Value of Middle Managers [*]

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Abstract

Why do modern organizations have middle managers? We address this question by analyzing the following hierarchical communication model. A subordinate has private information regarding the profitability of a new project and sends a costly signal about it (e.g., technical reports). Although the boss has to decide whether to approve the new project, she cannot freely observe the signal. Instead, the boss has two channels to learn it: (i) indirect communication with the biased (middle) manager who directly observes the signal, and (ii) direct observation of the signal by conducting costly investigations. We show that commitments to the investigation (i.e., organizations without managers) are always suboptimal for the boss even if the investigation cost is sufficiently small and the manager is sufficiently biased, providing a rationale for hierarchical organizations. By decomposing the gain from having managers, we clarify that indirect communication and costly investigation are complements if and only if the manager's bias is sufficiently large. Furthermore, we characterize the optimal direction of the manager's bias for the boss.

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1 Introduction

Modern organizations, especially large ones, have hierarchical structures with many middle managers who are neither at the top of organizations nor front-line workers.[1] Middle managers are broadly considered villains that make organizations inefficient, along with their ubiquitousness. For example, conventional wisdom has an impression that "[m]iddle managers are empire builders who do little useful work for their organizations" and "[t]he work of middle management has been de-skilled by restructuring" (Osterman, 2008). Consistent with this perspective, middle managers tend to be dismissed upon organizational restructuring. Rajan and Wulf (2006) and Wulf (2012), for instance, report a trend from the late 1980s that organizations have become flatter and CEOs have had more span of control. Specifically, CEOs eliminate COO positions, whereas they increase the number of functional managers (e.g., CFO, CHRO, and CIO) who directly communicate with them.[2] Furthermore, the trend of dismissing middle managers is accelerated by advancing information technologies.[3]

Although flat/downsized organizations are positively perceived, delayering of hierarchical structures never promises success. The disadvantage of downsizing is highlighted in the management literature. For example, Cascio (1993) reports that, although downsizing was expected to reduce people costs and make organizations less bureaucratic, more than half of the conducted companies did not materialize those benefits because downsizing made or-

[1]According to the U.S. Bureau of Labor Statistics survey, there were $2,347,420$ general and operations managers in May 2020, which was about 1.6% of the total U.S. employment.

[2]Wulf (2012) reports that the number of firms with COOs had decreased by about 20% from 1986 to 1998, and this trend had continued through 2011. Furthermore, Guadalupe et al. (2014) report that CEOs' span of control in U.S. firms increased from about 5 over the 1986 - 1990 period to almost 10 in 2006.

[3]Pinsonneault and Kraemer (1993) demonstrate that middle managers are substituted with IT when organizations are highly centralized. Furthermore, Bloom et al. (2014) show that decline in information-processing costs makes organizations flatter.

ganizational communication dysfunctional.[4] Specifically, as top managers had to manage unfamiliar tasks to fill the void, they bear large information-processing costs. Consequently, "vital information may not be available to help the chief executive and other top managers make decisions that only they can take" (Cascio, 1993). As another example, Google experimented with "a completely flat organization, eliminating engineering managers in an effort to break down barriers to rapid idea development" in the early 2000s (Garvin, 2013). However, the experiment failed within a few months. According to Gavin (2013), many employees "went directly [Larry] Page with questions about expense reports, interpersonal conflicts, and other nitty-gritty issues," which was one reason for the failure. These examples demonstrate that the cost of information processing is a bottleneck and makes flat organizations dysfunctional. Further, middle managers seem more helpful in reasonably providing information to the boss than conventional wisdom images.

We propose a model that explains such dysfunctional flat organizations, and demonstrate the benefits of middle managers, referred to as *value of middle managers*. Specifically, we consider a hierarchical communication model with three parties: a principal, a (middle) manager, and an agent.[5] The principal decides whether to implement a new project depending on the state. As an agent with strong bias toward a new project has private information regarding the state, the principal attempts to obtain it through the following two channels. The first channel is "costly direct communication." Here, the agent sends a costly signal regarding the state and the principal directly observes it by paying investigation/information-processing costs. The second channel is "costless indirect communication." Here, the manager who

[4] Specifically, according to a 1991 survey conducted by Wyatt Company of 1005 firms, downsizing induced that (i) 46% of the companies could reduce expenses, and (ii) 21% of the companies were satisfactory on improvements in shareholders' return on investments (Cascio, 1983).

[5] Throughout the paper, we treat the manager and the agent as male and the principal as female.

directly observes the agent's signal sends a cheap-talk message (e.g., endorsement of a new project) to the principal. The principal observes the manager's message without paying any costs. Notice that direct (resp. indirect) communication channel is associated with flat (resp. hierarchical) organizations. We emphasize the value of middle managers in the investigation of this model.

By characterizing the principal-optimal equilibrium for each investigation cost and the manager's bias, we show that commitments to direct communication are never optimal. Importantly, the suboptimality of direct communication is true despite the sufficiently small investigation cost and sufficiently large conflict between the principal and the manager. Although direct communication seems optimal in such a scenario, our result indicates that having the manager and trusting some message without investigation, referred to as *message-contingent investigation*, is more beneficial. That is, because the possibility of investigation enhances credible information transmission, the principal can save a part of the investigation cost. This implication confirms the value of middle managers.

Owing to the full characterization of the optimal equilibrium, we can further investigate the value of middle managers by clearly decomposing it. Specifically, it is decomposed into *information value* and *cost-saving values.* These values are the benefits of learning information via cheap talk and saving the investigation cost by conducting a message-contingent investigation, respectively. We then show that, if the manager's bias is small, then indirect communication and costly investigation are substitutes because gains from the communication with the manager are replaceable with those from the costly investigation. Conversely, if bias is large, then they are complements because gains from the communication cannot be replaced with that of the costly investigation only. These values under large bias are materialized only if the principal has the option of the investigation. Therefore, the value of middle managers mainly relies on the information (resp. cost-saving) value when the manager's bias is small (resp. large).

Furthermore, our decomposition uncovers the effects of bias direction. When the bias magnitude is small, the *anti-change-biased* manager is better than the *pro-change-biased* one because the former enhances the information value by facilitating information transmission from the agent as a "tough gatekeeper." Conversely, when the bias magnitude is large, bias toward an ex ante suboptimal project is better than that toward an ex ante optimal project. That is, as the manager who is biased toward the ex ante suboptimal project has stronger incentives to transmit credible information to change the principal's initial perspective, the principal can further save from the costly investigation more, enhancing the cost-saving value.

The remainder of this paper is organized as follows. The following subsection briefly reviews the related literature. Section 2 defines and discusses the formal model. Given the analysis of the three benchmarks in Section 3, Section 4 characterizes the principal-optimal equilibrium, and Section 5 investigates the value of the middle manager through its decomposition. Section 6 concludes the paper.

1.1 Related literature

This paper belongs to the strands of organizational economics and political science that rationalizes hierarchical organizations from the perspective of information transmission.[6] Fol-

[6]The literature on organizational economics regarding hierarchies has other strands, for example, focusing on (i) efficient resource/task allocation within organizations (Garicano, 2000), (ii) physical constraints and bounded rationality of the top (Radner, 1993), and (iii) collusion with supervisors (Tirole, 1986). See, for

lowing Dessein (2002), several papers, such as Boehmke et al. (2006), Ambrus et al. (2013b), Yang and Zhang (2019), Chakraborty et al. (2020), Celik et al. (2021), and Murtazashvili and Palida (2022) point out that (a broad sense of) intermediaries facilitate communication because they mitigate conflicts between informed players and an uninformed decision-maker. Mitusch and Strausz (2005) regard intermediaries as devices adding noise into communication, disciplining informed players' manipulation incentives.[7] Recently, Migrow (2021) shows that intermediaries as information aggregators facilitate information transmission. In these papers, intermediaries work as commitment devices for the uninformed decision-maker not to abuse obtained information.[8] In contrast, we shed light on the relationship between communication and investigation as a key mechanism justifying hierarchical organizations in terms of information transmission.

Unlike our conclusion on middle managers under costly investigation, the disadvantage of hierarchical communication with the uninformed decision-maker, who could obtain additional information, tends to be featured in skip-level communication and whistleblowing. Prendergast (2003) and Friebel and Raith (2004) demonstrate that skip-level communication might harm the decision-maker because it may aggravate intermediaries' moral hazard problem. Similarly, Ting (2008) argues that whistleblowing may make the decision-maker worse off by discouraging employees' effort investment. While these papers assume that the decision-maker is passive in obtaining additional information, she voluntarily chooses whether to obtain it in our setup, which is relevant to our positive result.[9]

instance, Garicano and Van Zandt (2013) and Mookherjee (2013) as overviews of each area.

[7]See also Goltsman et al. (2009) and Ivanov (2010).

[8]Intermediaries can also work as commitment devices for (i) encouraging ex ante investments (Rotemberg and Saloner, 2000; Nayeem, 2014, 2017) and (ii) increasing the decision-maker's "bargaining power" (Gailmard and Patty, 2013; Hirsch and Shotts, 2018).

[9]Kofman and Lawarree (1993) and Wang (2020) also demonstrate that external auditing and skip-level

This paper is also related to the literature on strategic communication. Li (2007), Li (2010a), and Ambrus et al. (2013a) investigate sequential or hierarchical communication games, wherein an intermediate sender only observes a message from his immediate predecessor, which are associated with our indirect communication mode.[10] Similarly, strategic communication with the receiver's information acquisition is investigated by Rantakari (2016), Le Quement (2016), Balbuzanov (2019), Miyahara and Sadakane (2020), and Sadakane and Tam (2022).[11] Our setup on the investigation is most closely related to that of Bijkerk et al. (2018), considering the scenario where the sender partially bears the receiver's investigation cost. This paper is different from existing literature by combining these two strands. Specifically, although the mechanism behind the complementarity between communication and investigation is shared with these papers, the comprehensive relationship between communication and investigation based on the intermediary's bias is newly derived owing to the hierarchical communication structure. Furthermore, our characterization provides several novel insights on the value of middle managers.

2 The Model

2.1 Setup

An organization has three parties, namely, a principal, a manager, and an agent, and they engage in the following hierarchical communication. The agent proposes a new project that

communication may benefit the decision-maker if she can commit to monetary incentives or decision rules.

[10]The literature also examines the scenarios wherein (i) senders share the same private information (Krishna and Morgan, 2001) and (ii) senders' private information is imperfectly correlated (Austen-Smith, 1993; Li, 2010b and 2012).

[11]See also Argenziano et al. (2016) for information acquisition by the sender and Levkun (2022) for information acquisition by a *fact-checker*.

he wants to be approved by the principal. The quality of the new project is represented by $\theta \in \Theta := [0, 1]$, which is the agent's private information. We assume that the parties share a common prior such that quality θ follows a uniform distribution over Θ. Given quality θ, the agent reports it to the manager by sending costly signal $s \in S := \mathbb{R}_+$. Specifically, the cost is increasing in signal s but decreasing in quality θ, which is given by $C(\theta, s) := s/(1 + \theta)$.[12]

The manager freely observes signal s. After receiving signal s, the manager provides an opinion regarding the new project for the principal. Specifically, the manager sends a cheap-talk message $m \in M := \{m_E, m_O\}$, where m_E and m_O mean that the manager endorses and opposes the new project, respectively.

The principal decides whether to approve the new project or reject it and continue the status quo project, following her investigation of the agent's reports. Specifically, in response to message m from the manager, the principal chooses action $r \in R := \{r_I, r_N\}$, where r_I and r_N denote that she conducts an investigation and does not, respectively. If the principal chooses $r = r_I$, then she also observes the agent's signal s by paying investigation cost $d > 0$. Hence, her project choice depends on signal s and message m. If the principal chooses $r = r_N$, then she does not observe signal s and pays nothing. This implies that the project choice solely depends on message m. Given observation $o \in O := M \times (S \cup \{\emptyset\})$ determined by action r, the principal chooses action $y \in Y := \{y_A, y_R\}$, where y_A and y_R denote that the principal approves the new project and rejects it (and continues the status quo), respectively.

The parties' preferences are defined as follows. The agent prefers for the principal to approve the new project irrelevant to its quality. Formally, $u : \Theta \times S \times Y \to \mathbb{R}$ represents

[12] Although we have the qualitatively same results without the cost function specification, we adopt it for deriving a clear characterization of the D1 equilibria. The specification does not explicitly appear in the subsequent analysis, except for the proofs in the appendixes.

the agent's utility function defined by

$$u(\theta, s, y) := \mathbb{1}(y = y_A) - C(\theta, s), \tag{1}$$

where $\mathbb{1}$ represents an indicator function. The principal intends to select a better project. Specifically, the principal's utility function $v : \Theta \times R \times Y \to \mathbb{R}$ is defined by

$$v(\theta, r, y) := \mathbb{1}(y = y_A)\theta + \mathbb{1}(y = y_R)\theta_{SQ} - \mathbb{1}(r = r_I)d, \tag{2}$$

where $\theta_{SQ} \in \Theta$ represents the quality of the status quo project. We assume that θ_{SQ} is common knowledge, and $\theta_{SQ} \neq \mathbb{E}[\theta] = 1/2$. The manager's utility function $w : \Theta \times R \times Y \to \mathbb{R}$ is defined by

$$w(\theta, r, s) := \mathbb{1}(y = y_A)(\theta + b) + \mathbb{1}(y = y_R)\theta_{SQ} - \mathbb{1}(r = r_I)\pi. \tag{3}$$

The manager has a similar preference to that of the principal, except for the following aspects. First, the manager has positive/negative bias toward the new project represented by $b \neq 0$. Positive (resp. negative) bias implies that the manager is biased toward changing (resp. continuing) the status quo. Hereafter, positive and negative biases are referred to as *pro-change* and *anti-change* biases, respectively. Second, if the principal investigates the signal, then the manager bears loss $\pi > 0$, as discussed below.

The timing of the game is summarized as follows:

1. Nature chooses quality θ following the common prior, and the agent directly observes

it.

2. Given quality θ, the agent sends signal $s \in S$ to the manager.

3. Given signal s, the manager sends cheap-talk message $m \in M$ to the principal.

4. Given message m, the principal chooses action $r \in R$, determining observation $o \in O$.

5. Given observation o, the principal chooses action $y \in Y$.

We adopt the perfect Bayesian equilibrium (PBE) satisfying the D1 criterion (Cho and Kreps, 1987) as our solution concept.[13] Intuitively, the D1 criterion requires that a type is eliminated from the support of the posterior if he is less likely to deviate than the other type. Then, in our environment, it implies that the manager/principal believes that the deviant type is either the discontinuous point of the agent's strategy or $\theta = 1$ for certain.[14] For example, if the agent's equilibrium strategy is given as the bold line in Figure 1, then the D1 criterion claims that (i) for any signal $s \in (0, s')$, the manager and principal believe that the state is $\theta = \theta'$ for certain, and (ii) for any signal $s > s'$, they believe that the state is $\theta = 1$ for certain. Owing to the implication of the D1 criterion, we can drastically simplify the analysis of hierarchical communication and then obtain a clear characterization. For easy reference, a PBE that satisfies the D1 criterion is hereafter referred to as the *D1 equilibrium*. We say that an equilibrium is *optimal* if it maximizes the principal's ex ante expected utility among the D1 equilibria.[15]

[13]The formal definition of strategies, beliefs, PBE, and the D1 criterion is found in Appendix A.

[14]See Lemma 1 in Appendix A for the formal statement.

[15]Due to our hierarchical communication structure, the D1 criterion cannot uniquely select an informative outcome. For example, there always exist uninformative D1 equilibria (Lemma 2). To obtain a unique prediction, we impose this optimality.

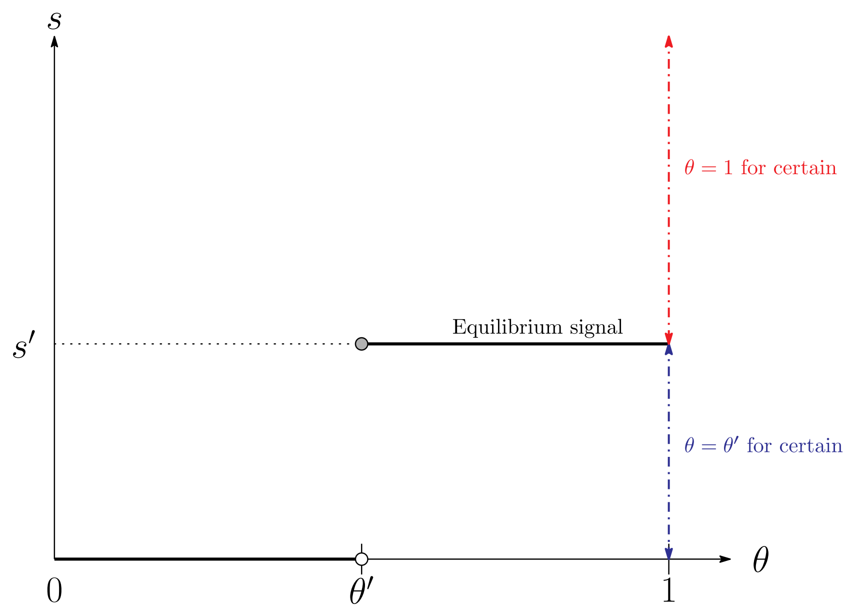


Figure 1: Implication of the D1 criterion

2.2 Discussion of the setup

2.2.1 Interpretation of the signals and costs

Signal s can be interpreted as the agent's observable effort investment for the presentation to the manager (e.g., Dewatripont and Tirole, 2005). For example, it represents the volume of materials (e.g., reports, slides, etc.) that the agent prepares in advance for the presentation, or it measures how the materials are well organized and impressive. Furthermore, our cost function implies that additional preparation is more costly, whereas the same level of preparation is less costly if the quality of the new project is better.

2.2.2 Interpretation of the investigation cost

Investigation cost d is interpreted as costs of information processing.[16] As reports from the agent would be technical (e.g., filled with terminology and figures that require expertise to understand), the principal must pay decoding costs when she directly reads them. Hence, the principal's investigation cost could be large when she has to manage unfamiliar tasks, as in the example in the introduction. Conversely, if the principal is promoted from a front-line worker and has expertise in that problem, then her investigation cost could be small.

2.2.3 Interpretation of the restricted message space

The binary-message setup reflects *categorization* by middle managers. According to Floyd and Wooldridge (1996), "[c]ategorization is one of the tangible results of the interpretation process and a powerful way middle managers influence strategic thinking in others." For instance, middle managers label an issue as either an "opportunity" or a "threat" depending on its potential gains/losses, captured by our binary-message setup. Furthermore, because of its simple structure, categorization drastically economizes information-processing costs. This corresponds to the assumption that the principal observes messages without costs.

2.2.4 Interpretation of signaling technologies

We assume that the agent sends costly signals, whereas the manager sends cheap-talk messages to clarify their fundamental differences in the hierarchical communication. On the one hand, although original reports from front-line workers are filled with details, their in-

[16] It can be regarded as a direct cost of information acquisition, e.g., a transaction cost of conducting skip-level communication with front-line workers.

formativeness decreases in passing layers within the hierarchies due to categorization. On the other hand, original reports require costly information processing, whereas categorized messages are easier to understand. We adopt asymmetric signaling technologies to highlight the trade-off between precise but costly signals and coarse but cheap messages.

2.2.5 Interpretation of investigation loss π

We assume that the manager bears loss π if the principal conducts an investigation, which has several interpretations. First, the investigation cost is shared by the manager à la Bijkerk et al. (2018). The investigation may also involve the manager and require his preparation. Hence, such an extra effort investment is costly for the manager. Second, it can be interpreted as the manager's psychological costs. Because conducting the investigation implies that the principal does not fully trust reports from the manager, it might bear his self-esteem. Similarly, as the investigation implies that the manager has less influence on the principal's decision-making, he might feel losses (Bartling et al., 2014). Finally, because the principal chooses this option when she suspects the manipulation by the manager, the loss can be interpreted as a broad sense of lying costs (Kartik et al., 2007; Kartik, 2009).

3 Benchmarks

As preliminaries, we consider the following three benchmarks wherein the principal (i) directly observes the quality (*first-best mode*), (ii) commits to investigate (*direct communication mode*), and (iii) commits not to investigate (*indirect communication mode*).

3.1 First-best mode

Because the principal directly observes quality θ, her decision-making is trivial: (i) $r = r_N$ for certain, and (ii) $y = y_A$ if and only if $\theta > \theta_{SQ}$. Her ex ante expected utility $\bar{V}$ is given by

$$\bar{V} := \int_0^{\theta_{SQ}} \theta_{SQ} d\theta + \int_{\theta_{SQ}}^1 \theta d\theta = \frac{1}{2}(1 + \theta_{SQ}^2), \tag{4}$$

which is the first-best outcome for her.

Proposition 0 *The principal's optimal utility in the first-best mode is* $\bar{V} = (1 + \theta_{SQ}^2)/2$.

3.2 Direct communication mode

Because the manager's message is uninformative in this scenario, the principal must conduct the costly investigation to obtain information. When she chooses $r = r_I$, the D1 criterion and the optimality uniquely determine the equilibrium structure as denoted in Figure 2: the agent signals whether the quality is higher than θ_{SQ}, and the principal's project choice is identical to that of the first-best mode.[17] Hence, she obtains utility $(1 + \theta_{SQ}^2)/2 - d$ in this continuation game. However, if she chooses $r = r_N$, then her project choice is only based on the prior. That is, $y = y_A$ is chosen if and only if $\theta_{SQ} < 1/2$, implying that she obtains the expected utility $\max\{1/2, \theta_{SQ}\}$ in this continuation game.

The characterization of the optimal D1 equilibrium depends on investigation cost d.

[17] The principal chooses action $y = y_I$ after observing signal $s \in (0, s')$ because she is indifferent between actions $y = y_A$ and y_I given the posterior satisfying the D1 criterion.

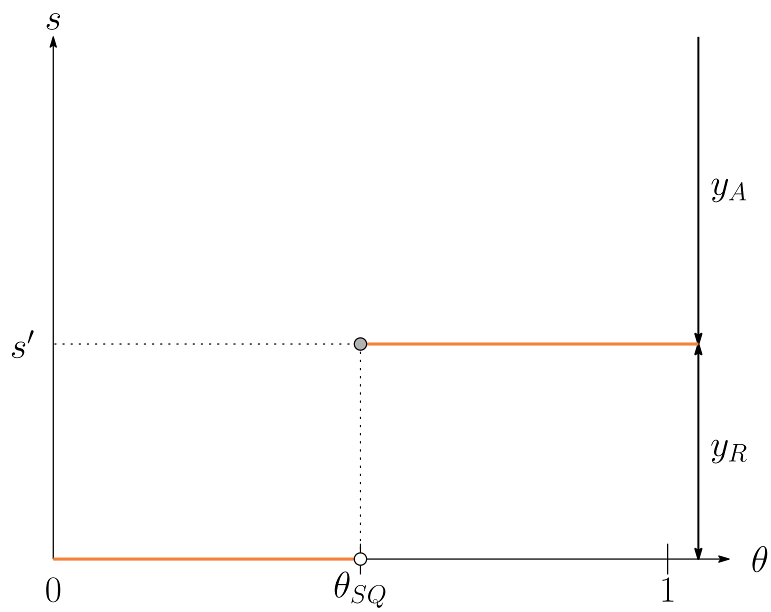


Figure 2: Full-investigation equilibrium

Specifically, she chooses $r = r_I$ if and only if

$$\frac{1}{2}(1+\theta_{SQ}^2) - d \geq \max\left\{\frac{1}{2}, \theta_{SQ}\right\} \iff d \leq \bar{d}(\theta_{SQ}), \tag{5}$$

where $\bar{d}(\theta) := \theta_{SQ}^2/2$ if $\theta_{SQ} < 1/2$ and $(1-\theta_{SQ})^2/2$ otherwise. That is, if the investigation cost is sufficiently small, then the principal certainly conducts the investigation and then correctly learns the quality on the optimal equilibrium, which is referred to as a *full-investigation equilibrium*. The principal's ex ante expected utility in the full-investigation equilibrium is given by

$$V^F := \int_0^{\theta_{SQ}} \theta_{SQ} d\theta + \int_{\theta_{SQ}}^1 \theta d\theta - d = \frac{1}{2}(1+\theta_{SQ}^2) - d. \tag{6}$$

Otherwise, an *uninformative equilibrium* is optimal, on which she never investigates and then chooses a project based on the prior. Hence, her ex ante expected utility is $V^U := \max\{1/2, \theta_{SQ}\}$. Note that any type of the agent chooses signal $s = 0$ in the uninformative equilibrium. We hereafter refer to an equilibrium wherein more than one signal is sent as an *informative equilibrium* for easy reference.

Proposition 1 *Consider the direct communication mode. If $d \leq \bar{d}(\theta_{SQ})$, then there exists the full-investigation equilibrium and it is optimal. Otherwise, the unique D1 equilibrium is uninformative.*

3.3 Indirect communication mode

In this benchmark, the principal can access the information regarding the agent's signal only through cheap-talk messages from the manager. As in cheap-talk games à la Crawford and Sobel (1982), the informativeness of cheap-talk communication depends on the manager's bias. Suppose that the manager has a pro-change bias, i.e., $b > 0$. When b is sufficiently small, an informative equilibrium exists wherein the (i) agent signals whether the quality is above threshold θ', (ii) manager endorses the new project if and only if the signal is weakly greater than threshold s', and (iii) principal approves the new project if and only if it is endorsed by the manager, as denoted in Figure 3. Otherwise, only uninformative equilibria exist.

The principal's welfare on the informative equilibrium increases as threshold θ' is closer to θ_{SQ}, but $\theta' = \theta_{SQ}$ is never attainable on D1 equilibria provided that the manager has a pro-change bias. The upper bound of the threshold is $\theta_{SQ} - b$. Intuitively, if the threshold is above the upper bound, then the agent has an incentive to deviate. Specifically, suppose that

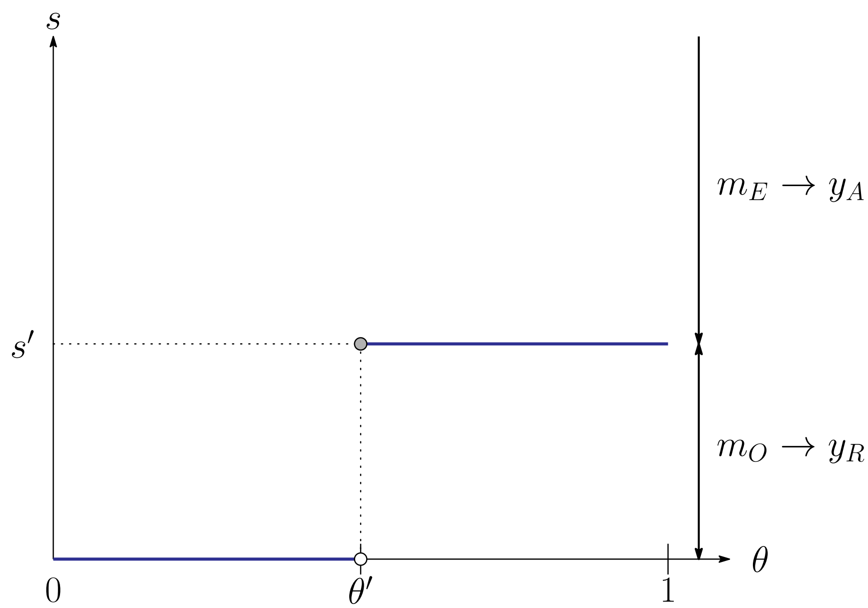


Figure 3: No-investigation equilibrium

$\theta' > \theta_{SQ} - b$ and the agent with type $\theta \in [\theta', 1]$ deviates from signal s' to signal $s'' \in (0, s')$. The D1 criterion requires that the manager believes that the quality is $\theta = \theta'$ for certain when he observes signal s''. Because the manager prefers the new project (i.e., $\theta' + b > \theta_{SQ}$) and his endorsement induces its approval, his best response to signal s'' is endorsing the new project. However, because signal s'' induces the principal's approval with lower costs, the agent has an incentive to deviate, indicating that the upper bound should be less than $\theta_{SQ} - b$.

This observation implies that the manager with anti-change biases may be more beneficial to the principal than the pro-change-biased one. As demonstrated previously, the first-best outcome is never attainable even if the bias is sufficiently small whenever the manager is pro-change biased. It is, however, attainable if the manager is anti-change biased. Specifically, if $b < 0$ and $|b| \leq b_{FB} := (1 - \theta_{SQ})/2$, then an informative equilibrium with threshold $\theta' = \theta_{SQ}$ exists. Intuitively, the superiority of the anti-change manager can be understood as follows. When the manager is pro-change biased, the manager and agent are easy to "collude." That is, because the manager tends to prefer the new project, he would endorse it even if the agent does not provide sufficient effort, which demotivates the agent's costly information transmission. Conversely, when the manager is anti-change biased, he behaves as a "tough gatekeeper" because he tends to oppose the new project if the agent does not provide sufficient effort. That is, the agent's bias toward the new project is mitigated by the anti-change manager as the tough gatekeeper, which disciplines information transmission from the agent.[18]

Hereafter, an informative D1 equilibrium is referred to as a *no-investigation equilibrium* if its threshold θ' is closest to θ_{SQ} among any informative D1 equilibria, and $\theta^*(b)$ represents the threshold of the no-investigation equilibrium. The principal's ex ante expected utility in the no-investigation equilibrium is given by

$$V^N := \int_0^{\theta^*(b)} \theta_{SQ} d\theta + \int_{\theta^*(b)}^1 \theta d\theta = \frac{1}{2} + \theta_{SQ}\theta^*(b) - \frac{1}{2}\theta^*(b)^2. \tag{7}$$

As mentioned in the following proposition, the no-investigation equilibrium is optimal whenever it exists in the indirect communication mode.

[18]This implication is reminiscent of Krishna and Morgan (2001), showing that the opposing biased experts could induce a fully revealing equilibrium in sequential cheap-talk games. See Miura (2014) for detail.

Proposition 2 *Consider the indirect communication mode with* $\theta_{SQ} > 1/2$.[19] *If* $b \in (-(1-\theta_{SQ}), 1-\theta_{SQ}]$*, then there exists the no-investigation equilibrium and it is optimal, where*

$$\theta^*(b) = \begin{cases} \theta_{SQ} - b & \text{if } b \in (0, 1-\theta_{SQ}], \\ \theta_{SQ} & \text{if } b \in [-b_{FB}, 0), \\ 2\theta_{SQ} - 2b - 1 & \text{if } b \in (-(1-\theta_{SQ}), -b_{FB}). \end{cases} \tag{8}$$

Otherwise, the unique D1 equilibrium is uninformative.

By comparing pro- and anti-change biases with the same absolute value, the superiority of anti-change bias is formally summarized as follows.

Corollary 1 *Under the indirect communication mode, the anti-change-biased manager is weakly better than the pro-change-biased one for the principal.*

It is worthwhile to mention the following as final remarks. First, the superiority of the anti-change manager appears by considering a hierarchical communication model. That is, this implication never appears once we consider a model where the manager directly observes state θ. Second, in the no-investigation equilibrium, the manager has *real authority* in the sense of Aghion and Tirole (1997). That is, the principal obeys the recommendation from the manager, which is beneficial for him. Specifically, when $b > 0$, the optimal informative equilibrium achieves the best outcome for the manager.

[19] The statement for $\theta_{SQ} < 1/2$ is found in Appendix B.5.2.

4 Value of Middle Managers

Now, we return to the model, where the principal potentially has two options to acquire the information regarding signals: (i) direct acquisition by costly investigation and (ii) indirect acquisition through the manager's cheap-talk messages. Hereafter, this scenario is referred to as a *hybrid communication mode*. First, we demonstrate that this mode has a new equilibrium that never appears in the benchmarks. Then, we characterize the optimal equilibrium based on investigation cost d and bias b.

4.1 Partial-investigation equilibrium

The interaction of these two options of information acquisition makes it possible for the principal to conduct the investigation based on messages. Suppose that the manager has a pro-change bias (i.e., $b > 0$). In this scenario, an equilibrium with the following structure exists: (i) the agent's strategy is a step function with two discontinuous points θ_+ and θ_{SQ}, (ii) the manager endorses the new project if and only if the observed signal is higher than s_1, (iii) the principal investigates only if the project is endorsed, and (iv) the principal approves the new project if and only if her observation is either $o = (m_E, \emptyset)$ or (m_E, s) with $s \geq s_2$, as depicted in Figure 4. A remarkable aspect of this equilibrium is that the principal randomizes $r = r_I$ and r_N given message m_E. This implies that investigation and no-investigation are indifferent for the principal when she receives the endorsement. The principal's indifference condition determines the first discontinuous point θ_+. Specifically,

$$\frac{1}{1-\theta_+}\left(\int_{\theta_+}^{\theta_{SQ}} \theta_{SQ} d\theta + \int_{\theta_{SQ}}^{1} \theta d\theta\right) - d = \frac{1}{1-\theta_+}\int_{\theta_+}^{1} \theta d\theta \Longleftrightarrow \theta_+ = \theta_{SQ} - \delta_+, \tag{9}$$

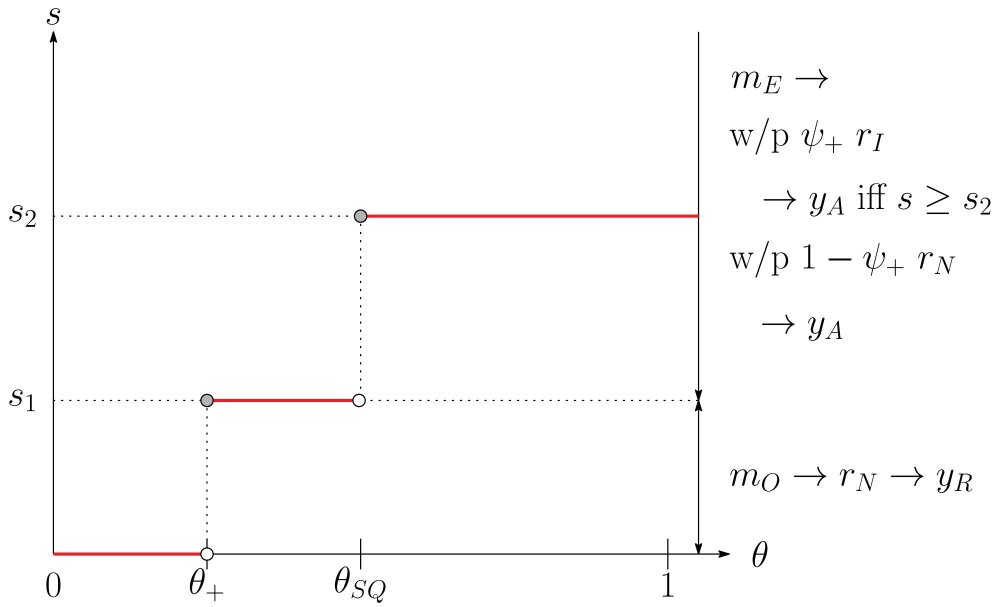


Figure 4: Partial-investigation equilibrium

where $\delta_+ := d + \sqrt{d^2 + 2(1-\theta_{SQ})d} > 0$. This equilibrium is hereafter referred to as a *partial-investigation equilibrium*.[20]

A partial-investigation equilibrium is a mixture of full- and no-investigation equilibria. That is, the principal basically follows the suggestion by the manager as in the no-investigation equilibrium. However, because the manager tends to endorse unqualified projects, the principal finds his affirmative suggestion less reliable. Thus, she investigates the original signal as in the full-investigation equilibrium when she receives the endorsement. Hence, this equilibrium demonstrates the coexistence of direct/indirect information acquisition by the principal, which is frequently observed in real organizations, as discussed in Section 4.3. Because of this coexistence, the existence of partial-investigation equilibria

[20]The partial-investigation equilibrium for $b < 0$ is similarly characterized. Specifically, (i) the agent's strategy is a step function with discontinuous points θ_{SQ}, and $\theta_- := \theta_{SQ} + \delta_-$, (ii) the manager endorses the new project if and only if $s \geq s_2$, (iii) the principal investigates only if the project is opposed, and (iv) the principal rejects the new project if and only if either $o = (m_O, \emptyset)$ or (m, s) with $s < s_1$ for any m, where signals 0, s_1, and s_2 with $0 < s_1 < s_2$ are sent on the equilibrium path and $\delta_- := d + \sqrt{d^2 + 2\theta_{SQ} d}$. Note that δ_- is determined, so as the principal observing message $m = m_O$ is indifferent between $r = r_I$ and r_N.

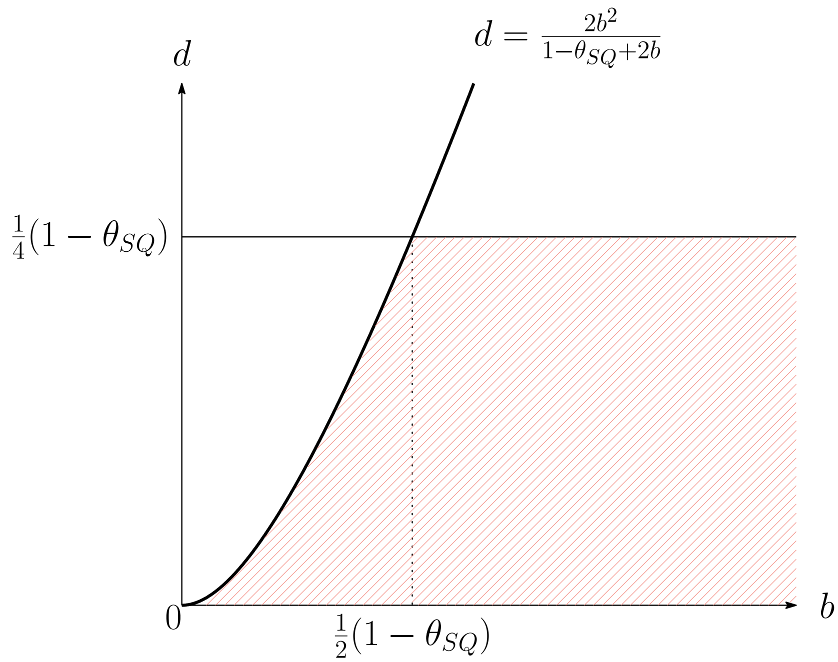


Figure 5: Existence of partial-investigation equilibrium for $b > 0$

depends on both investigation cost d and bias b, as indicated in the following proposition.

Proposition 3 *Consider the hybrid communication mode with $\theta_{SQ} > 1/2$.*[21]

(i) Suppose that $b > 0$. Then, a partial-investigation equilibrium exists if and only if either one of the following holds: (a) $b < (1-\theta_{SQ})/2$ and $d < 2b^2/(1-\theta_{SQ}+2b)$ or (b) $b \geq (1-\theta_{SQ})/2$ and $d \leq (1-\theta_{SQ})/4$.

(ii) Suppose that $b < 0$. Then, a partial-investigation equilibrium exists if and only if either one of the following holds: (a) $|b| \leq 1-\theta_{SQ}$ and $d < |b|^2/[2(\theta_{SQ}+|b|)]$ or (b) $|b| > 1-\theta_{SQ}$ and $d < (1-\theta_{SQ})^2/2$.

A partial-investigation equilibrium exists for $b > 0$ in the shaded region of Figure 5. As denoted in the diagram, the following two incentive conditions are potentially binding: (i)

[21] The characterization for $\theta_{SQ} < 1/2$ can be found in Appendix B.6.

$d \leq (1-\theta_{SQ})/4$ and (ii) $d < 2b^2/(1-\theta_{SQ}+2b)$. Condition (i) is equivalent to $\theta_{SQ} \leq 1-\delta_+$, guaranteeing that the principal chooses $y = y_A$ if her observation is $o = (m_E, \emptyset)$. Condition (ii) is equivalent to $b > \delta_+/2$, which is necessary for the manager sending $m = m_E$ when he observes signal $s = s_1$. Intuitively, a large bias is necessary for endorsing relatively unqualified projects.[22] Note that Conditions (i) and (ii) coincide when $b = (1-\theta_{SQ})/2$.[23]

The following are our remarks regarding partial-investigation equilibria. First, partial-investigation equilibria could exist even if the manager is sufficiently biased. Specifically, a sufficiently biased manager who intends to mislead the principal is necessary to induce her voluntary investigation. In this equilibrium, she investigates when she receives a suggestion towards the manager's preferred direction. However, when the manager is insufficiently biased, she is reluctant to investigate because the manager's suggestion can transmit credible information without any intervention, as in the no-investigation equilibrium. Therefore, a sufficiently biased manager is essential for providing investigation incentives.

Second, cheap-talk messages transmit credible information in partial-investigation equilibria even though the manager is sufficiently biased, which might appear counterintuitive. This is because the principal's investigation disciplines the manager's information transmission. When the manager is sufficiently biased, he has few incentives to tell the truth. Once the principal expects the manager's suggestion to be useless, she directly investigates the agent's signal. Note that the manager prefers her not to investigate because the investi-

[22] The manager's incentive condition of sending $m = m_O$ when the observed signal is $s \in (0, s_1)$ is also potentially binding. Under Condition (ii), the manager's all incentive conditions are satisfied by appropriately choosing the investigation probability under the endorsement.

[23] Similarly, the binding incentive conditions for $b < 0$ are understood as follows. First, $d < |b|^2/[2(\theta_{SQ} + |b|)]$ is equivalent to $|b| > \delta_-$, which is necessary for the manager sending $m = m_O$ when he observes signal $s \in (s_1, s_2)$. As in Condition (ii) of the pro-change-biased manager, a sufficiently large magnitude is necessary for opposing qualified projects. Second, $d < (1-\theta_{SQ})^2/2$ is equivalent to $\theta_- < 1$, which is necessary for the well-defined agent's equilibrium strategy. These conditions coincide when $|b| = 1-\theta_{SQ}$.

gation prevents the manager's distortion and generates loss $\pi > 0$.[24] Hence, the manager has incentives to refrain from information distortion to less likely induce the investigation, making the cheap-talk communication informative.[25]

Third, large π is unnecessary for the existence of partial-investigation equilibria. To support the above equilibrium structure, the manager should oppose the new project if he observes off-the-equilibrium-path signal $s' \in (0, s_1)$. When the investigation cost is small (i.e., $d \leq b^2/[2(1-\theta_{SQ}+b)]$), θ_+ is so close to θ_{SQ} that the manager prefers to endorse the new project under signal s'. In this scenario, positive loss is necessary for incentivizing the manager not to induce the agent's deviation.[26] Note that small but positive loss is sufficient for the above argument. Conversely, if the cost is moderate (i.e., $b^2/[2(1-\theta_{SQ}+b)] < d < 2b^2/(1-\theta_{SQ}+2b)$), then θ_+ is so far from θ_{SQ} that the manager observing signal s' prefers to oppose the new project. This implies that the manager's incentive condition is never binding. Therefore, a partial-investigation equilibrium exists for any nonnegative loss π.

Finally, the principal's ex ante expected utility in the partial-investigation equilibrium is relevant to investigation cost d and the sign of bias b whereas the magnitude of bias b is irrelevant. Specifically, it is represented by

$$V^P := \int_0^{\theta_{SQ}-\delta(b)} \theta_{SQ} d\theta + \int_{\theta_{SQ}-\delta(b)}^{1} \theta d\theta = \frac{1}{2}(1+\theta_{SQ}^2 - \delta(b)^2), \tag{10}$$

where $\delta(b) := \delta_+$ if $b > 0$ and $-\delta_- := -\left(d + \sqrt{d^2 + 2\theta_{SQ} d}\right)$ otherwise. Note that V^P is not

[24]Given this interpretation, loss π can be understood as a lying cost for the manager.

[25]Information acquisition by the receiver enhancing information transmission by the sender is pointed out in the literature on cheap-talk games. See Miyahara and Sadakane (2020) and Sadakane and Tam (2022).

[26]If $\pi = 0$, then the principal must conduct full investigation. However, it implies that the agent of type $\theta \in [\theta_+, \theta_{SQ})$ deviates to signal $s = 0$, which breaks down the partial-investigation equilibrium.

dependent on $|b|$, which is also a contrast to the no-investigation equilibrium.

4.2 Optimal equilibria

We characterize the optimal equilibrium in the hybrid communication mode. As candidates of the optimal equilibria, without loss of generality, we can restrict our attention to the full-, no-, and partial-investigation equilibria, although other equilibria might exist.[27] Note that, even in the hybrid communication mode, the existence of the full- and no-investigation equilibria is identically characterized as in Propositions 1 and 2, respectively.[28] Define $d_+ := b^2/[2(1-\theta_{SQ}+b)]$, and $d_- := (2|b|-1+\theta_{SQ})^2/[2(2|b|-1+2\theta_{SQ})]$. The optimal equilibrium is characterized as follows.

Proposition 4 *Consider the hybrid communication mode with $\theta_{SQ} > 1/2$.*[29]

(i) Suppose that $b > 0$.

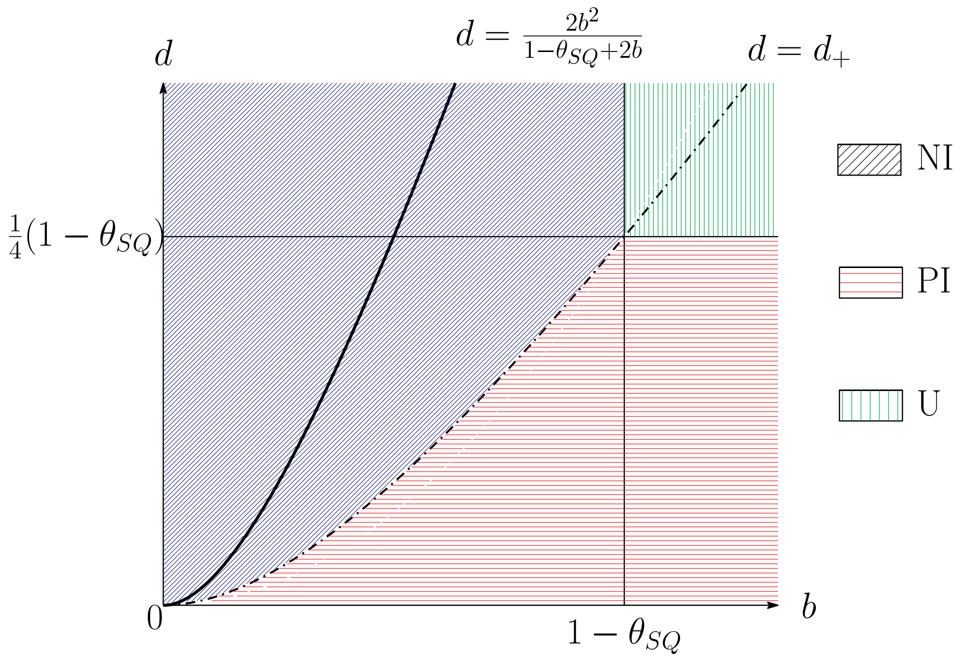
- *(a) If $b \leq 1-\theta_{SQ}$ and $d \geq d_+$, then the no-investigation equilibrium is optimal.*
- *(b) If either $[b < 1-\theta_{SQ}$ and $d < d_+]$ or $[b \geq 1-\theta_{SQ}$ and $d \leq (1-\theta_{SQ})/4]$, then a partial-investigation equilibrium is optimal.*
- *(c) Otherwise, an uninformative equilibrium is optimal.*

(ii) Suppose that $b < 0$.

[27] For example, a *non-monotonic equilibrium* exists, wherein the sender's strategy is a step function with two discontinuous points, and the manager sends the same message if he observes signals associated with the left and right intervals. However, this equilibrium is never optimal. The detail is available from the authors upon request.

[28] Specifically, an informative D1 equilibrium exists such that the manager adopts a babbling strategy, e.g., $\phi^*(s) = m_E$ for any s, which is associated with the full-investigation equilibrium. Similarly, an informative D1 equilibrium also exists such that the agent's strategy is a step function with a unique discontinuous point and the manager's strategy is a surjection, which is associated with the no-investigation equilibrium. Detailed proofs are available from the authors upon request.

[29] The statement for $\theta_{SQ} < 1/2$ is found in Appendix B.7.



(Note: NI, PI, and U mean no-investigation, partial-investigation, and uninformative equilibria, respectively.)

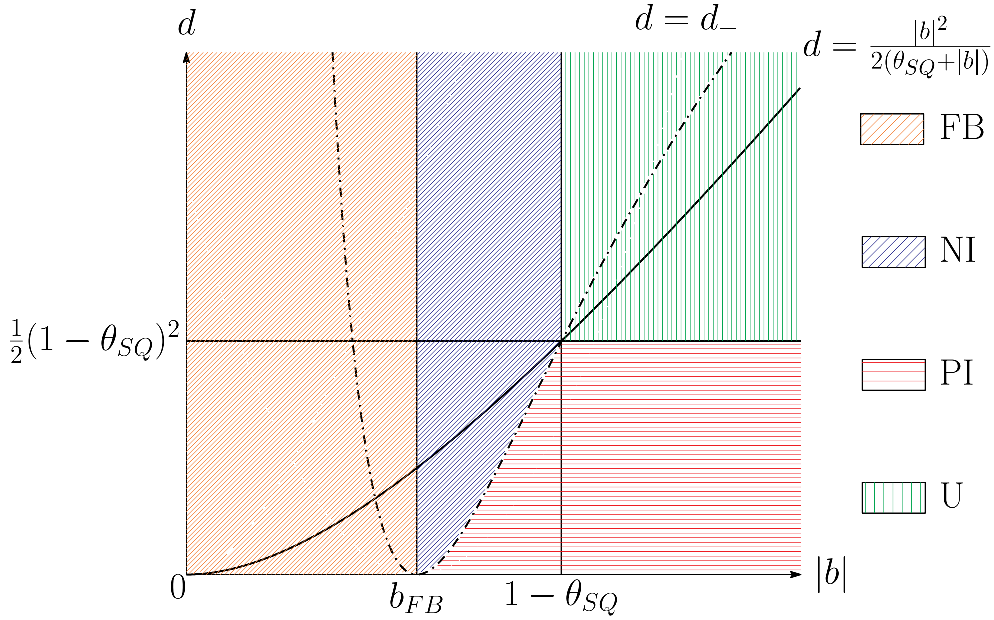
Figure 6: Optimal equilibria for $b > 0$ and $\theta_{SQ} > 1/2$

(a) If either $[|b| \leq b_{FB}]$ *or* $[|b| \in (b_{FB}, 1-\theta_{SQ}]$ *and* $d > d_-]$*, then the no-investigation equilibrium is optimal.*

(b) If either $[|b| \in (b_{FB}, 1-\theta_{SQ}]$ *and* $d \leq d_-]$ *or* $[|b| > 1-\theta_{SQ}$ *and* $d < (1-\theta_{SQ})^2/2]$*, then a partial-investigation equilibrium is optimal.*

(c) Otherwise, an uninformative equilibrium is optimal.

Proposition 4 is summarized in Figures 6 and 7. In the northwest region (of the diagram), the no-investigation equilibrium is optimal. The indirect acquisition relying on the manager's messages is optimal because the manager's bias is relatively small compared with the investigation cost. Conversely, a partial-investigation equilibrium is optimal in the southeast region, where the investigation costs are relatively small compared with the manager's bias. Because an investigation is a reasonable option and the manager's message tends to



(Note: FB, NI, PI, and U mean the no-investigation equilibrium with the first-best outcome, no-investigation, partial-investigation, and uninformative equilibria, respectively.)

Figure 7: Optimal equilibria for $b < 0$ and $\theta_{SQ} > 1/2$

be incredible, involving the direct acquisition is beneficial for the principal. Finally, when the manager's bias and the investigation cost are sufficiently large, i.e., the northeast region, the direct acquisition is too costly, and the indirect acquisition is incredible. Consequently, no information is transmitted. Proposition 4 derives the following implication.

Corollary 2 *Consider the hybrid communication mode. Then, the full-investigation equilibrium is never optimal.*

The suboptimality of the full-investigation equilibrium is understood as follows. First, note that there is no region where only the full-investigation equilibrium exists. That is, when this equilibrium exists, either the no- or a partial-investigation equilibrium also exists. Second, the full-investigation equilibrium is dominated by a partial-investigation equilibrium when both equilibria exist, e.g., $\theta_{SQ} > 1/2$, $b > 0$, and $d < \min\{2b^2/(1 - \theta_{SQ} +$

$2b), (1-\theta_{SQ})^2/2\}$. Intuitively, a partial-investigation equilibrium is more "efficient" than the full-investigation equilibrium. In the full-investigation equilibrium, the principal has to pay investigation cost d for certain. Contrary, a partial-investigation equilibrium involves the message-contingent investigation; that is, the principal randomizes the investigation only when she receives a less credible message. In summary, she can save the investigation cost by simply following the manager's suggestion when receiving a relatively credible message, which derives the superiority of partial-investigation equilibria. Finally, the full-investigation equilibrium is dominated by the no-investigation equilibrium when all but partial-investigation equilibria exist, e.g., $\theta_{SQ} > 1/2$ and $d \in [2b^2/(1-\theta_{SQ}+2b), (1-\theta_{SQ})^2/2]$. Because the manager's bias is relatively small, the cheap-talk communication sufficiently transmits information. Although the principal is more likely to select the correct project under the investigation, the gain is too small to compensate for the investigation cost because d is sufficiently large in this parameter range. Consequently, the full-investigation equilibrium is never optimal.

Corollary 2 has the following implications. First, it provides a rationale for having middle managers in organizations.[30] On the one hand, the full-investigation equilibrium is regarded as a "flat organization," where the boss and the subordinates communicate directly. On the other hand, the no- or a partial-investigation equilibrium is associated with a "hierarchi-

[30]Corollary 2 highlights the contrast from Garicano (2000). He adopts a team-theoretical approach and shows that the trade-off between *communication* and *knowledge acquisition costs* determines the optimal organizational structure. Specifically, if communication costs are relatively small (resp. large) compared with knowledge-acquisition costs, then the optimal organization has hierarchical (resp. flat) structures. Although communication costs are exogenously given in Garicano (2000), they are endogenously determined as losses due to the manager's manipulation in this paper. The suboptimality of flat organizations suggests that communication costs are smaller than knowledge acquisition costs measured by d, which is uncovered by our game-theoretical analysis. This observation is consistent with Colombo and Grilli (2013), which report that shifting toward hierarchical organizations is more supported by the information-processing-cost argument rather than the agency-costs argument.

cal organization," where the boss and subordinates indirectly communicate through middle managers. Given these interpretations, Corollary 2 claims that having middle managers is beneficial for the boss, which could explain why many real organizations still have hierarchical structures. We refer to the benefit as *the value of middle managers* (hereafter, VoM). Furthermore, we would like to emphasize that the VoM always appears irrelevant to managers' biases and investigation costs, which is rarely highlighted in the literature.[31]

Second, it claims that the boss with expertise should also have middle managers. As mentioned in Section 2.2.2, the principal with low investigation costs is interpreted as the boss with expertise. If middle managers are regarded as "translators" of the agent's technical reports, then they seem unnecessary for the boss who can directly understand them. However, Corollary 2 disagrees with this statement; that is, middle managers are still beneficial for the boss with expertise.[32]

4.3 A case study

British private sectors provide a notable example of the coexistence of direct and indirect communication channels within a broad sense of organization. Specifically, employee voice is regarded as communication between employees and employers. As its outlets, the firms are often equipped with either one of the following two communication channels. The first channel is the conventional *union voice*, where unions aggregate employee voice and tell it

[31]Bloom et al. (2014) claim that reducing information acquisition costs makes organizations flatter. The discrepancy between our and their arguments can be understood as follows. First, their theory is orthogonal to ours in terms of strategic interactions. Furthermore, because their empirical analysis is not time series, it does not exclude the possibility that flattered firms reinstall hierarchical structures after recognizing their suboptimality, as in the Google example mentioned previously.

[32]It is reminiscent of Crémer et al. (2007). They show that hierarchical organizations are optimal when a broad sense of investigation costs is not extreme. Our result enhances the benefits of hierarchical organizations by incorporating strategic communication within organizations.

to employers (e.g., bargaining between management and trade unions). The second one is the *non-union voice*, involving a direct two-way communication between employees and employers (e.g., problem-solving groups that "solve specific problems or discuss aspects of performance or quality" (Bryson, 2004)). The former and latter are associated with the indirect communication through the manager and direct investigation by the principal in our setup, respectively. According to Bryson et al. (2013), about 20% of the British private firms in 1980–2004 adopted both channels, which is referred to as the *dual-voice regime*. Because both communication channels are active in the dual-voice regime, it corresponds to a partial-investigation equilibrium.

The dual-voice regime shows (weakly) better performance than the alternatives. Bryson et al. (2013) compare the performance of four regimes: no-voice, union-only-voice, non-union-only-voice, and dual-voice regimes. Note that the first three regimes represent firms where employees and employers do not communicate, communicate only through unions, and communicate directly without unions, respectively. Hence, in our framework, these four regimes are interpreted as uninformative, no-investigation, full-investigation, and partial-investigation equilibria, respectively. Bryson et al. (2013) show that firms with the dual-voice regime demonstrate the highest usage of human management resource practices, indicating the highest organizational performance.[33] Assuming that unions have biases toward employees and sufficiently disagree with employers seems natural, which might be associated with the manager with $b > 0$ and its magnitude being not small in our model. Given this interpretation, the superiority of the dual-voice regime represents the VoM.

[33] In the literature on human resource management, the positive relationship between the usage of human resource management practices and organizational performance is well documented. See, for example, Huselid (1995).

5 Decomposition of the VoM

In this section, we investigate the VoM from the following two perspectives. First, we specify the dominant reason for generating the VoM. Second, we clarify the optimal direction of managers' biases.

5.1 Information and cost-saving values

The benefit of hierarchical communication can be decomposed into two-folds. The first benefit is choosing the correct project by learning information via cheap-talk communication. The second benefit arises from saving the investigation cost by conducting a message-contingent investigation. In this subsection, we clarify which sub-value is dominant in the VoM. For easy exposition, the first and second benefits are referred to as *information* and *cost-saving values*, respectively.

Formally, these sub-values are defined as follows. We consider the following four scenarios and compare the principal's optimal equilibrium payoffs. The first scenario is a counterfactual case in which the principal has no option to obtain information. The second scenario is the indirect communication mode, where only the communication with the manager is available to the principal. The third scenario is the direct communication mode, where only the investigation is available to the principal. Finally, the fourth scenario is the hybrid communication mode, where both communication and investigation are available. Let V_i represent the principal's optimal equilibrium payoff in the ith scenario, and define $\Lambda_I := V_2 - V_1$ and $\Lambda_C := V_4 - V_3$, respectively.

When investigation cost d is sufficiently small, Λ_I and Λ_C represent the information and cost-saving values, respectively. Specifically, we assume that $d < \min\{\theta_{SQ}^2/2, (1-\theta_{SQ})^2/2\}$, guaranteeing that $V_3 = V^F$ and $V_4 = V^N$ or V^P. Because Λ_I represents the payoff difference between the indirect communication mode and the no-information scenario, it measures the gain on learning more precise information irrelevant to the investigation effects, which is the information value. Similarly, Λ_C represents the payoff difference between the hybrid and the direct communication modes. Because the principal obtains full information by paying the maximum investigation cost on the full-investigation equilibrium, the superiority of V^N and V^P over V^F comes from the reduction of investigation costs. Hence, we regard Λ_C as a measurement of the cost-saving value. By comparing Λ_I and Λ_C, we obtain the following proposition: the information value is dominant if and only if the manager is not sufficiently biased.

Proposition 5 *Suppose that* $d < \min\{\theta_{SQ}^2/2, (1-\theta_{SQ})^2/2\}$*. Then, there exists* $\beta(\theta_{SQ}, b, d)$ *such that* $|b| \leq \beta(\theta_{SQ}, b, d)$ *if and only if* $\Lambda_I \geq \Lambda_C$*.*

Regarding the relationship between communication and investigation, Proposition 5 can be restated as follows. Note that Λ_I and Λ_C represent the marginal gains of communication when the investigation is unavailable and available, respectively. As an analogy of supermodularity, we say that communication and investigation are *substitutes* (resp. *complements*) if $\Lambda_I > \Lambda_C$ (resp. $\Lambda_I < \Lambda_C$), which derives the restatement of Proposition 5.

Corollary 3 *Suppose that* $d < \min\{\theta_{SQ}^2/2, (1-\theta_{SQ})^2/2\}$*. Then, communication and the investigation are substitutes (resp. complements) if* $|b| < \beta^*(\theta_{SQ}, b, d)$ *(resp.* $|b| > \beta^*(\theta_{SQ}, b, d)$*).*

Because the manager has little incentive to manipulate when bias is small, communication is sufficiently credible even without the investigation. Thus, the principal obtains sufficiently precise information. Consequently, communication and the investigation are substitutes because gains from communication are "overlapped" with those of the investigation. Conversely, when bias is large, communication is useless without the investigation because the manager has much incentive to execute manipulation. However, as mentioned previously, the investigation makes communication more credible. Furthermore, owing to the credible communication enhanced through the investigation, the principal conducts the investigation more efficiently via the message-dependent investigation. That is, the options are complements in the sense of the "mutual enhancing relationship": the investigation enhances the benefit of communication, which also enhances the benefit of the investigation.

The following is the intuition of Proposition 5. Note that the information value can be replaced by the investigation, but the cost-saving value cannot be. In this sense, the information value is mitigated by the gain from the investigation, whereas the cost-saving value is reinforced with it. Given this interpretation, the substitutability under small bias suggests that the information value, namely the competitive value, is the main source of the VoM. In contrast, the complementarity under large bias suggests that the cost-saving value, namely, the non-competitive one, is dominant.

5.2 Pro-change bias vs. anti-change bias

So far, we have demonstrated the asymmetry between the pro-change- and anti-change-biased manager. This subsection clarifies the optimal direction of the bias for the principal.

Specifically, we compare the principal's optimal equilibrium payoffs under positive and negative biases with the same absolute value. Note that both no- and partial-investigation equilibria have threshold θ' at which the equilibrium project choice is switched, and then distance $|\theta' - \theta_{SQ}|$ represents the length of the "incorrect-project-choice region."[34] The principal's equilibrium payoff is decreasing in $|\theta' - \theta_{SQ}|$. Then, by comparing the lengths under biases $b = |b'|$ and $-|b'|$, we obtain the following proposition. Let $b_+ := (1 - \theta_{SQ} + \delta_+)/2$.

Proposition 6 *Consider the hybrid communication mode with $\theta_{SQ} > 1/2$.*[35]

(i) Suppose that $d > (1-\theta_{SQ})/4$. Then, the anti-change-biased manager is always (weakly) better for the principal.

(ii) Suppose that $d \leq (1 - \theta_{SQ})/4$. Then, the anti-change-biased manager is better for the principal if and only if $|b| \leq b_+$.

Proposition 6 is summarized in Figures 8 and 9.[36] When the investigation cost is so large that the investigation is never conducted in equilibrium irrelevant to the direction of the bias (i.e., $d > (1 - \theta_{SQ})/4$), the anti-change-biased manager is better for the principal. The reason for the superiority of the anti-change-biased manager is owing to the enhancement of information transmission from the agent. Specifically, the anti-change-biased manager and the agent disagree with the acceptance of the new project, which disciplines the agent's costly information transmission, as mentioned in Corollary 1.

However, when the investigation cost is sufficiently small to investigate in equilibrium

[34] $\theta' = \theta^*(b)$ for the no-investigation equilibrium, and $\theta' = \theta_+$ or θ_- for the partial-investigation equilibrium.

[35] The statement for $\theta_{SQ} < 1/2$ can be found in Appendix B.9.

[36] The diagram for $(1 - \theta_{SQ})^2/2 \leq d \leq (1 - \theta_{SQ})/4$ is qualitatively equivalent to that in Figure 9.

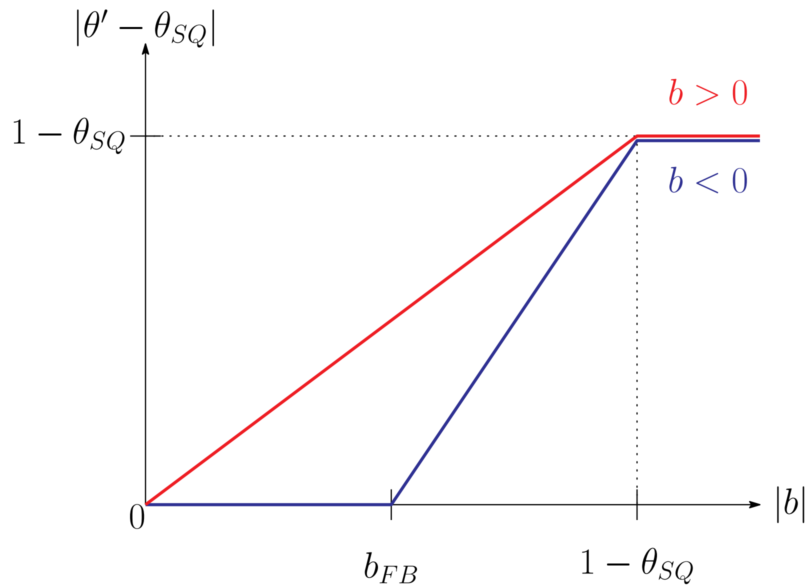


Figure 8: Pro- and anti-change biases for $d > (1 - \theta_{SQ})/4$

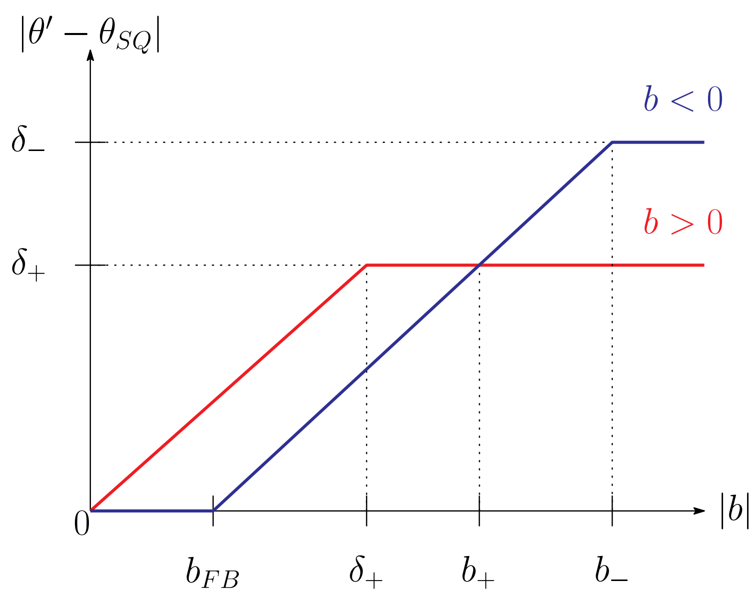


Figure 9: Pro- and anti-change biases for $d < (1 - \theta_{SQ})^2/2$

(i.e., $d \leq (1 - \theta_{SQ})/4$), the pro-change-biased manager may be optimal. Note that the pro-change bias under $\theta_{SQ} > 1/2$ can be interpreted as bias toward an ex ante suboptimal project (referred to as the *underdog project*). The reason for the superiority of the pro-underdog-biased manager is owing to the cost-saving value. If the manager has bias for the ex ante optimal project, then he has a strong incentive to execute manipulation because the preferred project is selected unless the principal obtains information. Hence, the principal conducts the investigation more frequently to suppress the agent's manipulation. On the other hand, the pro-underdog-biased manager has an incentive to transmit credible information to change the principal's prior evaluation. As the message transmits credible information, the principal refrains the investigation, which benefits her.

With the decomposition of the VoM defined above, we have implications regarding the optimal direction of the bias. First, anti-change bias enhances the information value of the VoM. When the investigation is sufficiently costly, or the bias magnitude is sufficiently small, the information value is dominant, as demonstrated in Proposition 5. Hence, in this scenario, the anti-change bias becomes optimal. Second, the pro-underdog bias enhances the cost-saving value of the VoM. Because the cost-saving value is dominant when the bias magnitude is sufficiently large, the optimal direction of the bias in this scenario should prioritize this sub-value. That is, the pro-underdog-biased manager is optimal.[37]

[37] When $\theta_{SQ} < 1/2$, the anti-change bias is also the pro-underdog bias. Consequently, the pro-change-biased manager is always optimal. The detail can be found in Appendix B.9.

6 Conclusion

We discussed the value of middle managers by considering a hierarchical communication model in which the principal has two channels to learn the agent's private information: communication with the manager and the costly investigation. The key mechanism behind our arguments is interaction of these two options. We show that commitments to a costly investigation (i.e., the full-investigation equilibrium) are always dominated by communication involving the manager. Specifically, when the investigation cost is sufficiently small and the manager is sufficiently biased, the costly investigation disciplines information transmission from the biased manager. Then, the principal can save the investigation cost by relying on the recommendation from the manager.

We then investigated the value of middle managers from the following two aspects. First, we decompose the value into information and cost-saving values and clarify which sub-value is the main source of the gain from having the manager. Because communication and the investigation are substitutes (resp. complements) when the manager's bias is small (resp. large), the information (resp. cost-saving) value is the main source of the gains from having the manager. Second, we characterize the optimal direction of the manager's bias. The anti-change-biased manager is optimal when the bias magnitude is small because that bias enhances the information value through facilitating information transmission from the agent. However, if the bias magnitude is large, then the pro-underdog-biased manager is optimal because that bias enhances the cost-saving value through providing credible messages.

Our results shed light on the advantage of hierarchies in terms of the interaction between communication and costly investigation. Although we focused on a particular context to clarify our argument, this perspective could be useful for understanding other phenomena that involve hierarchical structures (e.g., communication between politicians and voters via mass media), which is left for future research.

A Appendix: Proofs

This section provides the definition of equilibrium notion and the proofs of the main results. The proofs of the auxiliary results are postponed in Appendix B.

A.1 Optimal D1 equilibria

A.1.1 Definition and properties

The strategies and beliefs are defined as follows. Let $\sigma : \Theta \to S$ represent the agent's strategy. Let $\phi : S \to M$ and $\mu : S \to \Delta(\Theta)$ represent the manager's strategy and posterior belief about the quality, respectively.[38] The principal's strategy is represented by double $\psi := (\psi_r, \psi_y)$, where $\psi_r : M \to \Delta(R)$ and $\psi_y : O \to \Delta(Y)$ denote her local strategies at the investigation and project-choice stages, respectively. The principal's posterior belief is also given by double $\nu := (\nu_r, \nu_y)$, where $\nu_r : M \to \Delta(S)$ and $\nu_y : O \to \Delta(\Theta)$ represent her beliefs about the signal at the investigation stage and the quality at the project-choice stage, respectively. Let $e := (\sigma, \phi, \psi; \mu, \nu)$ represent an assessment.

As auxiliary notation, let $\rho : S \times M \times R \to O$ represent the principal's observation at

[38] For simplification, we restrict our attention to equilibria where the agent and manager adopt pure strategies. While allowing their mixed strategies might quantitatively change the results, qualitative properties seem to remain.

the project-choice stage, defined by

$$\rho(s,m,r) := \begin{cases} (m,s) & \text{if } r = r_I, \\ (m,\emptyset) & \text{otherwise.} \end{cases} \tag{A.1}$$

With some abuse of notation $\rho^{-1} : O \to R$ specifies the principal's behavior at the investigation stage inducing observation o, defined by

$$\rho^{-1}(o) := \begin{cases} r_I & \text{if } o \in M \times S, \\ r_N & \text{otherwise.} \end{cases} \tag{A.2}$$

Furthermore, let $\text{supp}(F)$ represent the support of distribution F.

Definition 1 *PBE*

Assessment $e^* = (\sigma^*, \phi^*, \psi^*; \mu^*, \nu^*)$ is a PBE if it satisfies the following conditions.

(i) For each $\theta \in \Theta$,

$$\sigma^*(\theta) \in \arg\max_{s\in S} \sum_{r\in R}\sum_{y\in Y} \psi_r^*(r \mid \phi^*(s))\psi_y^*(y \mid \rho(s,\phi^*(s),r))\, u(\theta,s,y). \tag{A.3}$$

(ii) For each $s \in S$,

$$\phi^*(s) \in \arg\max_{m\in M} \sum_{r\in R}\sum_{y\in Y} \psi_r^*(r \mid m)\psi_y^*(y \mid \rho(s,m,r)) \int_0^1 w(\theta,r,y)d\mu^*(\theta \mid s). \tag{A.4}$$

(iii) For each $m \in M$, (a) $\psi_r^*(r_I \mid m) > 0$ implies that

$$\int_{s \in S} \sum_{y \in Y} \psi_y^*(y \mid m, s) \int_0^1 v(\theta, r_I, y) d\nu_y^*(\theta \mid m, s) d\nu_r^*(s \mid m)$$

$$\geq \sum_{y \in Y} \psi_y^*(y \mid m, \phi) \int_0^1 v(\theta, r_N, y) d\nu_y^*(\theta \mid m, \phi), \tag{A.5}$$

and (b) if (A.5) holds with strict inequality, then $\psi_r^*(r_I \mid m) = 1$ holds.

(iv) For each $o \in O$ and $y \in \mathrm{supp}(\psi_y^*(o))$,

$$y \in \arg\max_{y' \in Y} \int_0^1 v\left(\theta, \rho^{-1}(o), y'\right) d\nu_y^*(\theta \mid o). \tag{A.6}$$

(v) μ^*, ν_r^*, and ν_y^* are derived from σ^* and ϕ^* by using Bayes' rule on the equilibrium path.

(vi) For each off-the-equilibrium-path observation $o = (m, \emptyset)$, $\nu_y^*(m, \emptyset)$ is derived from σ^* and ϕ^* by using Bayes' rule whenever possible.

(vii) For each off-the-equilibrium-path observation $o = (m, s)$, $\nu_y^*(m, s) = \mu^*(s)$.

The definition of PBE is standard except for conditions (vi) and (vii).[39] Especially, condition (vii) requires that if the principal observes both signal s and message m, then her belief about the quality only depends on the signal. It imposes a kind of "consistency" of beliefs in the sense that the principal and the manager share the same beliefs if the principal knows what the manager knows. This property is referred to as the *signal-priority condition.*

[39] Because our model is a multi-stage signaling game, off-the-equilibrium-path beliefs should be derived using Bayes' rule whenever possible (Fudenberg and Tirole, 1991). This requirement is associated with Condition (vi).

Given PBE e^*, let $U^*(\theta)$ represent the agent's equilibrium utility when the quality is θ, which is defined by

$$U^*(\theta) := \sum_{r \in R} \sum_{y \in Y} \psi_r^* \left(r \mid \phi^*(\sigma^*(\theta))\right) \psi_y^* \left(y \mid \rho(\sigma^*(\theta), \phi^*(\sigma^*(\theta)), r)\right) u(\theta, \sigma^*(\theta), y). \tag{A.7}$$

With some abuse of notation, define

$$\begin{aligned} U^*(\theta, s) &:= \sum_{r \in R} \psi_r^*(r \mid \phi^*(s)) \psi_y^*(y_A \mid \rho(s, \phi^*(s), r)) - C(\theta, s) \\ &:= G(s) - C(\theta, s) \end{aligned} \tag{A.8}$$

for each $\theta \in \Theta$ and $s \in S$. Note that $G(s)$ represents the probability of approving a new project under signal s (i.e., the agent's gross payoff from sending signal s), and $U^*(\theta, \sigma^*(\theta)) = U^*(\theta)$ holds. Similarly, let V^* represent the principal's ex ante equilibrium utility defined by

$$V^* := \int_0^1 \sum_{r \in R} \sum_{y \in Y} \psi_r^*(r \mid \phi^*(\sigma^*(\theta))) \psi_y^*(y \mid \rho(\sigma^*(\theta), \phi^*(\sigma^*(\theta)), r)) v(\theta, r, y) d\theta. \tag{A.9}$$

Define $\gamma(\theta, s) := U^*(\theta) + C(\theta, s)$, which represents a compensated gain for type-θ agent obtaining her equilibrium payoff $U^*(\theta)$ by sending signal s. Let $S^+(\sigma^*) := \{\, s \in S \mid \exists \theta \in \Theta \text{ s.t. } \sigma^*(\theta) = s \,\}$ and $S^-(\sigma^*) := S \setminus S^+(\sigma^*)$ represent the set of on-the-equilibrium-path and off-the-equilibrium-path signals, respectively. Let $M^+(\sigma^*, \phi^*) := \{\, m \in M \mid \exists \theta \in \Theta \text{ s.t. } \phi^*(\sigma^*(\theta)) = m \,\}$ represent the set of on-the-equilibrium-path messages under equilibrium e^*. Let $\#(X)$ denote the cardinality of set X. Let $\mathcal{U}(X)$ and $\mathcal{D}(x)$ represent a uniform distribution on set X and a degenerate distribution on point x, respectively.

Definition 2 *D1 Criterion (Cho and Kreps, 1987)*

PBE e^* satisfies the D1 criterion if it satisfies the following condition: for any $s \in S^-(\sigma^*)$ and $\theta \in \Theta$, if there exists $\theta' \in \Theta$ such that $\gamma(\theta, s) > \gamma(\theta', s)$, then $\theta \notin \operatorname{supp}(\mu^*(\cdot \mid s))$.

Intuitively, the D1 criterion eliminates a type from the support of the posterior if he is less likely to deviate in terms of the compensated gain. Because $\gamma(\theta, s)$ represents the minimal gain forcing type θ to deviate to signal s, $\gamma(\theta, s) > \gamma(\theta', s)$ means that type θ' is "easier" to deviate than type θ. The D1 criterion requires that, after observing deviation to signal s, type θ is never the deviant. The implication of the D1 criterion in our framework is summarized as follows.

Lemma 1 *Suppose that e^* is a PBE. Then, e^* satisfies the D1 criterion if and only if the following conditions hold.*

(i) If σ^* is discontinuous at θ', then for any off-the-equilibrium-path signal $s \in [\lim_{\theta \uparrow \theta'} \sigma^*(\theta), \lim_{\theta \downarrow \theta'} \sigma^*(\theta)]$ and message $m \in M$, $\mu^*(\theta' \mid s) = \nu_y^*(\theta' \mid m, s) = 1$ hold.

(ii) For any off-the-equilibrium-path signal $s > \sigma^*(1)$ and message $m \in M$, $\mu^*(1 \mid s) = \nu_y^*(1 \mid m, s) = 1$.

In contrast to the standard signaling games, our hierarchical communication structure prevents the D1 criterion from selecting a unique outcome, demonstrated as follows.

Lemma 2 *There always exists an uninformative D1 equilibrium.*

Hence, to obtain sharp predictions, we additionally impose the following restriction.

Definition 3 *Optimality*

A D1 equilibrium is optimal if there exists no D1 equilibrium in which the principal's ex ante equilibrium utility is strictly greater.

Finally, we introduce a useful fact for characterizing equilibria.

Claim 1 *For any PBE e^*, if there exist $s, s' \in S^+(\sigma^*)$ with $s < s'$, then $G(s) < G(s')$ holds.*

A.1.2 Proof of Lemma 1

A.1.2.1 Preliminaries

Lemma 3 *Suppose that e^* is a PBE. Then, the following holds.*

(i) σ^* is nondecreasing in θ.

(ii) σ^* is differentiable in θ almost everywhere.

(iii) σ^* is constant whenever differentiable.

(iv) U^* is continuous in θ.

Lemma 4 *Suppose that e^* is a PBE.*

(i) Suppose that σ^* is discontinuous at θ'. Then, for any off-the-equilibrium-path signal $s \in [\lim_{\theta \uparrow \theta'} \sigma^*(\theta), \lim_{\theta \downarrow \theta'} \sigma^*(\theta)]$ and $\theta \neq \theta'$, $\gamma(\theta, s) > \gamma(\theta', s)$ holds.

(ii) For any off-the-equilibrium-path signal $s > \sigma^*(1)$ and $\theta < 1$, $\gamma(\theta, s) > \gamma(1, s)$ holds.

Proof of Lemma 4. (i) Fix off-the-equilibrium-path signal $s \in [\lim_{\theta \uparrow \theta'} \sigma^*(\theta), \lim_{\theta \downarrow \theta'} \sigma^*(\theta)]$, arbitrarily. By Lemma 3-(ii) and (iii), there exist a set of open intervals $\{(0, \theta_1), (\theta_1, \theta_2), \ldots, (\theta_n, 1)\}$ such that σ^* is constant on each interval.[40] Without loss of generality, assume that $\theta_i = \theta'$. First, we consider interval (θ_{i-1}, θ'). Let $s' := \sigma^*(\theta)$ for any $\theta \in (\theta_{i-1}, \theta')$. Note that, by Lemma 3-(i), we have $s' < s$. By Lemma 3-(iv), we have $U^*(\theta') = \lim_{\theta \uparrow \theta'} U^*(\theta) = G(s') - C(\theta', s')$ and $U^*(\theta_{i-1}) = \lim_{\theta \downarrow \theta_{i-1}} U^*(\theta) = G(s') - C(\theta_{i-1}, s')$. Now, fix $\theta'' \in [\theta_{i-1}, \theta')$, arbitrarily. Hence,

$$\begin{aligned} \gamma(\theta'', s) - \gamma(\theta', s) &= (G(s') - C(\theta'', s') + C(\theta'', s)) - (G(s') - C(\theta', s') + C(\theta', s)) \\ &= (C(\theta'', s) - C(\theta'', s')) - (C(\theta', s) - C(\theta', s')) > 0, \end{aligned} \tag{A.10}$$

where the inequality comes from the fact that $\theta'' < \theta'$ and $C_{\theta s} < 0$. Next, consider interval $[\theta_{i-2}, \theta_{i-1})$. By applying the same argument, we can show that $\gamma(\theta''', s) > \gamma(\theta_{i-1}, s)$ holds for any $\theta''' \in [\theta_{i-2}, \theta_{i-1})$, which implies that $\gamma(\theta''', s) > \gamma(\theta', s)$. By inductively applying the above argument to intervals $[0, \theta_1), \ldots, [\theta_{i-3}, \theta_{i-2})$, we can show that $\gamma(\theta, s) > \gamma(\theta', s)$ holds for any $\theta < \theta'$. Similarly, we can show that $\gamma(\theta, s) > \gamma(\theta', s)$ holds for any $\theta > \theta'$.

(ii) By applying the similar argument used in the proof of Part (i), we can show the statement. ■

A.1.2.2 Proof of Lemma 1

(Sufficiency) Suppose that conditions (i) and (ii) hold. By combining the hypothesis and Lemma 4, we immediately conclude that e^* satisfies the D1 criterion.

[40] We can show that the number of intervals is at most finite by the similar argument used in Section B.3.4.

(Necessity) Suppose, in contrast, that e^* satisfies the D1 criterion, but either Condition (i) or (ii) is violated. First, suppose that Condition (i) is violated; that is, there exists discontinuous point θ' of σ^* and off-the-equilibrium-path signal $s \in [\lim_{\theta \uparrow \theta'} \sigma^*(\theta), \lim_{\theta \downarrow \theta'} \sigma^*(\theta)]$ such that $\mu^*(\theta' \mid s) \neq 1$. However, because Lemma 4-(i) implies that $\gamma(\theta, s) > \gamma(\theta', s)$ for any $\theta \neq \theta'$, the D1 criterion requires that $\mu^*(\theta' \mid s) = 1$, which is a contradiction. For the scenario where Condition (ii) is violated, we can derive a contradiction by using a similar argument. Therefore, both conditions must hold. ■

A.2 Proof of Proposition 1

Without loss of generality, we assume that the manager commits to strategy $\phi^*(s) = m_E$ for any $s \in S$ in this scenario.

A.2.1 Preliminaries

Lemma 5 *Consider the direct communication mode. If e^* is an informative D1 equilibrium, then (i) $\#(S^+(\sigma^*)) = 2$, (ii) $\theta^* \leq \theta_{SQ}$, and (iii) $\psi_r^*(r_I \mid m_E) > 0$ hold, where θ^* is a discontinuous point of σ^*.*

Note that for any $\theta \in (0, 1)$, there exists an informative PBE where σ^* is discontinuous at θ. Nevertheless, the D1 criterion eliminates ones whose discontinuous points are higher than θ_{SQ}. Furthermore, there are still multiple informative D1 equilibria.

A.2.2 Proof of Proposition 1

Suppose that $\theta_{SQ} > 1/2$ and $d \le \bar{d}(\theta_{SQ})$.[41] First, we show that the following constitutes the full-investigation equilibrium:

$$
\begin{aligned}
\sigma^*(\theta) &= \begin{cases} 0 & \text{if } \theta < \theta_{SQ}, \\ 1+\theta_{SQ} & \text{otherwise,} \end{cases} \\
\phi^*(s) &= m_E \text{ for any } s, \\
\psi_r^*(m) &= r_I \text{ for any } m, \\
\psi_y^*(o) &= \begin{cases} y_A & \text{if } o=(m,s) \text{ with } s \ge 1+\theta_{SQ} \text{ for any } m, \\ y_R & \text{otherwise,} \end{cases} \\
\mu^*(s) &= \begin{cases} \mathcal{U}([0,\theta_{SQ})) & \text{if } s=0, \\ \mathcal{U}([\theta_{SQ},1]) & \text{if } s=1+\theta_{SQ}, \\ \mathcal{D}(\theta_{SQ}) & \text{if } s \in (0, 1+\theta_{SQ}), \\ \mathcal{D}(1) & \text{otherwise,} \end{cases} \\
\nu_r^*(s \mid m) &= \begin{cases} \theta_{SQ} & \text{if } s=0 \text{ for any } m, \\ 1-\theta_{SQ} & \text{if } s=1+\theta_{SQ} \text{ for any } m, \end{cases} \\
\nu_y^*(o) &= \begin{cases} \mu^*(s) & \text{if } o=(m,s), \\ \mathcal{U}([0,1]) & \text{otherwise.} \end{cases}
\end{aligned}
\tag{A.11}
$$

Given ν_y^*, the optimality of ψ_y^* is obvious. Given ν_r^* and ψ_y^*, the principal's expected payoffs from $r = r_I$ and r_N are $(1+\theta_{SQ}^2)/2 - d$ and θ_{SQ}, respectively. Because $d \le \bar{d}(\theta_{SQ})$, the

[41] The proof for the scenario of $\theta_{SQ} < 1/2$ is omitted because it is essentially equivalent to that presented here.

former is higher, implying that ψ_r^* is optimal.

The optimality of σ^* is as follows. Note that for type $\theta = \theta_{SQ}$, signals $s = 0$ and $1 + \theta_{SQ}$ are indifferent, i.e., $1 - C(\theta_{SQ}, 1 + \theta_{SQ}) = 0$. Because $C_\theta < 0$, type $\theta \in [0, \theta_{SQ})$ (resp. $[\theta_{SQ}, 1]$) has no incentive to deviate to $s = 1 + \theta_{SQ}$ (resp. 0). It remains to show that any type never deviates to off-the-equilibrium-path signals. For type $\theta \in [0, \theta_{SQ})$, he has no incentive to deviate to signal $s \in (0, 1 + \theta_{SQ})$ because action $y = y_R$ is induced with paying more signaling costs. He also has no incentive to deviate to signal $s > 1 + \theta_{SQ}$ because $U^*(\theta) > U^*(\theta, 1 + \theta_{SQ}) > U^*(\theta, s)$. Similarly, for type $\theta \in [\theta_{SQ}, 1]$, he has no incentive to deviate to signal $s \in (0, 1 + \theta_{SQ})$ because $U^*(\theta) \geq U^*(\theta, 0) > U^*(\theta, s)$. He also has no incentive to deviate to signal $s > 1 + \theta_{SQ}$ because $y = y_A$ is induced with paying more signaling costs. Obviously, μ^*, ν_r^*, and ν_y^* satisfy Conditions (v), (vi), and (vii) of the definition of PBE. Therefore, e^* is a PBE. Furthermore, Lemma 1 immediately implies that e^* is a D1 equilibrium.

Next, we show that the full-investigation equilibrium is optimal when $d \leq \bar{d}(\theta_{SQ})$. Suppose, in contrast, that there exists D1 equilibrium e' that strictly dominates full-investigation equilibrium e^*; that is, $V' > V^F$, where V' represents the principal's ex ante expected payoff on equilibrium e'. Because $V^F \geq V^U$ holds when $d \leq \bar{d}(\theta_{SQ})$, $V' > V^F$ implies that e' is also an informative equilibrium. Hence, by Lemma 5, $\#(S^+(\sigma')) = 2$, $\theta' \leq \theta_{SQ}$, and $\psi_r'(r_I \mid m_E) > 0$ must hold, where θ' is the discontinuous point of σ'. Hence,

$$
\begin{aligned}
V' &= \psi_r'(r_I \mid m_E) \left[\int_0^{\theta'} \theta_{SQ} d\theta + \max \left\{ \int_{\theta'}^1 \theta_{SQ} d\theta, \int_{\theta'}^1 \theta d\theta \right\} - d \right] + \left(1 - \psi_r'(r_I \mid m_E)\right) V^U \\
&:= \psi_r'(r_I \mid m_E) V'' + \left(1 - \psi_r'(r_I \mid m_E)\right) V^U. \qquad \text{(A.12)}
\end{aligned}
$$

As $V' > V^F \geq V^U$, we should have $V'' > V^F$. However,

$$V'' < \int_0^{\theta_{SQ}} \theta_{SQ} d\theta + \int_{\theta_{SQ}}^1 \theta d\theta - d = V^F, \tag{A.13}$$

which is a contradiction. Therefore, we conclude that full-investigation equilibrium is optimal.

Finally, suppose that $d > \bar{d}(\theta_{SQ})$. While informative equilibria never exist because investigation cost d exceeds the first-best outcome $\bar{V}$, uninformative equilibria still exist by Lemma 2. Thus, an uninformative equilibrium is optimal. ■

A.3 Proof of Proposition 2

A.3.1 Preliminaries

Lemma 6 *Consider the indirect communication mode. If e^* is an informative D1 equilibrium, then the following holds.*

(i) $\#(S^+(\sigma^*)) = 2$.

(ii) $M^+(\sigma^*, \phi^*) = M$.

(iii) $\psi_y^*(y_A \mid m_E, \emptyset) \neq \psi_y^*(y_A \mid m_O, \emptyset)$.

(iv) $\theta' \leq \theta_{SQ} - b$, *where* θ' *is a discontinuous point of* σ^*.

(v) If $b < 0$, *then* $|b| < 1 - \theta_{SQ}$.

By Claim 1 and Lemmas 3-(i) and 6, without loss of generality, we hereafter assume that informative D1 equilibrium e^* has the following structure in the indirect communication mode: (a) $\sigma^*(\theta) = s_1$ if $\theta < \theta'$ and s_2 otherwise, (b) $\psi_y^*(y_A \mid m_E, \emptyset) > \psi_y^*(y_A \mid m_O, \emptyset)$, and (c) $\phi^*(s_1) = m_O$ and $\phi^*(s_2) = m_E$, where $\theta' \in (0,1)$ and $s_1 < s_2$.

A.3.2 Proof of Proposition 2

Suppose that $b \in (-(1-\theta_{SQ}), 1-\theta_{SQ}]$. First, show that the following constitutes an informative D1 equilibrium:

$$
\begin{aligned}
\sigma^*(\theta) &= \begin{cases} 0 & \text{if } \theta < \theta^*(b), \\ 1+\theta^*(b) & \text{otherwise,} \end{cases} \\
\phi^*(s) &= \begin{cases} m_O & \text{if } s < 1+\theta^*(b), \\ m_E & \text{otherwise,} \end{cases} \\
\psi_r^*(m) &= r_N \text{ for any } m, \\
\psi_y^*(o) &= \begin{cases} y_A & \text{if } o = (m_E, \emptyset), \\ y_R & \text{if } o = (m_O, \emptyset), \end{cases} \\
\mu^*(s) &= \begin{cases} \mathcal{U}([0,\theta^*(b))) & \text{if } s = 0, \\ \mathcal{U}([\theta^*(b),1]) & \text{if } s = 1+\theta^*(b), \\ \mathcal{D}(\theta^*(b)) & \text{if } s \in (0, 1+\theta^*(b)), \\ \mathcal{D}(1) & \text{otherwise,} \end{cases} \\
\nu_y^*(o) &= \begin{cases} \mathcal{U}([0,\theta^*(b))) & \text{if } o = (m_O, \emptyset), \\ \mathcal{U}([\theta^*(b),1]) & \text{if } o = (m_E, \emptyset). \end{cases}
\end{aligned}
\tag{A.14}
$$

The optimality of ψ_y^* given ν_y^* is as follows. Given $o = (m_O, \emptyset)$, $\mathbb{E}_{\nu_y^*(m_O,\emptyset)}[\theta] = \theta^*(b)/2$. Note that $\theta^*(b)/2 = (\theta_{SQ} - b)/2$ if $b \in (0, 1-\theta_{SQ}]$, $\theta_{SQ}/2$ if $b \in [-b_{FB}, 0)$, and $\theta_{SQ} - b - 1/2$

otherwise. Hence, because $\theta^*(b)/2 < \theta_{SQ}$ for any $b \in (-(1-\theta_{SQ}), 1-\theta_{SQ}]$, $y = y_R$ is optimal. Given $o = (m_E, \emptyset)$, $\mathbb{E}_{\nu_y^*(m_E,\emptyset)}[\theta] = (1 + \theta^*(b))/2$. Note that $(1 + \theta^*(b))/2 = (1 + \theta_{SQ} - b)/2$ if $b \in (0, 1 - \theta_{SQ}]$, $(1 + \theta_{SQ})/2$ if $b \in [-b_{FB}, 0)$, and $\theta_{SQ} - b$ otherwise. Hence, because $(1 + \theta^*(b))/2 > \theta_{SQ}$ for any $b \in (-(1 - \theta_{SQ}), 1 - \theta_{SQ}]$, $y = y_A$ is optimal.

The optimality of ϕ^* given μ^* and ψ_y^* is as follows. Given $s = 0$, $\mathbb{E}_{\mu^*(0)}[\theta+b] = \theta^*(b)/2+b < \theta_{SQ}$ holds for any $b \in (-(1 - \theta_{SQ}), 1 - \theta_{SQ}]$, implying that $m = m_O$ is optimal. Given $s = 1+\theta^*(b)$, $\mathbb{E}_{\mu^*(1+\theta^*(b))}[\theta+b] = (1+\theta^*(b))/2+b \geq \theta_{SQ}$ holds for any $b \in (-(1-\theta_{SQ}), 1-\theta_{SQ}]$, implying that $m = m_E$ is optimal. Given $s \in (0, 1 + \theta^*(b))$, $\mathbb{E}_{\mu^*(s)}[\theta + b] = \theta^*(b) + b \leq \theta_{SQ}$ holds for any $b \in (-(1-\theta_{SQ}), 1-\theta_{SQ}]$, implying that $m = m_O$ is optimal. Given $s > 1+\theta^*(b)$, $\mathbb{E}_{\mu^*(s)}[\theta + b] = 1 + b > \theta_{SQ}$ holds for any $b \in (-(1 - \theta_{SQ}), 1 - \theta_{SQ}]$, implying that $m = m_E$ is optimal.

The optimality of σ^* given ϕ^* and ψ_y^* is as follows. First, show that any type never mimics the other types. Fix $\theta \in [0, \theta^*(b))$, arbitrarily, implying that $U^*(\theta) = 0$. By Lemma 3-(iv), $U^*(\theta^*(b)) = 0$ holds. Hence, if type θ deviates to signal $s = 1 + \theta^*(b)$, then $U^*(\theta, 1 + \theta^*(b)) < U^*(\theta^*(b), 1 + \theta^*(b)) = U^*(\theta^*(b)) = 0 = U^*(\theta)$. Similarly, if $\theta \in [\theta^*(b), 1]$, then $U^*(\theta) \geq U^*(\theta^*(b), 1 + \theta^*(b)) = 0 = U^*(\theta, 0)$. That is, any type has no incentive to mimic the other types. Next, show that any type never deviates to off-the-equilibrium-path signals. Fix $\theta \in [0, 1]$, arbitrarily. If he deviates to signal $s \in (0, 1 + \theta^*(b))$, then action $y = y_R$ is induced, which means that $U^*(\theta, s) < 0 \leq U^*(\theta)$. If he deviates to signal $s > 1 + \theta^*(b)$, then action $y = y_A$ is induced, which means that $U^*(\theta, s) < U^*(\theta, 1 + \theta^*(b)) \leq U^*(\theta)$. We then conclude that σ^* is optimal.

Because it is obvious that μ^* and ν_y^* satisfy conditions (iv), (v), (vi) of the definition of PBE, we say that the above constitutes a PBE. Furthermore, Lemma 1 assures that μ^*

satisfies the D1 criterion, implying that e^* is an informative D1 equilibrium.

Next, we show that e^* is the no-investigation equilibrium. It is obvious for $b \in [-b_{FB}, 0)$ because $\theta^*(b) = \theta_{SQ}$. For $b \in (0, 1-\theta_{SQ}]$, if e^* is not the no-investigation equilibrium, then there exists another informative D1 equilibrium on which the agent's strategy is discontinuous at $\theta' > \theta_{SQ} - b$. However, it is impossible by Lemma 6-(iv). Note that for $b \in (-(1-\theta_{SQ}), -b_{FB})$, $\theta^*(b) > \theta_{SQ}$ and $\mathbb{E}[\theta \mid \theta \geq \theta^*(b)] = \theta_{SQ} - b$ hold. Hence, if e^* is not the no-investigation equilibrium, then there exists another informative D1 equilibrium e', on which (i) properties (a), (b), and (c) mentioned above are satisfied, and (ii) $\theta' < \theta^*(b)$. Because $\mathbb{E}[\theta \mid \theta \geq \theta^*(b)] = \theta_{SQ} - b$, $\mathbb{E}[\theta \mid \theta \geq \theta'] < \theta_{SQ} - b$ must hold. However, it implies that $\phi'(s_1) = \phi'(s_2) = m_O$, which is a contradiction to Lemma 6-(ii). Therefore, e^* should be the no-investigation equilibrium.

Finally, we show that there exists no informative D1 equilibrium when $b \notin (-(1-\theta_{SQ}), 1-\theta_{SQ}]$. Suppose, in contrast, that there exists an informative D1 equilibrium e' satisfying properties (a) to (c) mentioned above in the parameter range. Note that for $b \leq -(1-\theta_{SQ})$, we have $|b| \geq 1-\theta_{SQ}$, which is a contradiction to Lemma 6-(v). Hence, $b > 1-\theta_{SQ}$ should hold. By Lemma 6-(iv), we should have $\theta' \leq \theta_{SQ} - b$. However, $\mathbb{E}_{\nu'_y(m_E,\emptyset)}[\theta] = \mathbb{E}[\theta \mid \theta \geq \theta'] \leq \mathbb{E}[\theta \mid \theta \geq \theta_{SQ} - b] < \mathbb{E}[\theta \mid \theta \geq 2\theta_{SQ} - 1] = \theta_{SQ}$, implying that $\psi'_y(m_E, \emptyset) = y_R$, which is a contradiction to property (b). Therefore, there never exists an informative D1 equilibrium when $b \notin (-(1-\theta_{SQ}), 1-\theta_{SQ}]$. Thus, an uninformative equilibrium is optimal in this parameter range. ■

A.4 Proof of Corollary 1

The proof for $\theta_{SQ} > 1/2$ is identical to that of Proposition 6-(i) mentioned below. We can show the statement for $\theta_{SQ} < 1/2$ by using a similar argument.[42] ■

A.5 Proof of Proposition 3

(i) (Sufficiency) We show that the following is a partial-investigation equilibrium under the assumed parameter range:

$$\sigma^*(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, \theta_+), \\ s_1 & \text{if } \theta \in [\theta_+, \theta_{SQ}), \\ s_2 & \text{otherwise}, \end{cases}$$
$$\phi^*(s) = \begin{cases} m_E & \text{if } s \geq s_1, \\ m_O & \text{otherwise}, \end{cases}$$
$$\psi_r^*(r_I \mid m) = \begin{cases} \psi_+ & \text{if } m = m_E, \\ 0 & \text{otherwise}, \end{cases} \tag{A.15}$$
$$\psi_y^*(o) = \begin{cases} y_A & \text{if } o = (m_E, \emptyset) \text{ or } o = (m_E, s) \text{ with } s \geq s_2, \\ y_R & \text{otherwise}, \end{cases}$$

[42] See Appendix B.9 for the detail.

$$\mu^*(s) = \begin{cases} \mathcal{U}([0,\theta_+)) & \text{if } s = 0, \\ \mathcal{U}([\theta_+,\theta_{SQ}]) & \text{if } s = s_1, \\ \mathcal{U}([\theta_{SQ},1]) & \text{if } s = s_2, \\ \mathcal{D}(\theta_+) & \text{if } s \in (0,s_1), \\ \mathcal{D}(\theta_{SQ}) & \text{if } s \in (s_1,s_2), \\ \mathcal{D}(1) & \text{otherwise,} \end{cases}$$

$$\nu_r^*(s \mid m) = \begin{cases} 1 & \text{if } m = m_O \text{ and } s = 0, \\ (\theta_{SQ}-\theta_+)/(1-\theta_+) & \text{if } m = m_E \text{ and } s = s_1, \\ (1-\theta_{SQ})/(1-\theta_+) & \text{if } m = m_E \text{ and } s = s_2, \\ 0 & \text{otherwise,} \end{cases}$$

$$\nu_y^*(o) = \begin{cases} \mathcal{U}([0,\theta_+)) & \text{if } o = (m_O,\emptyset), \\ \mathcal{U}([\theta_+,1]) & \text{if } o = (m_E,\emptyset), \\ \mu^*(s) & \text{otherwise,} \end{cases}$$

where $\psi_+ := (2b-\delta_+)/(2b-\delta_++2\pi)$, $s_1 := (1-\psi_+)(1+\theta_+)$, and $s_2 := (1+\theta_{SQ})-(1-\psi_+)\delta_+$. We have the following observations. First, $2b^2/(1-\theta_{SQ}+2b) \le (1-\theta_{SQ})/4$ is equivalent to $b \le (1-\theta_{SQ})/2$, which implies that $\delta_+ \le 2b$, and then $\psi_+ \in (0,1)$ holds. Second, in this parameter range, $0 < s_1 < s_2$ holds. Finally, because $\theta_{SQ} > 1/2$, as long as $d \le (1-\theta_{SQ})/4$, $\theta_{SQ} \le 1-\delta_+$ and $\theta_{SQ} > \delta_+$ hold.

The optimality of ψ_y^* under ν_y^* is as follows. It is obvious that ψ_y^* represents a best response for any $o \neq (m_E,\emptyset)$. Given $o = (m_E,\emptyset)$, the principal's expected payoff from

$y = y_A$ is $(1+\theta_+)/2$. Hence, choosing action $y = y_A$ is a best response if and only if $(1+\theta_+)/2 \geq \theta_{SQ}$, which is equivalent to $\theta_{SQ} \leq 1-\delta_+$. As mentioned above, this inequality holds in this parameter range.

The optimality of ψ_r^* under ν_r^* and ψ_y^* is as follows. Given $m = m_O$, certainly choosing action $r = r_N$ is optimal. For $m = m_E$, by construction of θ_+, the principal is indifferent between r_I and r_N. Hence, any randomization over these actions is optimal.

The optimality of ϕ^* given μ^*, ψ_r^*, and ψ_y^* is as follows. The manager's expected payoffs from $m = m_O$ and m_E are θ_{SQ} and

$$\begin{cases} \psi_+(\theta_{SQ}-\pi)+(1-\psi_+)(\mathbb{E}_{\mu^*(s)}[\theta+b]) & \text{if } s < s_2, \\ \mathbb{E}_{\mu^*(s)}[\theta+b]-\psi_+\pi & \text{otherwise,} \end{cases} \tag{A.16}$$

respectively. Hence, ϕ^* represents a best response if and only if the following holds:

$$\theta_{SQ} \geq \psi_+(\theta_{SQ}-\pi)+(1-\psi_+)\left(\frac{1}{2}\theta_+ + b\right), \tag{A.17}$$

$$\theta_{SQ} \geq \psi_+(\theta_{SQ}-\pi)+(1-\psi_+)(\theta_+ + b), \tag{A.18}$$

$$\theta_{SQ} \leq \psi_+(\theta_{SQ}-\pi)+(1-\psi_+)\left(\frac{1}{2}(\theta_+ + \theta_{SQ}) + b\right), \tag{A.19}$$

$$\theta_{SQ} \leq \psi_+(\theta_{SQ}-\pi)+(1-\psi_+)(\theta_{SQ} + b), \tag{A.20}$$

$$\theta_{SQ} \leq \frac{1}{2}(1+\theta_{SQ})+b-\psi_+\pi, \tag{A.21}$$

$$\theta_{SQ} \leq 1+b-\psi_+\pi. \tag{A.22}$$

Note that only (A.18) and (A.19) are potentially binding. With simple algebra, we can show that (A.18) and (A.19) are satisfied when $\psi_+ = (2b-\delta_+)/(2b-\delta_+ + 2\pi)$.

The optimality of σ^* given ϕ^*, ψ_r^*, and ψ_y^* are as follows. Note that for any off-the-equilibrium-path signal $s \in S^-(\sigma^*)$, the induced probability of approval is

$$G(s) = \begin{cases} G(0) & \text{if } s \in (0, s_1), \\ G(s_1) & \text{if } s \in (s_1, s_2), \\ G(s_2) & \text{if } s > s_2, \end{cases} \tag{A.23}$$

which means that deviations to off-the-equilibrium-path signals are dominated by mimicking the other types. Hence, it is sufficient to show that any type has no incentive to mimic the other types. First, for type $\theta \in [0, \theta_+)$, $U^*(\theta) = 0$ holds. If he deviates to signal $s = s_1$, then $U^*(\theta, s_1) < G(s_1) - C(\theta_+, s_1) = U^*(\theta_+) = 0$, where the last equality comes from Lemma 3-(iv). Similarly, if he deviates to signal $s = s_2$, then $U^*(\theta, s_2) = G(s_2) - C(\theta, s_2) < G(s_1) - C(\theta, s_1) = U^*(\theta, s_1) < U^*(\theta)$, where the first inequality comes from the fact that $G(s_2) - C(\theta_{SQ}, s_2) = G(s_1) - C(\theta_{SQ}, s_1)$ and $\theta < \theta_{SQ}$. Next, for $\theta \in [\theta_+, \theta_{SQ})$, note that $U^*(\theta) = G(s_1) - C(\theta, s_1) \geq G(s_1) - C(\theta_+, s_1) = 0 = U^*(\theta, 0)$. If he deviates to signal $s = s_2$, then $U^*(\theta, s_2) = G(s_2) - C(\theta, s_2) < G(s_1) - C(\theta, s_1) = U^*(\theta)$, where the inequality comes from the fact that $G(s_2) - C(\theta_{SQ}, s_2) = G(s_1) - C(\theta_{SQ}, s_1)$ and $\theta < \theta_{SQ}$. Finally, for $\theta \in [\theta_{SQ}, 1]$, note that $U^*(\theta) = 1 - C(\theta, s_2) > 1 - C(\theta, 1 + \theta_{SQ}) \geq 0 = U^*(\theta, 0)$. If he deviates to signal $s = s_1$, then $U^*(\theta, s_1) = G(s_1) - C(\theta, s_1) \leq G(s_2) - C(\theta, s_2) = U^*(\theta)$, where the inequality comes from the fact that $G(s_2) - C(\theta_{SQ}, s_2) = G(s_1) - C(\theta_{SQ}, s_1)$ and $\theta \geq \theta_{SQ}$. Therefore, we conclude that any type has no incentive to mimic the other types.

Because it is straightforward that μ^*, ν_r^*, and ν_y^* satisfy Conditions (v), (vi), and (vii), e^* is a PBE. Furthermore, Lemma 1 implies that e^* is a D1 equilibrium; that is, e^* is a partial-investigation equilibrium.

(Necessity) Suppose, in contrast, that there exists partial-investigation equilibrium e' when either (a') $b < (1-\theta_{SQ})/2$ and $d \geq 2b^2/(1-\theta_{SQ}+2b)$ or (b') $b \geq (1-\theta_{SQ})/2$ and $d > (1-\theta_{SQ})/4$. By definition of a partial-investigation equilibrium, the first discontinuous point of σ' is also given by θ_+. First, consider Scenario (a'). Note that $d \geq 2b^2/(1-\theta_{SQ+2b})$ is equivalent to $\delta_+ \geq 2b$. Because e' is a partial-investigation equilibrium, the manager endorses the new project if he observes signal $s = s'_1$, where $S^+(\sigma') := \{s'_0, s'_1, s'_2\}$ with $s'_0 < s'_1 < s'_2$. That is, $\theta_{SQ} \leq \psi'_+(\theta_{SQ}-\pi) + (1-\psi'_+)((\theta_+ + \theta_{SQ})/2 + b)$, or equivalently, $\psi'_+(\delta_+ - 2b - 2\pi) \geq \delta_+ - 2b$ must hold, where $\psi'_+ := \psi'_r(r_I \mid m_E)$. However, as long as $\delta_+ \geq 2b$, it is never satisfied, which is a contradiction. Next, consider scenario (b'). Because $d > (1-\theta_{SQ})/4$, $\theta_{SQ} > 1-\delta_+$ holds. However, it means that the principal chooses $y = y_R$ under $o = (m_E, \emptyset)$, which is a contradiction.

(ii) The proof is essentially equivalent to that of Part (i). Hence, we omit the proof except for the characterization of equilibrium strategies.[43]

$$\sigma^*(\theta) = \begin{cases} 0 & \text{if } \theta \in [0, \theta_{SQ}), \\ s_1 & \text{if } \theta \in [\theta_{SQ}, \theta_-), \\ s_2 & \text{if } \theta \in [\theta_-, 1], \end{cases}$$

$$\phi^*(s) = \begin{cases} m_O & \text{if } s < s_2, \\ m_E & \text{otherwise}, \end{cases} \tag{A.24}$$

[43]Because the characterization of equilibrium beliefs are straightforward, it is also omitted.

$$\psi_r^*(r_I \mid m) = \begin{cases} \psi_- & \text{if } m = m_O, \\ 0 & \text{otherwise,} \end{cases}$$

$$\psi_y^*(o) = \begin{cases} y_R & \text{if } o = (m_O, \emptyset) \text{ or } (m_O, s) \text{ with } s < s_1, \\ y_A & \text{otherwise,} \end{cases}$$

where $\delta_- := d + \sqrt{d^2 + 2\theta_{SQ} d}$, $\theta_- := \theta_{SQ} + \delta_-$, $\psi_- := (|b| - \delta_-)/(|b| - \delta_- + \pi)$, $s_1 := \psi_-(1 + \theta_{SQ})$, and $s_2 := (1 + \theta_{SQ}) + (1 - \psi_-)\delta_-$. ■

A.6 Proof of Proposition 4

(i) Note that $V^F = (1 + \theta_{SQ}^2)/2 - d$, $V^N = (1 + \theta_{SQ}^2 - b^2)/2$, and $V^P = (1 + \theta_{SQ}^2 - \delta_+^2)/2$ whenever each equilibrium exists. First, we compare V^F and V^P when both the full- and the partial-investigation equilibria exist, i.e., $d < \min\{(1 - \theta_{SQ})^2/2, 2b^2/(1 - \theta_{SQ} + 2b)\}$. Note that

$$\begin{aligned} V^P \geq V^F &\iff d \geq \frac{1}{2}\delta_+^2 \\ &\iff d \leq \frac{1}{2}\theta_{SQ}^2, \end{aligned} \tag{A.25}$$

which is always satisfied in the parameter range because $(1 - \theta_{SQ})^2/2 < \theta_{SQ}^2/2$ for $\theta_{SQ} > 1/2$. That is, the full-investigation equilibrium is dominated by the partial-investigation equilibrium whenever both equilibria exist.

Next, we compare V^N and V^F when the all but the partial-investigation equilibrium exist, i.e., $b \le 1-\theta_{SQ}$ and $d \in [2b^2/(1-\theta_{SQ}+2b), (1-\theta_{SQ})^2/2]$. Note that

$$V^N \ge V^F \iff d \ge \frac{1}{2}b^2, \tag{A.26}$$

which is always satisfied because $2b^2/(1-\theta_{SQ}+2b) > b^2/2$ holds in this parameter range. Hence, we say that the full-investigation equilibrium is dominated by the no-investigation equilibrium when the partial-investigation equilibrium does not exist.

Finally, we compare V^P and V^N whenever both equilibria exist, i.e., $b \le \theta_{SQ}$ and $d < \min\{(1-\theta_{SQ})/4, 2b^2/(1-\theta_{SQ}+2b)\}$. Note that

$$\begin{aligned} V^P \ge V^N &\iff b^2 \ge \delta_+^2 \\ &\iff d \le d_+ = \frac{b^2}{2(1-\theta_{SQ}+b)}. \end{aligned} \tag{A.27}$$

That is, the no-investigation equilibrium dominates the partial-investigate equilibrium if and only if $d \in (d_+, \min\{(1-\theta_{SQ})/4, 2b^2/(1-\theta_{SQ}+2b)\})$.[44] Furthermore, in the other regions, either exactly one of the three equilibria exists or only uninformative equilibria exist.

(ii) Because Proposition 2 implies that the no-investigation equilibrium implements the first-best outcome when $|b| \in (0, b_{FB}]$, it is straightforward that the no-investigation equilibrium is optimal in this scenario. Hence, hereafter, we restrict our attention to the scenario of $|b| > b_{FB}$. Note that $V^F = (1+\theta_{SQ}^2)/2 - d$, $V^N = \theta_{SQ} - 2|b|^2 + 2(1-\theta_{SQ})|b|$, and $V^P = (1+\theta_{SQ}^2 - \delta_-^2)/2$ whenever each equilibrium exists. First, we compare V^F and V^P whenever both the full- and the partial-investigation equilibria exist, i.e., $d <$

[44] As $d_+ < 2b^2/(1-\theta_{SQ}+2b)$ and $d_+ \le (1-\theta_{SQ})/4$ for $b \le 1-\theta_{SQ}$ hold, this interval is well defined.

$\min\{|b|^2/[2(\theta_{SQ}+|b|)], (1-\theta_{SQ})^2/2\}$. Note that

$$\begin{aligned} V^P \geq V^F &\iff d \geq \frac{1}{2}\delta_-^2 \\ &\iff d \leq \frac{1}{2}(1-\theta_{SQ})^2, \end{aligned} \tag{A.28}$$

which is always satisfied in this parameter range. That is, the partial-investigation equilibrium dominates the full-investigation equilibrium whenever both equilibria exist.

Next, we compare V^N and V^F when all but the partial-investigation equilibrium exist, i.e., $|b| \in (b_{FB}, 1-\theta_{SQ}]$ and $d \in [|b|^2/[2(\theta_{SQ}+|b|)], (1-\theta_{SQ})^2/2]$. Note that

$$V^N \geq V^F \iff d \geq \frac{1}{2}(2|b|-1+\theta_{SQ})^2. \tag{A.29}$$

Hence, it is sufficient to show that $|b|^2/[2(\theta_{SQ}+|b|)] > (2|b|-1+\theta_{SQ})^2/2$ holds in this parameter range to claim that the no-investigation equilibrium dominates the full-investigation equilibrium. Specifically,

$$\begin{aligned} \frac{|b|^2}{2(\theta_{SQ}+|b|)} > \frac{1}{2}(2|b|-1+\theta_{SQ})^2 &\iff 4|b|^2 + (4\theta_{SQ}-1)|b| - \theta_{SQ}(1-\theta_{SQ}) > 0 \\ &\iff |b| > \frac{1}{8}\left(1-4\theta_{SQ}+\sqrt{8\theta_{SQ}+1}\right), \end{aligned} \tag{A.30}$$

which always holds because $|b| > b_{FB} > (1-4\theta_{SQ}+\sqrt{8\theta_{SQ}+1})/8$.

Finally, we compare V^P and V^N when both equilibria exist, i.e., $|b| \in (b_{FB}, 1-\theta_{SQ}]$ and

$d < \min\{|b|^2/[2(\theta_{SQ}+|b|)], (1-\theta_{SQ})^2/2\}$. Note that

$$V^P \geq V^N \iff d \leq d_- = \frac{(2|b|-1+\theta_{SQ})^2}{2(2|b|-1+2\theta_{SQ})}. \tag{A.31}$$

That is, the no-investigation equilibrium dominates the partial-investigate equilibrium if and only if $d \in (d_-, \min\{|b|^2/[2(\theta_{SQ}+|b|)], (1-\theta_{SQ})^2/2\})$.[45] Furthermore, in the other regions, either exactly one of the three equilibria exists or only uninformative equilibria exist. ■

A.7 Proof of Corollary 2

It is straightforward from Proposition 4. ■

A.8 Proof of Proposition 5

Suppose that $b > 0$ and $\theta_{SQ} > 1/2$.[46] By the hypothesis, we have $V_1 = \theta_{SQ}$, $V_2 = \max\{V^N, \theta_{SQ}\}$, $V_3 = V^F$, and $V_4 = \max\{V^N, V^P\}$. Note that $d \geq d_+$ is equivalent to $b \leq \delta_+$. Hence,

$$\Lambda_I = \begin{cases} ((1-\theta_{SQ})^2 - b^2)/2 & \text{if } b \leq 1-\theta_{SQ}, \\ 0 & \text{otherwise,} \end{cases} \tag{A.32}$$

$$\Lambda_C = \begin{cases} d - b^2/2 & \text{if } b \leq \delta_+ \\ d - \delta_+^2/2 & \text{otherwise.} \end{cases} \tag{A.33}$$

[45] As $d_- < |b|^2/(\theta_{SQ}+|b|)$ holds, this interval is well defined.

[46] By the similar argument used here, we can show the statement for the other cases. The detail is in Appendix B.8.

Because $d < (1-\theta_{SQ})^2/2 < (1-\theta_{SQ})/4$ implies that $\delta_+ < 1-\theta_{SQ}$, there are the following three cases. First, suppose that $0 < b \leq \delta_+$. Note that $\Lambda_I - \Lambda_C = (1-\theta_{SQ})^2/2 - d > 0$, where the inequality comes from the hypothesis. Second, suppose that $\delta_+ < b \leq 1-\theta_{SQ}$. Let $\lambda(b) := \Lambda_I - \Lambda_C = ((1-\theta_{SQ})^2 - b^2)/2 - d + \delta_+^2/2$. Because $\lambda(\delta_+) = (1-\theta_{SQ})^2/2 - d > 0$, $\lambda(\theta_{SQ}) = -d + \delta_+^2/2 < 0$, and $\lambda'(b) = -b < 0$, the intermediate value theorem implies that there exists $\beta^*(\theta_{SQ}, b, d) \in (\delta_+, \theta_{SQ})$ such that $\Lambda_I \geq \Lambda_C$ if and only if $b \leq \beta^*(\theta_{SQ}, b, d)$.[47] Finally, suppose that $b > 1-\theta_{SQ}$. Note that $\Lambda_I - \Lambda_C = -d + \delta_+^2/2 < 0$. ■

A.9 Proof of Corollary 3

It is straightforward from Proposition 5. ■

A.10 Proof of Proposition 6

(i) Suppose that $d > (1-\theta_{SQ})/4$. Note that a partial-investigation equilibrium never exists irrelevant to the direction of the bias. Define $\zeta(b, \theta_{SQ}) := |\theta^*(b) - \theta_{SQ}|$, implying that

$$\zeta(b,\theta_{SQ}) = \begin{cases} b & \text{if } 0 < b \leq 1-\theta_{SQ}, \\ 0 & \text{if } -b_{FB} \leq b < 0, \\ 2|b| - (1-\theta_{SQ}) & \text{if } -(1-\theta_{SQ}) \leq b < -b_{FB}, \\ 1-\theta_{SQ} & \text{otherwise.} \end{cases} \tag{A.34}$$

By simple algebra, we immediately obtain that $\zeta(|b|, \theta_{SQ}) \geq \zeta(-|b|, \theta_{SQ})$ for any $b \neq 0$.[48]

(ii) Suppose that $d \leq (1-\theta_{SQ})/4$. Define $\zeta(b, d, \theta_{SQ}) := |\theta' - \theta_{SQ}|$, where θ' is the

[47] Specifically, $\beta^*(\theta_{SQ}, b, d) = \sqrt{(1-\theta_{SQ})^2 + \delta_+^2 - 2d}$.

[48] $\zeta(b, d)$ is represented as in Figure 8.

threshold of the optimal equilibrium, and $b_- := (1-\theta_{SQ}+\delta_-)/2$. Then, we have the following representation:

$$\zeta(b,d,\theta_{SQ}) = \begin{cases} b & \text{if } 0 < b \leq \delta_+, \\ \delta_+ & \text{if } b > \delta_+, \\ 0 & \text{if } -b_{FB} \leq b < 0, \\ 2|b| - (1-\theta_{SQ}) & \text{if } -b_- \leq b < -b_{FB}, \\ \delta_- & \text{otherwise.} \end{cases} \tag{A.35}$$

To complete the proof, we show the following lemma.

Lemma 7 *Suppose that $d \leq (1-\theta_{SQ})/4$ and $\theta_{SQ} > 1/2$.*

(i) $b_- > \delta_+$.

(ii) There exists a unique $|b'| \in [\delta_+, b_-)$ such that $\zeta(|b'|,d,\theta_{SQ}) = \zeta(-|b'|,d,\theta_{SQ})$.

Proof of Lemma 7. (i) Note that $\delta_- > \delta_+$ holds because $\theta_{SQ} > 1/2$. Hence,

$$b_- - \delta_+ > \frac{1}{2}(1-\theta_{SQ}+\delta_+) - \delta_+ = \frac{1}{2}(1-\theta_{SQ}-\delta_+) \geq 0, \tag{A.36}$$

where the first and the last inequalities comes from the facts that $\delta_- > \delta_+$ and $1-\theta_{SQ} \geq \delta_+$, respectively.[49]

(ii) Define $|b'|$ by $\zeta(|b'|,d,\theta_{SQ}) = \zeta(-|b'|,d,\theta_{SQ})$ and suppose its existence. First, suppose, in contrast, that $|b'| \in (0,b_{FB}] \cup [b_-,\infty)$. If $|b'| \in (0,b_{FB}]$, then $\zeta(-|b'|,d,\theta_{SQ}) = 0 < \zeta(|b'|,d,\theta_{SQ})$ must hold, which is a contradiction. Likewise, if $|b'| \in [b_-,\infty)$, then

[49]Note that $\delta_+ \leq 1-\theta_{SQ}$ is equivalent to $d \leq (1-\theta_{SQ})/4$.

$\zeta(|b'|, d, \theta_{SQ}) = \delta_+ < \zeta(-|b'|, d, \theta_{SQ}) = \delta_-$ because of $\theta_{SQ} > 1/2$, which is a contradiction. Hence, $|b'| \in (b_{FB}, b_-)$ should hold, implying that $\zeta(-|b'|, d, \theta_{SQ}) = 2|b'| - (1 - \theta_{SQ})$. Second, suppose, in contrast, that $|b'| \in (0, \delta_+)$. Because $\zeta(|b'|, d, \theta_{SQ}) = |b'|$, $\zeta(|b'|, d, \theta_{SQ}) = \zeta(-|b'|, d, \theta_{SQ})$ implies that $|b'| = 1 - \theta_{SQ}$. However, because $d \leq (1 - \theta_{SQ})/4$, $\delta_+ \leq 1 - \theta_{SQ}$ holds, which is a contradiction to $|b'| < \delta_+$. Therefore, $|b'| \in [\delta_+, b_-)$ must hold. Finally, we guarantee the existence of $|b'|$ in this region. Define $\xi(|b|) := \zeta(-|b|, d, \theta_{SQ}) - \zeta(|b|, d, \theta_{SQ}) = 2|b| - (1 - \theta_{SQ}) - \delta_+$. Because $\xi(\delta_+) = \delta_+ - (1 - \theta_{SQ}) \leq 0 < \xi(b_-) = \delta_- - \delta_+$ and $\xi'(|b|) > 0$, the intermediate value theorem assures that there exists a unique $|b'| \in [\delta_+, b_-)$ such that $\xi(|b'|) = 0$. Specifically, $|b'| = (1 - \theta_{SQ} + \delta_+)/2$. □

Lemma 7 immediately implies the statement.[50] ■

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[50] $\zeta(|b|, d, \theta_{SQ})$ can be represented as in Figure 9.

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B Supplementary Materials (for Online Appendix)

This appendix provides the omitted proofs of Appendix A and supplementary results. For any $s \in S$, let $\Theta(s) := \{\, \theta \in \Theta \mid \sigma^*(\theta) = s \,\}$ represent the set of types who send signal s under equilibrium strategy σ^*. Likewise, for any m, let $S(m) := \{\, s \in S^+(\sigma^*) \mid \phi^*(s) = m \,\}$ represent the set of on-the-equilibrium-path signals inducing message m under equilibrium strategy ϕ^*.

B.1 Proof of Lemma 2

Suppose that $\theta_{SQ} < 1/2$. We then show that the following is a D1 equilibrium:

$$\begin{aligned}
\sigma^*(\theta) &= 0 \text{ for any } \theta, \\
\phi^*(s) &= m_E \text{ for any } s, \\
\psi_r^*(m) &= r_N \text{ for any } m, \\
\psi_y^*(o) &= y_A \text{ for any } o, \\
\mu^*(s) &= \begin{cases} \mathcal{U}([0,1]) & \text{if } s = 0, \\ \mathcal{D}(1) & \text{if } s > 0, \end{cases} \\
\nu_r^*(m) &= \mathcal{D}(0) \text{ for any } m, \\
\nu_y^*(o) &= \begin{cases} \mathcal{U}([0,1]) & \text{if } o = (m, \emptyset) \text{ or } (m, 0) \text{ for any } m, \\ \mathcal{D}(1) & \text{otherwise.} \end{cases}
\end{aligned} \tag{B.1}$$

Because $\theta_{SQ} < 1/2$, it is obvious that ψ_y^* is optimal given ν_y^*. Given ν_r^*, ν_y^* and ψ_y^*, the principal's expected payoffs from $r = r_I$ and r_N are $1/2 - d$ and $1/2$, respectively. Hence, we say that ψ_r^* is optimal. Because the principal's behaviors are irrelevant to message m given ψ_r^* and ψ_y^*, ϕ^* is obviously optimal. Note that $U^*(\theta) = 1$ for any θ. If the agent with type θ deviates to signal $s' > 0$, then this deviation is not recognized by the principal given ϕ^* and ψ_r^*. It means that $U^*(\theta, s') < U^*(\theta)$, implying that σ^* is optimal. Finally, it is obvious that μ^*, ν_r^*, and ν_y^* satisfy Conditions (v), (vi), and (vii) of the definition of PBE. Therefore, we say that e^* is a PBE. Furthermore, Lemma 1 implies that e^* is a D1 equilibrium. We can guarantee the existence of an uninformative D1 equilibrium for $\theta_{SQ} > 1/2$ by using a similar argument. ■

B.2 Proof of Claim 1

Suppose, in contrast, that there exist PBE e^* and $s, s' \in S^+(\sigma^*)$ such that $s < s'$ and $G(s) \geq G(s')$. Fix $\theta \in \Theta(s')$, arbitrarily. However,

$$\begin{aligned} U^*(\theta) &= G(s') - C(\theta, s') \\ &< G(s) - C(\theta, s) = U^*(\theta, s), \end{aligned} \tag{B.2}$$

which is a contradiction to that $\sigma^*(\theta) = s'$. Therefore, $G(s) < G(s')$ must hold. ■

B.3 Proof of Lemma 3

B.3.1 Part (i)

Suppose, in contrast, that there exists a PBE e^* such that $\sigma^*(\theta') < \sigma^*(\theta)$ for some $\theta, \theta' \in \Theta$ with $\theta < \theta'$. Because σ^* is an equilibrium strategy, the following IC conditions should hold:

$$U^*(\theta) \geq U^*(\theta, \sigma^*(\theta')) \iff G(\sigma^*(\theta)) - G(\sigma^*(\theta')) \geq C(\theta, \sigma^*(\theta)) - C(\theta, \sigma^*(\theta')); \tag{B.3}$$

$$U^*(\theta') \geq U^*(\theta', \sigma^*(\theta)) \iff C(\theta', \sigma^*(\theta)) - C(\theta', \sigma^*(\theta')) \geq G(\sigma^*(\theta)) - G(\sigma^*(\theta')). \tag{B.4}$$

Because $C_{s\theta} < 0$, $\theta < \theta'$, and $\sigma^*(\theta') < \sigma^*(\theta)$, we have

$$C(\theta, \sigma^*(\theta)) - C(\theta, \sigma^*(\theta')) > C(\theta', \sigma^*(\theta)) - C(\theta', \sigma^*(\theta')). \tag{B.5}$$

However, (B.3), (B.4), and (B.5) imply that

$$\begin{aligned} C(\theta, \sigma^*(\theta)) - C(\theta, \sigma^*(\theta')) &> G(\sigma^*(\theta)) - G(\sigma^*(\theta')) \\ &\geq C(\theta, \sigma^*(\theta)) - C(\theta, \sigma^*(\theta')), \end{aligned} \tag{B.6}$$

which is a contradiction. Therefore, σ^* must be nondecreasing in θ. ■

B.3.2 Part (ii)

It is straightforward from Lemma 3-(i) and the Lebesgue Theorem. ■

B.3.3 Auxiliary claims for Part (iii)

To show the second part, it is useful to show the following claims.

Claim 2 *For any PBE e^*, if there exists $s \in S^+(\sigma^*)$ such that $\psi_y^*(y_A \mid m, s) \in (0,1)$ for any $m \in M$, then $\psi_y^*(y_A \mid m, s') \notin (0,1)$ holds for any $s' \in S^+(\sigma^*)$ with $s' \neq s$ and $m \in M$.*

Proof of Claim 2. Suppose that there exists $s \in S^+(\sigma^*)$ such that $\psi_y^*(y_A \mid m, s) \in (0,1)$ for any $m \in M$. It means that the principal is indifferent between y_A and y_R given signal s, implying that $\mathbb{E}[\theta \mid \theta \in \Theta(s)] = \theta_{SQ}$. By Lemma 3-(i), $\Theta(s')$ should be an interval for any $s' \in S^+(\sigma^*)$. Hence, $\theta_{SQ} \in \Theta(s)$ and $\theta_{SQ} \notin \Theta(s')$ for any $s' \in S^+(\sigma^*)$ with $s' \neq s$ must hold. Because $\Theta(s')$ is an interval and $\theta_{SQ} \notin \Theta(s')$, we have $\mathbb{E}[\theta \mid \theta \in \Theta(s')] \neq \theta_{SQ}$, which implies that $\psi_y^*(y_A \mid m, s') \notin (0,1)$ holds for any $m \in M$. ■

Claim 3 *For any PBE e^* and $m \in M^+(\sigma^*, \phi^*)$, if there exist $s, s' \in S(m)$ with $s < s'$, then (i) $\psi_r^*(r_I \mid m) > 0$ and (ii) $\psi_y^*(y_A \mid m, s) < \psi_y^*(y_A \mid m, s')$ hold.*

Proof of Claim 3. Suppose, in contrast, that there exist PBE e^*, on-the-equilibrium-path message $m \in M^+(\sigma^*, \phi^*)$, and on-the-equilibrium-path signals $s, s' \in S(m)$ with $s < s'$ such that either (i) $\psi_r^*(r_I \mid m) = 0$ or (ii) $\psi_y^*(y_A \mid m, s) \geq \psi_y^*(y_A \mid m, s')$ holds. Fix $\theta \in \Theta(s')$, arbitrarily. First, consider case (i). Note that

$$\begin{aligned} U^*(\theta) &= \psi_y^*(y_A \mid m, \emptyset) - C(\theta, s') \\ &< \psi_y^*(y_A \mid m, \emptyset) - C(\theta, s) \\ &= U^*(\theta, s). \end{aligned} \tag{B.7}$$

However, it means that type θ has an incentive to deviate to signal s, which is a contradiction. That is, $\psi_r^*(r_I \mid m) > 0$ must hold. Next, consider case (ii). Because of Lemma 3-(i), $\mathbb{E}_{\nu_y^*(m,s)}[\theta] < \mathbb{E}_{\nu_y^*(m,s')}[\theta]$ holds. Hence, $\psi_y^*(y_A \mid m, s) \leq \psi_y^*(y_A \mid m, s')$ holds, implying that $\psi_y^*(y_A \mid m, s) = \psi_y^*(y_A \mid m, s)$. However, it implies that $G(s) = G(s')$, which is a contradiction to Claim 1. Therefore, $\psi_y^*(y_A \mid m, s) < \psi_y^*(y_A \mid m, s')$ must hold. ■

B.3.4 Part (iii)

Fix PBE e^*, arbitrarily. By Lemma 3-(i) and (ii), σ^* is either constant or strictly increasing whenever differentiable. Suppose, in contrast, that there exists subset $\Theta' \subseteq \Theta$ on which σ^* is differentiable and strictly increasing. Hence, there exists on-the-equilibrium-path message $m \in M^+(\sigma^*, \phi^*)$ such that $\#(S(m)) > 4$. Without loss of generality, assume that $s_1, s_2, s_3, s_4 \in S(m)$ with $s_1 < s_2 < s_3 < s_4$. By Claim 3-(ii), we have $\psi_y^*(y_A \mid m, s_1) < \psi_y^*(y_A \mid m, s_2) < \psi_y^*(y_A \mid m, s_3) < \psi_y^*(y_A \mid m, s_4)$ must hold. To hold this order, it is necessary that $\psi_y^*(y_A \mid m, s_2) \in (0, 1)$ and $\psi_y^*(y_A \mid m, s_3) \in (0, 1)$. However, by Claim 2, it is impossible, which is a contradiction. Therefore, σ^* should be constant whenever differentiable. ■

B.3.5 Auxiliary claims for Part (iv)

Claim 4 *For any PBE e^*, $G(\sigma^*)$ is continuous almost everywhere.*

Proof of Claim 4. By Lemma 3-(ii) and (iii), σ^* is differentiable almost everywhere and constant whenever differentiable. Fix subset $\Theta' \subseteq \Theta$ on which σ^* is constant, arbitrarily. Because $\sigma^*(\theta) = \sigma^*(\theta')$ holds for any $\theta, \theta' \in \Theta'$, $G(\sigma^*(\theta)) = G(\sigma^*(\theta'))$ holds, implying that $G(\sigma^*)$ is continuous on Θ'. ■

B.3.6 Part (iv)

Suppose, in contrast, that there exists $\theta' \in \Theta$ such that $U^*(\theta') \neq \lim_{\theta \uparrow \theta'} U^*(\theta)$. Because of Lemma 3-(i), Claim 4 and the continuity of C, there exists an open interval $(\underline{\theta}, \theta')$ on which U^* is continuous. Hence, there exists $\theta'' \in (\underline{\theta}, \theta')$ such that θ'' is so close to θ' that $|U^*(\theta'') - \lim_{\theta \uparrow \theta'} U^*(\theta)|$ is sufficiently close to 0. Suppose, in contrast, that $U^*(\theta') < \lim_{\theta \uparrow \theta'} U^*(\theta)$. However, it implies that $U^*(\theta') < U^*(\theta'') < G(\sigma^*(\theta'')) - C(\theta', \sigma^*(\theta'')) = U^*(\theta', \sigma^*(\theta''))$, which is a contradiction. Hence, $U^*(\theta') > \lim_{\theta \uparrow \theta'} U^*(\theta)$ should hold. Because θ'' is sufficiently close to θ', it implies that $U^*(\theta') = G(\sigma^*(\theta')) - C(\theta', \sigma^*(\theta')) > G(\sigma^*(\theta')) - C(\theta'', \sigma^*(\theta')) = U^*(\theta'', \sigma^*(\theta')) > U^*(\theta'')$. However, it means that type θ'' has an incentive to mimic type θ', which is a contradiction. Therefore, $U^*(\theta') = \lim_{\theta \uparrow \theta'} U^*(\theta)$ holds. Because $U^*(\theta') = \lim_{\theta \downarrow \theta'} U^*(\theta)$ also should hold by the similar argument, U^* must be continuous. ∎

B.4 Proof of Lemma 5

(i) By Lemma 3-(i), (ii) and (iii), σ^* should be a step function. Furthermore, by using the same argument used in the proof of Lemma 3-(iii), we say that $\#(S^+(\sigma^*)) \leq 3$. Now, suppose, in contrast, that there exists D1 equilibrium e^* such that $S^+(\sigma^*) = S(m_E) = \{s_1, s_2, s_3\}$ with $s_1 < s_2 < s_3$. Claims 2 and 3 imply that $\psi_r^*(r_I \mid m_E) > 0$ and $\psi_y^*(y_A \mid m_E, s_1) = 0 < \psi_y^*(y_A \mid m_E, s_2) < \psi_y^*(y_A \mid m_E, s_3) = 1$. That is, $\mathbb{E}_{\nu_y^*(m_E, s_2)}[\theta] = \theta_{SQ}$ must hold. Furthermore, because σ^* is nondecreasing by Lemma 3-(i), we assume that $\Theta(s_1) = [0, \theta_1)$, $\Theta(s_2) = [\theta_1, \theta_2)$, and $\Theta(s_3) = [\theta_2, 1]$ without loss of generality. Because of $\mathbb{E}_{\nu_y^*(m_E, s_2)}[\theta] = \theta_{SQ}$, $\theta_{SQ} \in (\theta_1, \theta_2)$ holds. Note that the equilibrium utility of the agent with $\theta = \theta_2$ is $U^*(\theta_2) = \psi_r^*(r_I \mid m_E) + \psi_r^*(r_N \mid m_E)\psi_y^*(y_A \mid m_E, \emptyset) - C(\theta_2, s_3)$. Now, consider a deviation of type θ_2 to signal

$s = s_2 + \varepsilon$, where $\varepsilon > 0$ is sufficiently small. By Lemma 1, the D1 criterion requires that $\mu^*(\theta_2 \mid s_2 + \varepsilon) = 1$, implying that $\psi_y^*(y_A \mid m_E, s_2 + \varepsilon) = 1$ because of $\theta_2 > \theta_{SQ}$. However, it means that

$$
\begin{aligned}
U^*(\theta_2, s_2 + \varepsilon) &= \psi_r^*(r_I \mid m_E) + \psi_r^*(r_N \mid m_E)\psi_y^*(y_A \mid m_E, \emptyset) - C(\theta_2, s_2 + \varepsilon) \\
&> U^*(\theta_2),
\end{aligned} \tag{B.8}
$$

which is a contradiction. Therefore, $\#(S^+(\sigma^*)) = 2$ must hold.

(ii) By Part (i), σ^* should be a step function with at most one discontinuous point. Hence, θ^* is well defined. Suppose, in contrast, that $\theta^* > \theta_{SQ}$. By using a similar argument that type θ_2 has an incentive to deviate in the proof of Part (i), we can derive a contradiction. Therefore, $\theta^* \leq \theta_{SQ}$ must hold.

(iii) The statement is immediately derived from Claim 3-(i). ■

B.5 Proposition 2

B.5.1 Proof of Lemma 6

(i) It is sufficient to show that $\#(S^+(\sigma^*)) \neq 3$ because $\#(S^+(\sigma^*)) \leq 3$ is guaranteed by the similar argument used in Section B.3.4. Suppose, in contrast, that $\#(S^+(\sigma^*)) = 3$, implying that there exists message $m \in M^+(\sigma^*, \phi^*)$ such that $S(m) = \{s_1, s_2\}$ with $s_1 < s_2$. However, by Claim 3-(i), $\psi_r^*(r_I \mid m) > 0$ must hold, which is a contradiction. Therefore, $\#(S^+(\sigma^*)) = 2$ must hold.

(ii) Suppose, in contrast, that $M^+(\sigma^*, \phi^*) = \{m_E\}$. Because $\#(S^+(\sigma^*)) = 2$ by Part (i), there exist signals $s_1, s_2 \in S(m_E)$ with $s_1 < s_2$. However, it implies that $\psi_r^*(r_I \mid m_E) > 0$ must hold by Claim 3-(i), which is a contradiction. Therefore, $M^+(\sigma^*, \phi^*) = M$ must hold.

(iii) Suppose, in contrast, that $\psi_y^*(y_A \mid m_E, \emptyset) = \psi_y^*(y_A \mid m_O, \emptyset)$. By Part (i), there exist signals $s_1, s_2 \in S^+(\sigma^*)$ with $s_1 < s_2$. Because the principal commits to $\psi_r^*(r_I \mid m) = 0$ for any m, $G(s_1) = G(s_2)$. However, Claim 1 requires that $G(s_1) < G(s_2)$, which is a contradiction. Therefore, $\psi_y^*(y_A \mid m_E, \emptyset) \neq \psi_y^*(y_A \mid m_O, \emptyset)$ must hold.

(iv) By Part (i), $S^+(\sigma^*) = \{s_1, s_2\}$ with $s_1 < s_2$ and σ^* has a unique discontinuous point θ'. Suppose, in contrast, that $\theta' > \theta_{SQ} - b$. By Lemma 3-(i) and (iv), $U^*(\theta') = U^*(\theta', s_2)$ holds. Because of Part (iii), without loss of generality, we assume that $\psi_y^*(y_A \mid m_E, \emptyset) > \psi_y^*(y_A \mid m_O, \emptyset)$. Furthermore, Claim 1 and Part (ii) imply that $\phi^*(s_2) = m_E$. That is, $U^*(\theta', s_2) = \psi_y^*(y_A \mid m_E, \emptyset) - C(\theta', s_2)$. Now, suppose that type θ' deviates to signal $s' \in (s_1, s_2)$. By Lemma 1, the D1 criterion requires that $\mu^*(s') = \mathcal{D}(\theta')$, implying that $\phi^*(s') = m_E$ because $\theta' + b > \theta_{SQ}$. However, it means that

$$\begin{aligned} U^*(\theta', s') &= \psi_y^*(y_A \mid m_E, \emptyset) - C(\theta', s') \\ &> \psi_y^*(y_A \mid m_E, \emptyset) - C(\theta', s_2) = U^*(\theta'), \end{aligned} \tag{B.9}$$

which is a contradiction. Therefore, $\theta' \leq \theta_{SQ} - b$ must hold.

(v) Suppose, in contrast, that $b < 0$ but $|b| \geq 1 - \theta_{SQ}$, i.e., $1 \leq \theta_{SQ} - b$. By Lemma 3-(i), Parts (i), (ii), and (iii), without loss of generality, we assume that (a) $\sigma^*(\theta) = s_1$ if $\theta < \theta'$ and s_2 otherwise, and (b) $\psi_y^*(y_A \mid m_E, \emptyset) > \psi_y^*(y_A \mid m_O, \emptyset)$, where $s_1 < s_2$. Note that $\mathbb{E}_{\mu^*(s_1)}[\theta + b] = \theta'/2 + b$. Because $1 \leq \theta_{SQ} - b$ and $\theta' < 1$, $\theta_{SQ} - (\theta'/2 + b) \geq 1 - \theta'/2 > 0$, implying that $\phi^*(s_1) = m_O$. Because $m_E \in M^+(\sigma^*, \phi^*)$, $\phi^*(s_2) = m_E$ must hold, implying that $\mathbb{E}_{\mu^*(s_2)}[\theta+b] \geq \theta_{SQ}$, or still $(1+\theta')/2 \geq \theta_{SQ}-b$. However, it implies that $1 > (1+\theta')/2 \geq \theta_{SQ} - b \geq 1$, which is a contradiction. Therefore, $|b| < 1 - \theta_{SQ}$ must hold. ∎

B.5.2 The statement for $\theta_{SQ} < 1/2$

Proposition 7 *Consider the indirect communication mode with $\theta_{SQ} < 1/2$. If $b \in [-1/2, \theta_{SQ})$, then the no-investigation equilibrium is optimal, where*

$$\theta^*(b) = \begin{cases} \theta_{SQ} - b & \text{if } b \in (0, \theta_{SQ}), \\ \theta_{SQ} & \text{if } b \in [-b_{FB}, 0), \\ 2\theta_{SQ} - 2b - 1 & \text{if } b \in [-1/2, -b_{FB}.) \end{cases} \tag{B.10}$$

Otherwise, an uninformative equilibrium is optimal.

Proof. Because the proof is essentially equivalent to that of Proposition 2, it is omitted. ∎

B.6 Proposition 3 for $\theta_{SQ} < 1/2$

Proposition 8 *Consider the hybrid communication mode with $\theta_{SQ} < 1/2$.*

(i) Suppose that $b > 0$. Then, a partial-investigation equilibrium exists if and only if either one of the following holds: (a) $b \leq \theta_{SQ}/2$ and $d < 2b^2/(1 - \theta_{SQ} + 2b)$ or (b) $b > \theta_{SQ}/2$ and $d < \theta_{SQ}^2/2$.

(ii) Suppose that $b < 0$. Then, a partial-investigation equilibrium exists if and only if either one of the following holds: (a) $|b| \leq \theta_{SQ}$ and $d < |b|^2/[2(\theta_{SQ} + |b|)]$ or (b) $|b| > \theta_{SQ}$ and $d \leq \theta_{SQ}/4$.

Proof. Because the proof is essentially equivalent to that of Proposition 3, it is omitted. ∎

B.7 Proposition 4 for $\theta_{SQ} < 1/2$

Proposition 9 *Consider the hybrid communication mode with $\theta_{SQ} < 1/2$.*

(i) Suppose that $b > 0$.

(a) If $b \leq \theta_{SQ}$ and $d \geq d_+$, then the no-investigation equilibrium is optimal.

(b) If either $[b < \theta_{SQ}$ and $d < d_+]$ or $[b \geq \theta_{SQ}$ and $d < \theta_{SQ}^2/2]$, then a partial-investigation equilibrium is optimal.

(c) Otherwise, an uninformative equilibrium is optimal.

(ii) Suppose that $b < 0$.

(a) If either $[|b| \leq b_{FB}]$ or $[|b| \in (b_{FB}, 1/2]$ and $d > d_-]$, then the no-investigation equilibrium is optimal.

(b) If either $[|b| \in (b_{FB}, 1/2]$ and $d \leq d_-]$ or $[|b| > 1/2$ and $d \leq \theta_{SQ}/4]$, then a partial-investigation equilibrium is optimal.

(c) Otherwise, an uninformative equilibrium is optimal.

Proof. Because the proof is essentially equivalent to that of Proposition 4, it is omitted. ■

B.8 Proof of Proposition 5 except for $b > 0$ and $\theta_{SQ} > 1/2$

Because the proofs of the other cases are identical to that mentioned in the body of the paper except for the characterization of $\Lambda_I - \Lambda_C$ and $\beta^*(\theta_{SQ}, b, d)$, we only provide the characterization.

(i) $b > 0$ and $\theta_{SQ} < 1/2$.

$$\Lambda_I = \begin{cases} (\theta_{SQ}^2 - b^2)/2 & \text{if } b \leq \theta_{SQ}, \\ 0 & \text{otherwise,} \end{cases}$$

$$\Lambda_C = \begin{cases} d - b^2/2 & \text{if } b \leq \delta_+, \\ d - \delta_+^2/2 & \text{otherwise,} \end{cases} \tag{B.11}$$

$$\beta^*(\theta_{SQ}, b, d) = \sqrt{\theta_{SQ}^2 + \delta_+^2 - 2d}.$$

(ii) $b < 0$ and $\theta_{SQ} < 1/2$.

$$\Lambda_I = \begin{cases} \theta_{SQ}^2/2 & \text{if } |b| \leq b_{FB}, \\ (1 - 2|b|)(2|b| - 1 + 2\theta_{SQ})/2 & \text{if } b_{FB} < |b| \leq 1/2, \\ 0 & \text{otherwise,} \end{cases}$$

$$\Lambda_C = \begin{cases} d & \text{if } |b| \leq b_{FB}, \\ d - (2|b| - 1 + \theta_{SQ})^2/2 & \text{if } b_{FB} < |b| \leq b_-, \\ d - \delta_-^2/2 & \text{otherwise,} \end{cases} \tag{B.12}$$

$$\beta^*(\theta_{SQ}, b, d) = \frac{1}{2}\left(1 - \theta_{SQ} + \sqrt{\theta_{SQ}^2 + \delta_-^2 - 2d}\right),$$

where $b_- := (1 - \theta_{SQ} + \delta_-)/2$, and $|b| \leq b_-$ is equivalent to $d \leq d_-$, or still $V^N \leq V^P$.

(iii) $b < 0$ and $\theta_{SQ} > 1/2$.

$$\Lambda_I = \begin{cases} (1-\theta_{SQ})^2/2 & \text{if } |b| \le b_{FB}, \\ 2|b|(1-\theta_{SQ}-|b|) & \text{if } b_{FB} < |b| \le 1-\theta_{SQ}, \\ 0 & \text{otherwise,} \end{cases}$$

$$\Lambda_C = \begin{cases} d & \text{if } |b| \le b_{FB}, \\ d - (2|b|-1+\theta_{SQ})^2/2 & \text{if } b_{FB} < |b| \le b_{-}, \\ d - \delta_{-}^2/2 & \text{otherwise,} \end{cases} \tag{B.13}$$

$$\beta^*(\theta_{SQ}, b, d) = \frac{1}{2}\left(1-\theta_{SQ}+\sqrt{(1-\theta_{SQ})^2+\delta_{-}^2-2d}\right).$$

■

B.9 Proposition 6 for $\theta_{SQ} < 1/2$

Proposition 10 *Consider the hybrid communication mode with $\theta_{SQ} < 1/2$. Then, the anti-change-biased manager is always better for the principal.*

Proof. There are the following two cases: (i) $d > \theta_{SQ}/4$, where a partial-investigation equilibrium never exists, and (ii) $d \le \theta_{SQ}/4$, where there exists a partial-investigation equilibrium at least for the anti-change-biased manager.

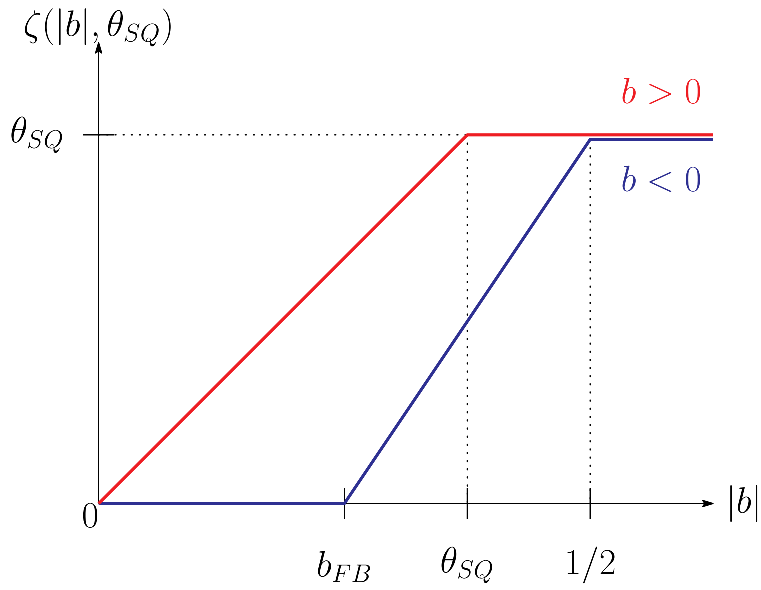

Figure 10: Optimal Direction for $d > \theta_{SQ}/4$ and $\theta_{SQ} < 1/2$

First, consider Case (i). Define $\zeta(b, \theta_{SQ}) := |\theta^*(b) - \theta_{SQ}|$, implying that

$$\zeta(b, \theta_{SQ}) = \begin{cases} b & \text{if } 0 < b \leq \theta_{SQ}, \\ 0 & \text{if } -b_{FB} \leq b < 0, \\ 2|b| - (1 - \theta_{SQ}) & \text{if } -1/2 \leq b < -b_{FB}, \\ \theta_{SQ} & \text{otherwise.} \end{cases} \tag{B.14}$$

By simple algebra, we can show that $\zeta(|b|, \theta_{SQ}) \geq \zeta(-|b|, \theta_{SQ})$ for any $b \neq 0$, as denoted in Figure 10.

Next, consider Case (ii). Define $\zeta(b, d, \theta_{SQ}) := |\theta' - \theta_{SQ}|$, where θ' is the threshold of the optimal equilibrium. The characterization of $\zeta(b, d, \theta_{SQ})$ is also given by (A.35) in the body of the paper. There are the following two subcases: (a) $d < \theta_{SQ}^2/2$, where a partial-investigation equilibrium exists irrelevant to the direction of bias, and (b) $\theta_{SQ}^2/2 \leq d \leq \theta_{SQ}/4$, where a partial-investigation equilibrium exists only when bias is negative. Now, consider Subcase (a), and show that $\zeta(|b|, d, \theta_{SQ}) \geq \zeta(-|b|, d, \theta_{SQ})$ for any $b \neq 0$. Note that if $|b| \in (0, b_{FB}]$, then $\zeta(-|b|, d, \theta_{SQ}) = 0 < \zeta(|b|, d, \theta_{SQ})$ holds. Similarly, if $|b| \in (\max\{\delta_+, b_-\}, \infty)$, then $\zeta(-|b|, d, \theta_{SQ}) = \delta_- < \zeta(|b|, d, \theta_{SQ}) = \delta_+$ holds because $\delta_- < \delta_+$ when $\theta_{SQ} < 1/2$. Hence, it remains to show the statement for $|b| \in (b_{FB}, \max\{\delta_+, b_-\}]$. If $\delta_+ < b_{FB}$, then $\max\{\delta_+, b_-\} = b_-$, implying that

$$\begin{aligned}
\zeta(|b|, d, \theta_{SQ}) - \zeta(-|b|, d, \theta_{SQ}) &= \delta_+ + (1 - \theta_{SQ}) - 2|b| \\
&\geq \delta_+ + (1 - \theta_{SQ}) - 2b_- \\
&= \delta_+ - \delta_- \\
&> 0,
\end{aligned} \tag{B.15}$$

where the first and the last inequalities come from the facts that $|b| \leq b_-$ and $\delta_+ > \delta_-$ when $\theta_{SQ} < 1/2$, respectively. If $\delta_+ > b_-$ and $|b| \in (b_{FB}, b_-]$, then we have

$$\begin{aligned}
\zeta(|b|, d, \theta_{SQ}) - \zeta(-|b|, d, \theta_{SQ}) &= (1 - \theta_{SQ}) - |b| \\
&\geq (1 - \theta_{SQ}) - b_- \\
&= \frac{1}{2}(1 - \theta_{SQ} - \delta_-) \\
&> 0,
\end{aligned} \tag{B.16}$$

where the first and the second inequalities come from the facts that $|b| \leq b_-$ and $1 - \theta_{SQ} > \delta_-$ when $d < \theta_{SQ}^2/2$ and $\theta_{SQ} < 1/2$, respectively. Similarly, if $\delta_+ > b_-$ and $|b| \in (b_-, \delta_+]$, then

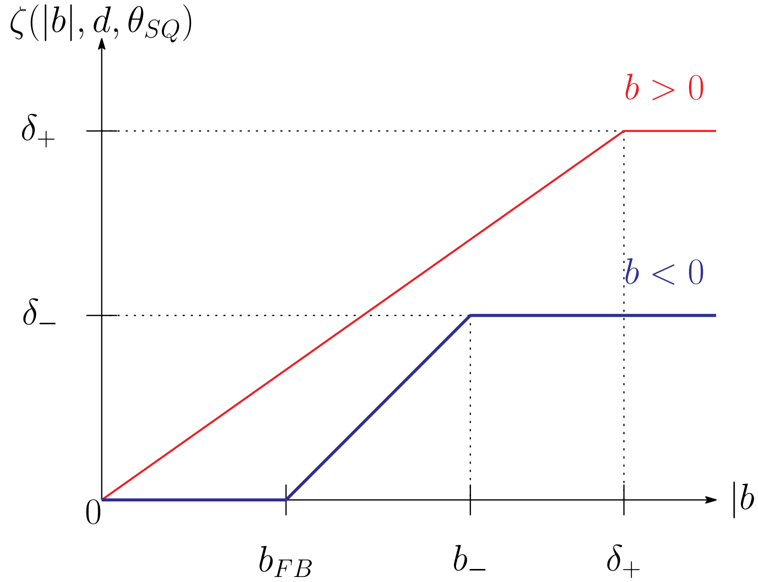

Figure 11: Optimal Direction for $d < \theta_{SQ}^2/2$ and $\theta_{SQ} < 1/2$

we have

$$\begin{aligned} \zeta(|b|, d, \theta_{SQ}) - \zeta(-|b|, d, \theta_{SQ}) &= |b| - \delta_- \\ &> b_- - \delta_- \\ &= \frac{1}{2}(1 - \theta_{SQ} - \delta_-) \\ &> 0, \end{aligned} \tag{B.17}$$

where the first and the second inequalities hold analogous to the above case. Finally, suppose that $b_{FB} \leq \delta_+ \leq b_-$. If $|b| \in (b_{FB}, \delta_+]$, we can show the statement by the same argument used in the case of $\delta_+ > b_-$ and $|b| \in (\delta_+, b_-]$. Similarly, if $|b| \in (\delta_+, b_-]$, we can show the statement by the same argument used in the case of $\delta_+ < b_{FB}$. Thus, we conclude that $\zeta(|b|, d, \theta_{SQ}) \geq \zeta(-|b|, d, \theta_{SQ})$ for any $b \neq 0$ in Case (a), as denoted in Figure 11.

Next consider Case (b). Note that $\zeta(|b|, d, \theta_{SQ})$ under Case (b) is weakly larger than that under Case (a) while $\zeta(-|b|, d, \theta_{SQ})$ is identical to that of Case (a). Hence, the inequality under Case (a) implies that the same inequality also holds under Case (b). As a result, $\zeta(|b|, d, \theta_{SQ}) \geq \zeta(-|b|, d, \theta_{SQ})$ holds for any $b \neq 0$.[51] ■

[51]The diagram of this scenario is similar to Figure 11.