-routine-skilled laborers, respectively, is given by

$$
\begin{aligned}
L^s &= \int_0^{\theta_1} h dF(h) \\
M^s &= \int_{\theta_1}^{\theta_2} h dF(h) \\
H^s &= \int_{\theta_1}^{1} h dF(h)
\end{aligned}
$$

The numbers of the unskilled, routine skilled, and non-routine-skilled workers are given by $n_L = F(\theta_1)$, $n_M = F(\theta_2) - F(\theta_1)$, and $n_H = 1 - F(\theta_2)$, respectively.

2.2 Firms

2.2.1 Services sector

For simplicity, we assume a linear technology of the services sector,

$$Y_s = f(L) = L \tag{5}$$

where L is unskilled labor, and Y_s is the output of services.

Perfect competition makes the wage rate of the unskilled labor equal to the price of services,

$$w_L = p_s \tag{6}$$

2.2.2 Final goods sector

The final consumption goods are produced by the routine-skilled labor and composite goods including intermediate goods. Following Dixit and Stiglitz (1977), the technology is specified by

$$
\begin{aligned}
Y_g &= M^{1-\beta} Z^{\beta} \\
Z &= \left[\int_0^{\phi} x(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}
\end{aligned}
$$

where Y_g is the output of final goods, and M and Z represent the inputs of the routine-skilled labor and composite goods, respectively. $x(j)$ represents the input of the intermediate good $j \in [0, \phi]$. The variety of intermediate goods $\phi > 0$ is assumed to be constant. $0 < \beta < 1$ is a constant production share of composite goods, and $\sigma > 1$ is a constant elasticity of substitution between the different intermediate goods.

The optimization problem is divided into two parts. First, firms minimize the expenditure for intermediated goods, taking the level of composite goods as given,

$$e(Z) = \min_{x(j)} \int_0^{\phi} p(j) x(j) dj \quad \text{subject to} \quad Z = \left[\int_0^{\phi} x(j)^{\frac{\sigma-1}{\sigma}} dj \right]^{\frac{\sigma}{\sigma-1}}$$

where $p(j)$ is the price of the intermediate good j, and $e(Z)$ is the expenditure function.

Solving the problem, the demand for the intermediate good j is given by

$$x(j) = Z \left[\frac{P}{p(j)} \right]^{\sigma} \tag{7}$$

where P represents the price index

$$P = \left[\int_0^{\phi} p(j)^{1-\sigma} dj \right]^{\frac{1}{1-\sigma}} \tag{8}$$

Eqs. (7) and (8) yield $e(Z) = PZ$, which implies that P represents the price of the composite goods.

Second, firms minimize the total cost of production, taking the level of output as given,

$$\min_{M,\, Z} w_M M + PZ \quad \text{subject to} \quad Y_g = M^{1-\beta} Z^{\beta}$$

where w_M is the wage rate of the routine-skilled labor.

The optimality condition is given by

$$\frac{w_M}{P} = \frac{1-\beta}{\beta}\frac{Z}{M} \tag{9}$$

which implies that the factor price ratio is equal to the marginal rate of transformation.

Solving the problem, the factor demand is given by

$$\begin{aligned} M &= \left(\frac{\beta}{1-\beta}\frac{w_M}{P}\right)^{-\beta} Y_g \\ Z &= \left(\frac{\beta}{1-\beta}\frac{w_M}{P}\right)^{1-\beta} Y_g \end{aligned}$$

Finally, the zero-profit condition yields the factor price frontier,

$$1 = \left(\frac{P}{\beta}\right)^{\beta}\left(\frac{w_M}{1-\beta}\right)^{1-\beta} \tag{10}$$

2.2.3 Intermediate goods sector

Intermediate goods are produced by the non-routine-skilled labor and capital. Focusing on the wage inequality, we assume that firms can access the international capital market.

The production technology in sector $j \in [0, \phi]$ is given by

$$x(j) = k(j)^{\gamma} l_H(j)^{1-\gamma}$$

where $x(j)$ is the output, and $k(j)$ and $l_H(j)$ represent the capital inputs and the non-routine-skilled labor, respectively. $0 < \gamma < 1$ is a constant production share of capital.

The optimization problem is divided into two parts. First, a firm in sector j minimizes the cost of production, taking the level of output as given,

$$\min_{k(j),\, l_H(j)} \quad Rk(j) + w_H l_H(j) \quad \text{subject to} \quad x(j) = k(j)^{\gamma} l_H(j)^{1-\gamma}$$

where R is a constant world interest rate, and w_H is the wage rate of the non-routine-skilled labor.

Solving the problem, we obtain the factor demand,

$$\begin{aligned} l_H(j) &= \left(\frac{\gamma}{1-\gamma}\frac{w_H}{R}\right)^{-\gamma} x(j) \\ k(j) &= \left(\frac{\gamma}{1-\gamma}\frac{w_H}{R}\right)^{1-\gamma} x(j) \end{aligned}$$

and the cost function $c(R, w_H)x(j)$, where

$$c(R, w_H) = \left(\frac{R}{\gamma}\right)^{\gamma}\left(\frac{w_H}{1-\gamma}\right)^{1-\gamma}$$

Second, the firm maximizes its profit by considering the factor demand function Eq. (7),

$$\pi(j) = \max_{p(j),\, x(j)} [p(j) - c(R, w_H)]x(j) \quad \text{subject to} \quad x(j) = Z\left[\frac{P}{p(j)}\right]^{\sigma}$$

Solving the problem, the optimal price is given by

$$p(j) = \frac{\sigma}{\sigma - 1} c(R, w_H)$$

for all $j \in [0, \phi]$.

Omitting the index of intermediate good j, the price index P and the output of intermediate good x are given by

$$P = \phi^{\frac{1}{1-\sigma}} \frac{\sigma}{\sigma-1} c(R, w_H) \tag{11}$$

$$x = \phi^{\frac{\sigma}{1-\sigma}} Z \tag{12}$$

respectively.

Finally, the aggregate profit is given by

$$\Pi = \int_0^{\phi} \pi(j) dj = \frac{\phi^{\frac{1}{1-\sigma}} c(R, w_H) Z}{\sigma - 1} \tag{13}$$

2.3 Market equilibrium

The labor market clearing conditions are given by

$$\begin{aligned} L^s &= L \\ M^s &= M \\ H^s &= \int_0^{\phi} l_H(j)dj \end{aligned}$$

and the services market clearing condition is given by

$$Y_s = \int_0^1 c_s(h)dF(h) \tag{14}$$

The goods market clearing condition,

$$Y_g = \int_0^1 c_g(h)dF(h) + (n_M + n_H)\theta_1(w_M - w_L) + n_H\theta_2(w_H - w_M) + R\int_0^{\phi} k(j)dj$$

can be derived by Walas's law.

3 Wage inequality

In this section, we derive the equilibrium wages. In Section 3.1, we analyze the top-end wage inequality, that is, the relative wage rate of the non-routine-skilled to routine-skilled labor. In Section 3.2, we analyze the low-end wage inequality, that is, the relative wage rate of the unskilled to routine-skilled labor.

3.1 Top-end wage inequality

The following proposition summarizes the top-end wage inequality.

Proposition 1 *The top-end wage inequality is given by*

$$\frac{w_H}{w_M} = \frac{\beta(1-\gamma)(\sigma-1)}{(1-\beta)\sigma} \times \frac{M}{H} \tag{15}$$

where $M = \int_{\theta_1}^{\theta_2} hdF(h)$ *and* $H = \int_{\theta_2}^{1} hdF(h)$. *The top-end wage inequality increases with* β, σ, *and* θ_2, *and decreases with* γ *and* θ_1.

Proof. See Appendix. ■

Eq. (15) can be intuitively interpreted by the diagram in Johnson (1997) (Figure 1, p.44), which shows the equilibrium wage premium of the skilled workers. On the one hand, a decrease in the cost of college education, denoted by θ_1, increases the supply of the routine-skilled labor, thereby decreasing their wages. On the other hand, a decrease in the cost of postgraduate education θ_2 increases the supply of the non-routine-skilled labor, thereby decreasing their wages. Therefore, the cost structure of higher education is a critical determiner of wage inequality among the college graduates.

The other comparative statics results are straightforward. If there is a decline in the production share of the non-routine-skilled labor β, then the routine-skilled labor will be displaced by composite goods, thereby decreasing the wage rate of the routine-skilled labor. If there is a decline in the production share of capital γ, then the management of the non-routine-skilled labor will gain more importance, thereby increasing the wage rate of the non-routine-skilled labor. If there is an increase in the elasticity of substitution between the intermediate goods σ, implying increased competitiveness in the intermediate goods sector, then the wage rate of the non-routine-skilled labor will increase because of a decline in the firm's monopoly power.

The following proposition explicitly shows the wage rate of the non-routine-skilled labor.

Proposition 2 *The wage rate of the non-routine-skilled labor is given by*

$$w_H = (1-\gamma)\left[\phi^{\frac{\beta}{\sigma-1}}\frac{\beta(\sigma-1)}{\sigma}\left(\frac{R}{\gamma}\right)^{-\beta\gamma}\left(\frac{M}{H}\right)^{1-\beta}\right]^{\frac{1}{1-\beta\gamma}} \tag{16}$$

which increases with ϕ, σ, *and* θ_2, *and decreases with* R *and* θ_1.

Proof. See Appendix. ■

By combining Eqs. (15) and (16), we obtain the wage rate of the routine-skilled labor, w_M. If there is an expansion in the variety of intermediate goods ϕ, then both w_H and w_M increase because of an increase in the demand for the non-routine-skilled labor. This will also increase the marginal product of the routine-skilled labor. However, the wage ratio w_H/w_M is not affected because the impact of variety expansion on w_M is the same as w_H. If there is a decline in the interest rate R, w_H will increase along the curve of the factor price frontier, but the wage ratio w_H/w_M will not be affected in the similar way as ϕ.

We focus on the effects of β and θ_1 on the top-end wage inequality. If there is a decline in the production share of the routine-skilled labor, then there will be an increase in the top-end wage inequality. This scenario is reflective of the skill-biased technical changes (Autor and Dorn 2013). If there is a decline in the cost of college education, then there will be an increase in the wage inequality among the college graduates. This scenario is consistent with the empirical evidence in Autor (2014).

3.2 Low-end wage inequality

The following proposition summarizes the low-end wage inequality.

Proposition 3 *Assume that*

$$(1-\alpha)L - \alpha(n_M + n_H)\theta_1 > 0$$

Then, the low-end wage inequality is given by

$$\frac{w_L}{w_M} = \frac{\alpha\left\{M - (n_M + n_H)\theta_1 + n_H\theta_2 + \left[\left(1 + \frac{1}{(1-\gamma)(\sigma-1)}\right)H - n_H\theta_2\right]\left(\frac{w_H}{w_M}\right)\right\}}{(1-\alpha)L - \alpha(n_M + n_H)\theta_1} \tag{17}$$

The coefficient of (w_H/w_M) is positive. Therefore, (w_L/w_M) is positively related to (w_H/w_M) irrespective of the skill distribution $F(h)$.

Proof. See Appendix. ■

The positive relationship between w_H and w_L can be interpreted by the spillover effect in Manning (2004), Mazzolari and Ragusa (2013) and Autor and Dorn (2013). Suppose that the wage rate of the non-routine-skilled labor w_H increases by one dollar. Two channels affect the wage rate of the unskilled labor. The first channel is the net labor income. For a high-type individual whose effective labor is $h > \theta_2$, his/her net labor income increases by the net rate of return on graduate education, $h - \theta_2$. Then, he/she will increase the expenditure for services by $\alpha(h - \theta_2)$. Aggregating the expenditure for all $h > \theta_2$ yields $\alpha(H - n_H\theta_2)$, which correspond to the first and third terms in the coefficient of (w_L/w_H) in Eq. (17). When the rich become richer, they increase the demand for services, thereby contributing to an increase in the price of services and the wage rate of the unskilled labor.

The second channel is the dividend accrued from the intermediate goods sector. The markup pricing implies that the average profit in sector j is given by $p(j) - c(R, w_H) = w_H l_H(j)/[(1-\gamma)(\sigma-1)]$. Aggregating the profit for all $j \in [0, \phi]$ yields $\Pi = w_H H/[(1-\gamma)(\sigma-1)]$, which is allocated among individuals proportional to their ownership ratio. Under the homothetic preferences, a one-dollar increase in w_H increases the expenditure for services by $\alpha H/[(1-\gamma)(\sigma-1)]$, which is the second term in the coefficient of w_H/w_M. In addition to the income effect of the increased labor income, the income effect of increased dividends further increases the wage rate of the unskilled labor.[1]

4 Numerical example

This section complements the outcomes of Propositions 1 and 3 by providing a numerical example of uniform distribution—$F(h) = h$, $h \in [0, 1]$. The values in this example should be treated cautiously because the skill distribution is generally skewed to the left, which implies that the uniform distribution could overestimate the effect of the cost of higher education on wage inequality.

[1] Under the homothetic preferences, the allocation of dividends is neutral to the aggregate demand effect. In a case where the marginal propensity to consume services is increasing in income, the prevalence of the performance-related pay such as bonuses and stock options would strengthen our results (Lemieux, MacLeod, and Parent 2009).

The distributions of population and effective labor are given by $(n_L, n_M, n_H) = (\theta_1, \theta_2 - \theta_1, 1 - \theta_2)$ and $(L, M, H) = (\theta_1^2/2, (\theta_2^2 - \theta_1^2)/2, (1 - \theta_2^2)/2)$, respectively. Substituting them into Eqs. (15) and (17), we obtain:

$$
\begin{aligned}
\frac{w_H}{w_M} &= \frac{\beta(1-\gamma)(\sigma-1)}{(1-\beta)\sigma} \times \frac{\theta_2^2 - \theta_1^2}{1-\theta_2^2} \\
\frac{w_L}{w_M} &= \frac{\alpha\,(\theta_2 - \theta_1)}{\frac{1}{2}\theta_1\left[(1+\alpha)\theta_1 - 2\alpha\right]} \left\{ 1 - \frac{1}{2}(\theta_1 + \theta_2) + \frac{\beta(\theta_1 + \theta_2)}{2(1-\beta)\sigma}\left[1 + (1-\gamma)(\sigma - 1)\frac{1-\theta_2}{1+\theta_2}\right]\right\}
\end{aligned}
$$

We need $\theta_1 > 2\alpha/(1+\alpha)$ to make the denominator of w_L/w_M positive. We assume the preference for services $\alpha = 0.2$, the share of composite goods $\beta = 0.5$, the capital share $\gamma = 0.3$, and the elasticity of substitution between intermediate goods $\sigma = 2$. As a benchmark, we assume the cost of college education $\theta_1 = 0.6$ and that of postgraduate education $\theta_2 = 0.95$. In this case, the population distribution is $(n_L, n_M, n_H) = (0.6, 0.395, 0.05)$, and the wage inequalities are given by $w_H/w_M = 1.95$ and $w_L/w_M = 0.45$.

Figure 1 illustrates the relationship between the cost of college education and wage inequality. When θ_1 decreases from 0.6 to 0.55, w_H/w_M increases by 10% (2.15) and w_L/w_M increases by 57% (0.71). An increase in the college enrollment rate results in wage polarization.

Figure 2 illustrates the relationship between the cost of postgraduate education and wage inequality. When θ_2 decreases from 0.95 to 0.94, w_H/w_M decreases by 20% (1.57) and w_L/w_M decreases by 2% (0.44). By combining Figures 1 and 2, one may conjecture that the polarization of wages could be attributed to a sizable reduction in the cost of college education relative to the cost of postgraduate education.

Figure 3 shows the relationship between the degree of the product differentiation of intermediated goods and wage inequality. If the elasticity of substitution between intermediate goods increases from 2 to 3, w_H/w_M will increase by 30% (2.60) and w_L/w_M will decrease by 20% (0.36). The more substitutable the intermediate goods, the higher will be the wage rate of the non-routine-skilled labor, given a smaller markup rate. However, the wage rate of the unskilled labor declines because the decreased profits have a negative income effect on the demand for services. In our model, a decrease in the product differentiation of intermediate goods is good and bad news for the non-routine-skilled and unskilled workers, respectively.

[Figures 2, 3 and 4 here]

5 Conclusions

In a simple model of higher education, we examined the relationship between the top-end and low-end wage inequalities. We showed that the top-end wage inequality increases and the low-end wage inequality decreases with a decline in the production share of the routine-skilled labor or a decline in the cost of college education. Our result suggests that the polarization of wages can be attributed not only to the skill-biased technical change but also to the accessibility to higher education.

Appendix

[Proof of Proposition 1]

On the one hand, Eqs. (9) and (11) yield the aggregate labor income of routine-skilled workers,

$$w_M M = \frac{1-\beta}{\beta}\phi^{\frac{1}{1-\sigma}}\frac{\sigma}{\sigma-1}c(R, w_H)Z$$

On the other hand, the wage income of non-routine-skilled workers in sector j is given by $w_H l_H(j) = (1-\gamma)c(R, w_H)x(j)$. Aggregating this for all $j \in [0, \phi]$ and using Eq. (12), we obtain

$$w_H H = (1-\gamma)\phi^{\frac{1}{1-\sigma}}c(R, w_H)Z \tag{18}$$

Dividing both sides of the two equations, we obtain Eq. (15). [Q.E.D.]

[Proof of Proposition 2]

Substituting Eqs. (11) and (15) into Eq. (10), we obtain

$$1 = \left[\phi^{\frac{1}{1-\sigma}}\frac{\sigma}{\beta(\sigma-1)}c(R, w_H)\right]^{\beta}\left[\frac{\sigma}{\beta(\sigma-1)}\frac{w_H}{1-\gamma}\frac{H}{M}\right]^{1-\beta}$$

Using $c(R, w_H) = (R/\gamma)^{\gamma}\left[w_H/(1-\gamma)\right]^{1-\gamma}$, we obtain Eq. (16). [Q.E.D.]

[Proof of Proposition 3]

Substituting Eq. (1) into Eq. (14) and using Eqs. (5) and (6), we obtain

$$L = \frac{\alpha}{w_L}\left[\int_0^1 y(h)dF(h) + \Pi\right]$$

On the one hand, the aggregate labor income, net of the cost of higher education, is given by

$$\int_0^h y(h)dF(h) = w_L L + w_M M + w_H H - (n_M + n_H)\theta_1(w_M - w_L) - n_H\theta_2(w_H - w_M)$$

On the other hand, Eqs. (13) and (18) yield

$$\Pi = \frac{w_H H}{(1-\gamma)(\sigma-1)}$$

Substituting them into the above equation, we obtain

$$w_L L = \alpha\left[w_L L + w_M M + w_H H - (n_M + n_H)\theta_1(w_M - w_L) - n_H\theta_2(w_H - w_M) + \frac{w_H H}{(1-\gamma)(\sigma-1)}\right]$$

Rearranging the terms, we obtain Eq. (17).

For proving that w_L/w_M is positively related to w_H/w_M, it is sufficient to show that $H - n_H\theta_2 > 0$. Let us define a function,

$$\varphi(\theta_2) = H - n_H\theta_2 = \int_{\theta_2}^1 hdF(h) - [1 - F(\theta_2)]\theta_2$$

Obviously, $\varphi(1) = 0$. Differentiating φ with respect to θ_2 and denoting the density function by $f(h)$, we obtain

$$\varphi'(\theta_2) = -\theta_2 f(\theta_2) - [1 - F(\theta_2)] + \theta_2 f(\theta_2) = F(\theta_2) - 1 < 0$$

because $n_H = 1 - F(\theta_2) > 0$. Therefore, we obtain $\varphi(\theta_2) > 0$ for all $\theta_2 < 1$. [Q.E.D.]

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Table 1. Production sectors

Output	Inputs	
Services	Unskilled (L)	
Final consumption goods	Routine-skilled (M)	Composite goods
Intermediate goods	non-routine-skilled (H)	Capital

Figure 1. Education choice

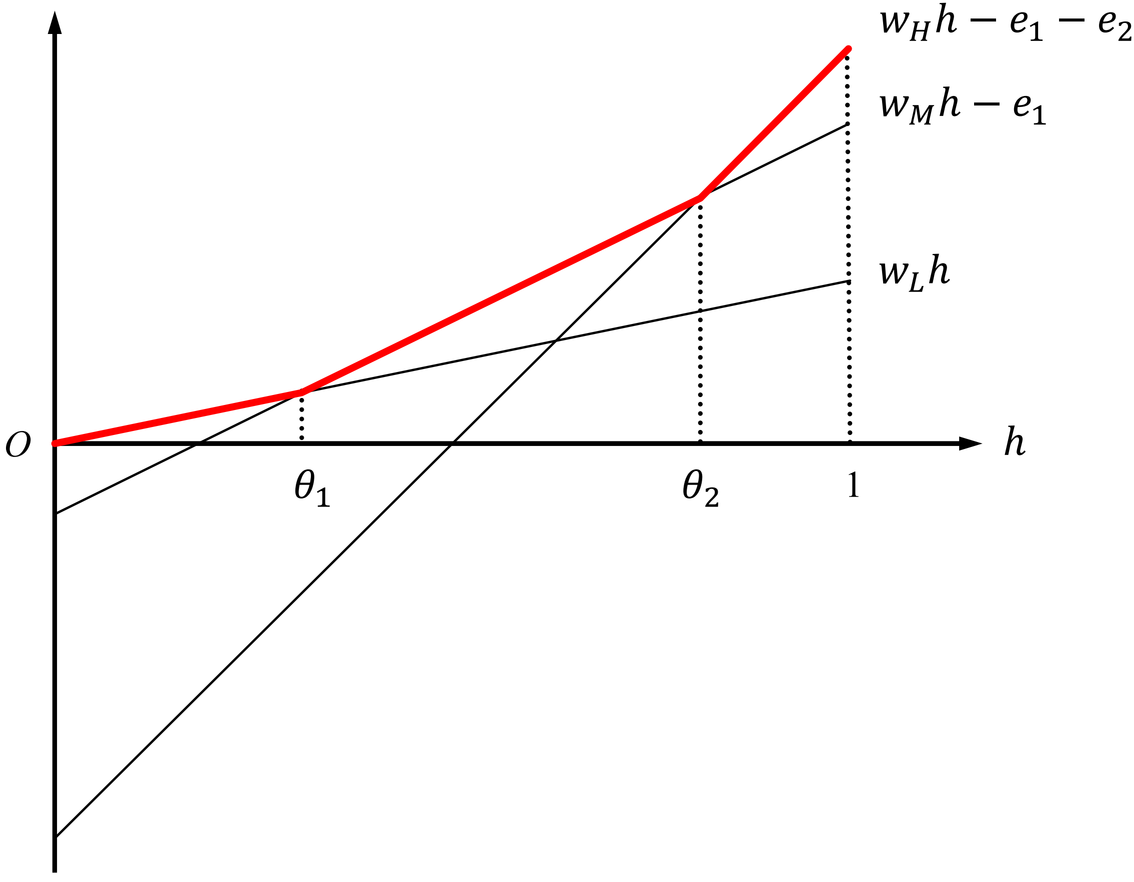
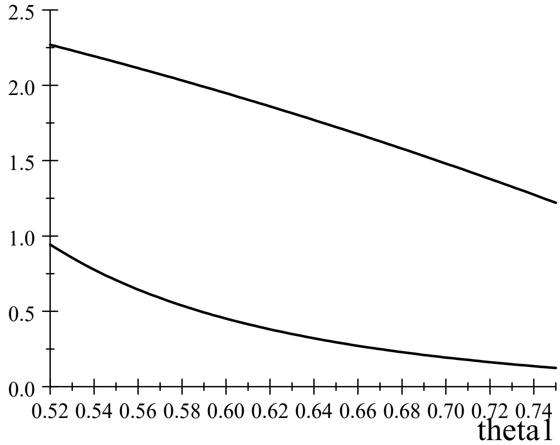
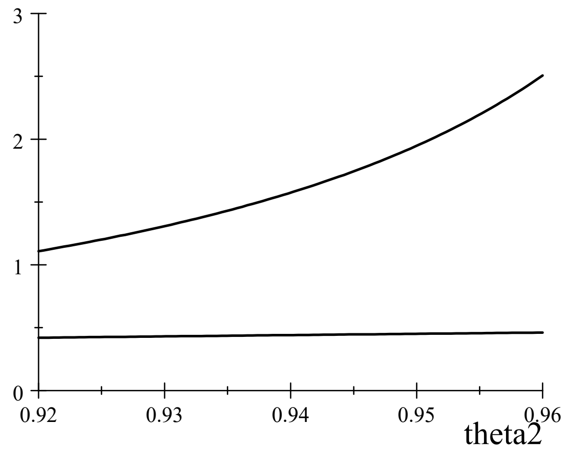


Figure 2. College education and wage inequality



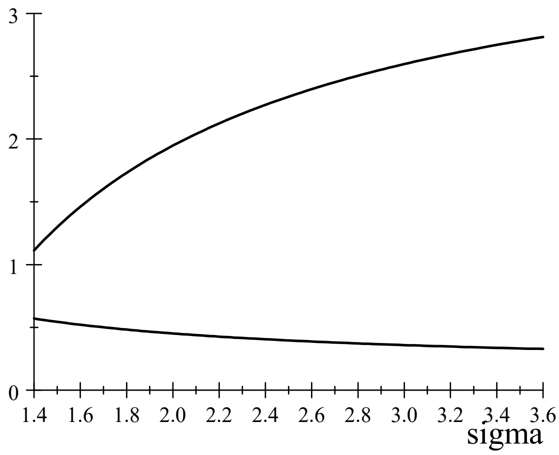
Note. The horizontal axis is the cost of college education, θ_1. The upper curve represents w_H/w_M, and the lower curve represents w_L/w_M. A reduction in θ_1 increases the top-end wage inequality and decreases the low-end wage inequality. $\alpha = 0.2$, $\beta = 0.5$, $\gamma = 0.3$, $\sigma = 2$ and $\theta_2 = 0.95$.

Figure 3. Postgraduate education and wage inequality



Note. The horizontal axis is the cost of postgraduate education, θ_2. The upper and lower curves represent w_H/w_M and w_L/w_M, respectively. An increase in θ_2 increases the top-end wage inequality and decreases the low-end wage inequality. $\alpha = 0.2$, $\beta = 0.5$, $\gamma = 0.3$, $\sigma = 2$ and $\theta_1 = 0.6$.

Figure 4. Factor substitution and wage inequality



Note. The horizontal axis is the elasticity of substitution between intermediate goods, σ. The upper and lower curves represent w_H/w_M and w_L/w_M, respectively. An increase in σ worsens both top-end and low-end wage inequalities. $\alpha = 0.2$, $\beta = 0.5$, $\gamma = 0.3$, $\theta_1 = 0.6$ and $\theta_2 = 0.95$.