Entrepreneurship and Growth in an Economy with Two Levels of Schooling

Koichi Yotsuya*

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Abstract

In a growth model in which the rate of technological progress is endogenously determined by entrepreneurs' investment in R&D, the effect of two levels of schooling (a high level and a low level) on economic growth are examined. Individuals completing a high level of schooling can directly contribute to technological progress by engaging in R&D. Individuals completing a low level of schooling indirectly contribute to it because an increase in workers adaptable for advanced technology increases the monopoly profit of successful entrepreneurs. However, this indirect effect may disappear if technology growth in their school age remains at a low rate, since accumulating specific knowledge prior to general knowledge in school could be preferable. We demonstrate that if regulation of accumulation of general knowledge is loose in the low level of schooling, individuals enrolled there fail to learn technological adaptability and thereby the economy might be caught in a low-growth trap.

Keywords: schooling, general knowledge, specific knowledge, entrepreneurship, technological progress, economic growth

JEL *Classification*: I20, J24, O33, O40

*Correspondence address: Faculty of Economics, Doshisha University, Karasuma-Imadegawa, Kamigyo-ku, Kyoto, 602-8580, E-mail: kyotsuya@mail.doshisha.ac.jp

1 Introduction

Individuals with different educational backgrounds play different roles in the economy because the amount and the composition of knowledge vary depending on the level and kind of education they receive. For example, if individuals go to a university or a graduate school, learning concept-based general knowledge would be central throughout their school age. Instead, learning vocational or specific knowledge may stand a large part if individuals end education by high school or go to a technical college. This suggests that different levels of education might affect macroeconomic outcomes through different channels, and thus, analyzing these channels as well as the interplay among them is an important issue when comprehending the overall relationship between education and economic growth.

The purpose of this paper is to present a theoretical framework in which the effect of interplay of different levels of education on economic growth can be examined. Based on Aghion and Howitt (1992) where technological progress brought about by entrepreneurs' profit-seeking R&D is the engine of growth, we construct an overlapping generations model in which the evolution of two levels of schooling (a high level and a low level) and the rate of technological progress are endogenously determined. In our model, following the seminal idea of Nelson and Phelps (1966) and Schultz (1975), we stress the positive role of education in adapting to a new technology, and the benefit from each level of education is supposed as follows: receiving at least a low level of schooling is required to operate new technologies, and only individuals with a high level of schooling have the capacity to become entrepreneurs. This is due to the fact that research and starting a new business rely heavily on individuals' ability, and naturally offers an own channel through which the level of schooling contributes to growth. A High level of schooling can directly contribute to technological progress by bringing along potential entrepreneurs, whereas a low level of schooling indirectly contributes to it by supplying workforce with technological adaptability that increases the monopoly profit of a successful entrepreneur.

The degree of technological adaptability is determined by the composition of the two types of knowledge (specific knowledge and general knowledge), both of which are accumulated through schooling. Like many studies classifying workers' skills by technological adaptability such as Galor and Moav (2000), Gould et al. (2001), and Krueger and Kumar (2004), we too defer to the idea of Nelson and Phelps (1966) and Schultz (1975). We assume that production under new technologies is general knowledge-intensive while specific knowledge is more effective under existing technologies. Then, to what extent would individuals in each level of schooling learn technological adaptability, or equivalently, which type of knowledge would they mainly accumulate, depend on the kind of technology they would access in their working age.

Here, we consider a regulation on the formation of the two types of knowledge in each level of schooling. We assume that individuals in schooling must accumulate two types of knowledge so that the aggregate amount and the share of the two types of knowledge reach the given values provided by each level of schooling. This regulation reflects one characteristic of actual schooling where teaching proceeds along with its own curriculum; and in our model, this plays an important role in binding individuals on the technology frontier. Because the relative importance of general knowledge increases with a rise in education level, the ratio of general knowledge to specific knowledge and the aggregate amount of both required for completing schooling is higher under a high level of schooling. In this sense, a high level of schooling does not do well against specific knowledge-intensive technologies, and thus, individuals enrolled there always energetically accumulate general knowledge to maximize accessibility to new technologies. This, in turn, helps the entrepreneurship of individuals in the same group. If a low level of schooling imposes a fairly ratio of general knowledge accumulation, the same scenario applies for individuals enrolled there. Individuals with a low level of schooling can also be a workforce with high technological adaptability for a successful entrepreneur in such a situation, and thus, attract individuals with a high level of schooling to R&D.

In contrast, when a low level of schooling permits a low ratio of general knowledge accumulation, accumulating specific knowledge prior to general knowledge and then using an old technology may be more preferable for individuals enrolled there. Failing to learn technological adaptability happens when the new technology developed during individuals' school age is not productive enough to be employed. Since a low level of schooling is needed merely for improving the knowledge required for old technologies in this situation, individuals enrolled there cannot be a profitable workforce for a successful entrepreneur in their working age. For this reason, the entrepreneurship of highly educated individuals in the same generation declines, and hence, technological progress in the future remains low.

We demonstrate that there exists a threshold ratio of general knowledge accumulation to prevent individuals with a low level of schooling from leaving the technology frontier. If a higher ratio of general knowledge accumulation than this threshold value is imposed, the economy can converge to a unique steady-state equilibrium in which a large number of individuals with a high level of schooling engage in R&D. Yet, the economy may be characterized by multiple steady-state equilibria if the regulation on general knowledge accumulation is too loose to satisfy this threshold value. In a "bad" steady-state equilibrium, technology grows only at a slow pace since most of the highly educated individuals hesitate to engage in R&D due to the lack of a technological adaptable workforce. As a result, as in Redding (1996) who shows the development trap in an R&D-based growth model by examining the comprementarity between R&D and education investments, an economy that is excessively friendly with the accumulation of specific knowledge in the early stage of education may be caught in a low-growth trap where individuals with a decent education do not serve to improve the entrepreneurship level in the economy.[1]

[1]Though many other human capital-based growth studies demonstrate the development trap or the multiplicity of equilibria, most of them employ accumulation or economy-wide externality of human capital, instead of entrepreneurs' R&D, as the driving force of economic growth, e.g., Azariadis and Drazen (1990), Galor and Zeira (1993), and Becker et al.

The paper is organized as follows. Section 2 presents a formal model. Section 3 examines the general equilibrium, and Section 4 analyzes the dynamical system of the economy.

2 The Model

2.1 Basic Setup

The world is populated by overlapping generations, indexed by t. Each generation lives for two periods and we refer to the generation living its second period in period t as generation t. Each generation consists of M numbers of individuals whose inherent abilities of educational attainment, a_i, are heterogeneous and uniformly distributed to $\{a_1, a_2, .., a_M\}$, where $a_{i+1} = a_i + m$ for all $1 \leq i \leq M - 1$ $(m > 0)$. Here m is assumed to be sufficiently small; that is, each a_i is densely distributed in $a_1 \leq a \leq a_M$, so it is permissible to treat it as differentiable with respect to a_i.

When young, individuals allocate one unit of time endowment to schooling and leisure, and do not work. When old, they are absorbed in production activities, that is, they decide whether to produce consumption goods using existing technologies or to become entrepreneurs who invest in profit-seeking R&D. In any case, all earnings are spent on consumption of that period, and the lifetime utility of an individual of generation t is given by the Cobb-Douglas utility function concerning leisure when young, Z_{t-1}, and consumption when old, C_t:

$$U = Z_{t-1}^{\gamma} C_t^{1-\gamma}, \quad (0 < \gamma < 1). \tag{1}$$

Consumption goods produced here are homogeneous, although technologies employed for production may vary across individuals. Goods are traded in a competitive market and their price is normalized to one. Individuals of generation t who decide to be producers of consumption goods devote all their time endowment toward consumption good production in period t. On the other hand, individuals who decide to be entrepreneurs engage in research to develop new technology. Research takes early part (λ) of their time endowment in period t, and research consequence turns out at the end of this research period $(1 < \lambda < 0)$. Denoting the leading-edge technology at the end of period $t-1$ by A_{t-1} and the number of entrepreneurs in period t by M_t^R, the output of each member of M_t^R is assumed to be given by

$$A_t = \left[1 + \phi M_t^{R^{\theta}}\right] A_{t-1}\ , \quad 0 < \phi\ , \quad 0 < \theta < 1. \tag{2}$$

The positive effect of the number of entrepreneurs, M_t^R, on the results of each entrepreneur indicates the existence of R&D spillovers. As stressed in the seminal exposition of Griliches (1979), knowledge spillovers mainly take place through two channels. One, which he terms "rent spillovers", is transmitted through trading products (e.g., investment or intermediary goods between firms) in which new knowledge

(1990). Maki et al. (2005) and Yotsuya (2002) are examples of studies investigating the role of the multi-dimensionality of skills on growth.

is embodied. The other concerns the fact that technology has important public goods aspects (non-rivalry and non-excludability) and so-called "pure knowledge spillovers", and is transmitted by channels such as the mobility of researchers or engineers, conferences and meetings, patent information, scientific literature, and reverse engineering. Many empirical studies present evidence on these sorts of R&D spillovers and emphasize the importance of these effects on productivity growth.[2] In our model, (2) plays a critical role in establishing the positive relationship between entrepreneurship and economic growth.

During the latter part $(1-\lambda)$ of period t, entrepreneurs who succeed in obtaining patents on A_t can earn monopoly profit, although the patent expires and there are A_t spillovers across generations at the end of period t. Therefore, during the early part (λ) of all periods, individuals have free access to all technologies and thus production occurs under perfect competition, whereas it becomes essentially imperfectly competitive during the latter part $(1-\lambda)$ of all periods.

Given the above flow of technological progress, $\{A_0, .., A_{t-2}, A_{t-1}\}$ and A_t are available in period t, while A_t joins in the middle of that period. Assuming that consumption goods are produced according to linear technology using efficiency units of labor (*work time* $\times$ *labor productivity*) and all existing technology, the period t aggregate output, Y_t, is given by

$$Y_t = \sum_{j=0}^{t} A_j H_{j,t}, \tag{3}$$

where we assume $A_0 > 0$ and $A_j \leq A_{j+1}$ holds for all $j \geq 0$ from (2). $H_{j,t}$ indicates the sum of efficiency units of labor combined with j-th technology in period t.

2.2 Two Types of Knowledge and Two Levels of Schooling

In this model, two different types of knowledge are considered. One is specific knowledge, s, which is mainly valuable for a particular field, occupation, or job. Another is general knowledge, x, which is useful for a broader environment and widens the adaptability or validity of specific knowledge. In addition to the amounts of these two types of knowledge, individuals' labor productivity depends on which technology they are associated with.

From the individual viewpoint, technologies can be classified according to whether they are developed before birth. For example, for individuals of generation t, $\{A_0, .., A_{t-2}\}$, which exist before birth, are well-known. In contrast, individuals might be unfamiliar with A_{t-1} and A_t since these newer technologies emerge in their school age and working age. As emphasized in many growth studies such as Galor and Moav (2000), Gould, et al. (2001), and Krueger and Kumar (2004), while specific knowledge may be directly valid and more valuable if working with familiar technologies, general knowledge plays an

[2] See, for instance, Griliches and Lichtenberg (1984), Mansfield (1980), Scherer (1982), and Goto and Suzuki (1989) regarding the former, and Jaffe (1988), Los and Verspagen (2000), Cincera (2005), and Harhoff (1998) regarding the latter.

important role when workers adapt to alien technologies.[3] Thus, we suppose that the labor productivity of individuals of generation t is given by

$$\begin{cases} x^{\alpha}s^{1-\alpha} & \text{if they use } A_{t-1} \text{ or } A_t \\ x^{\beta}s^{1-\beta} & \text{if they use one of } \{A_0, .., A_{t-2}\} \end{cases},$$

$$\text{where} \quad 0 < \beta < \frac{1}{2} < \alpha < 1.$$

That is, technologies developed before birth are specific knowledge-intensive, while those developed in their lives are general knowledge-intensive.

The only way for individuals to accumulate the above two types of knowledge is to invest in education while young. We suppose that there exist a high level and a low level of schooling that differ on two points: one concerns the sum of two types of knowledge that students must accumulate to complete the course, and the other concerns their ratio.

To finish a high level of schooling, individuals must accumulate the sum of two types of knowledge, Q, i.e., $x + s = Q$, whereas only ηQ is required to finish a low level of schooling ($0 < \eta < 1$). We denote the time allocated to acquire general and specific knowledge by ε_x and ε_s, respectively, and assume that the accumulation of each type of knowledge is as follows:

$$x = a_i \varepsilon_x \ , \quad s = a_i \varepsilon_s.$$

Then, an individual of ability a_i must sacrifice $\varepsilon_x + \varepsilon_s = \frac{Q}{a_i}$ of leisure when young to finish a high level of schooling, whereas $\frac{\eta Q}{a_i}$ of time must be devoted to finish a low level of schooling. To ensure the possibility that all individuals can complete a high level of schooling within a period, we assume $a_1 = Q$.[4]

In addition, to complete each level of schooling, the ratio of general knowledge to the aggregate amount of knowledge must reach a given value according to schooling levels. In general, learning academic and comprehensive knowledge is necessary in advanced courses of study. Fundamental scholastic ability would be essential for acquiring sophisticated technical or special knowledge. Thus, the relative importance of general knowledge increases with a rise in education level. Based on this, we simply assume that more than half of the aggregate amount of knowledge must be general knowledge to complete high-level schooling, whereas this ratio falls to σ in the case of low-level schooling:

$$\frac{x}{Q} \geq \frac{1}{2} \ , \quad \frac{x}{\eta Q} \geq \sigma,$$

where $0 < \sigma < \frac{1}{2}$. These, together with $x = a_i \varepsilon_x$, yield $\varepsilon_x \geq \frac{1}{2}\frac{Q}{a_i}$ and $\varepsilon_x \geq \sigma \frac{\eta Q}{a_i}$, respectively. This means that individuals enrolling in high-level schooling (resp. low-level schooling) must allocate at least half (resp. σ of) their schooling period to the acquisition of general knowledge.

[3]For empirical evidence on this point, see, e.g., Bartel and Lichtenberg (1987), Berman et al. (1994), and Autor et al. (1998).

[4]Though it might be more suitable to assume that $a_1 \geq Q$, this relaxation does not affect the main properties of our model.

2.3 Lifetime Behavior of a Consumption Goods Producer

Under the above setting, if we except the option of being an entrepreneur for a while, the lifetime decisions of generation t are summarized as follows. At the beginning of period $t-1$, individuals decide which level of schooling they receive and the amounts of the types of knowledge they accumulate, i.e., x and s. Then, at the beginning of period t, they decide whether to employ one of technologies among $\{A_0, .., A_{t-2}\}$ with labor productivity $x^{\beta}s^{1-\beta}$ or A_{t-1} with labor productivity $x^{\alpha}s^{1-\alpha}$ under a given x and s, where A_{t-2} is necessarily selected in the former from $A_j \leq A_{j+1}$ for all $j \geq 0$. Thus, to see which level of schooling is the best for individuals, lifetime utilities when individuals optimally choose their technology and x and s (equivalently, ε_x and ε_s) under a given schooling level must be examined.

First consider the case of finishing high-level schooling. If an individual i employs new technology A_{t-1} in period t, he allocates $\frac{Q}{a_i}$ of schooling time in period $t-1$ to the accumulation of each type of knowledge under $\varepsilon_x \geq \frac{1}{2}\frac{Q}{a_i}$ to maximize consumption in period t, $A_{t-1}(a_i\varepsilon_x)^{\alpha}(a_i\varepsilon_s)^{1-\alpha}$. That is, he solves

$$\max_{\varepsilon_x, \varepsilon_s} \quad A_{t-1}\left(a_i\varepsilon_x\right)^{\alpha}\left(a_i\varepsilon_s\right)^{1-\alpha}$$
$$\text{s.t.} \quad \varepsilon_x + \varepsilon_s = \frac{Q}{a_i}\ , \quad \varepsilon_x \geq \frac{1}{2}\frac{Q}{a_i}$$

in period $t-1$. From $\frac{1}{2} < \alpha < 1$, unique interior solution is guaranteed at $\varepsilon_x = \alpha\frac{Q}{a_i}$ and $\varepsilon_s = (1-\alpha)\frac{Q}{a_i}$. Hence, irrespective of the inherent ability of educational attainment, all individuals accumulate $x = \alpha Q$ and $s = (1-\alpha)Q$ of general and specific knowledge, respectively, and consume $C_t = A_{t-1}\alpha^{\alpha}(1-\alpha)^{1-\alpha}Q$ of goods. On the other hand, if an individual i employs A_{t-2} in period t, he solves

$$\max_{\varepsilon_x, \varepsilon_s} \quad A_{t-2}\left(a_i\varepsilon_x\right)^{\beta}\left(a_i\varepsilon_s\right)^{1-\beta}$$
$$\text{s.t.} \quad \varepsilon_x + \varepsilon_s = \frac{Q}{a_i}\ , \quad \varepsilon_x \geq \frac{1}{2}\frac{Q}{a_i}$$

in period $t-1$. From $0 < \beta < \frac{1}{2}$, the solution is bound at $\varepsilon_x = \varepsilon_s = \frac{1}{2}\frac{Q}{a_i}$, and thus the amounts of knowledge and consumption are the same for all individuals at $x = s = \frac{1}{2}Q$ and $C_t = A_{t-2}\frac{1}{2}Q$. Since we can easily see that $A_{t-1}\alpha^{\alpha}(1-\alpha)^{1-\alpha}Q > A_{t-2}\frac{1}{2}Q$ from $A_{t-1}\alpha^{\alpha}(1-\alpha)^{1-\alpha}Q > A_{t-1}\left(\frac{1}{2}Q\right)^{\alpha}\left(\frac{1}{2}Q\right)^{1-\alpha} = A_{t-1}\frac{1}{2}Q \geq A_{t-2}\frac{1}{2}Q$, employing A_{t-1} in period t is always preferable for all individuals. Hence, from (1) and (2), the A_{t-2}-adjusted lifetime utility of an individual i is given by

$$u^H(a_i, M_{t-1}^R) = \left(1 - \frac{Q}{a_i}\right)^{\gamma}\left[(1 + \phi M_{t-1}^{R^\theta})\alpha^{\alpha}(1-\alpha)^{1-\alpha}Q\right]^{1-\gamma}, \tag{4}$$

where $u = \frac{U}{A_{t-2}^{1-\gamma}}$ and $\phi M_{t-1}^{R^\theta}$ is the rate of technological progress in period $t-1$, i.e., $\phi M_{t-1}^{R^\theta} = \frac{A_{t-1}}{A_{t-2}} - 1$.

Next, consider the case of finishing low-level schooling. Similarly to the case of high-level schooling, an individual i adopts the solution of

$$\max_{\varepsilon_x, \varepsilon_s} \quad A_{t-1}\left(a_i\varepsilon_x\right)^{\alpha}\left(a_i\varepsilon_s\right)^{1-\alpha}$$

$$\text{s.t.} \qquad \varepsilon_x + \varepsilon_s = \frac{\eta Q}{a_i} \ , \ \ \varepsilon_x \geq \sigma \frac{\eta Q}{a_i}$$

if employing A_{t-1} in period t. Unique interior solution $\varepsilon_x = \alpha \frac{\eta Q}{a_i}$ and $\varepsilon_s = (1-\alpha)\frac{\eta Q}{a_i}$ is maintained from $\sigma < \frac{1}{2} < \alpha$, and the amount of each type of knowledge and consumption are, respectively, $x = \alpha\eta Q$, $s = (1-\alpha)\eta Q$, and $C_t = A_{t-1}\alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta Q$. In contrast,

$$\max_{\varepsilon_x, \varepsilon_s} \qquad A_{t-2} \left(a_i \varepsilon_x\right)^{\beta} \left(a_i \varepsilon_s\right)^{1-\beta}$$

$$\text{s.t.} \qquad \varepsilon_x + \varepsilon_s = \frac{\eta Q}{a_i} \ , \ \ \varepsilon_x \geq \sigma \frac{\eta Q}{a_i}$$

is solved in period $t-1$ if employing A_{t-2} in period t. When $\sigma > \beta$ — that is, when low-level schooling requires more than $\beta\eta Q$ of general knowledge accumulation — the solution is bound at $\varepsilon_x = \sigma\frac{\eta Q}{a_i}$ and $\varepsilon_s = (1-\sigma)\frac{\eta Q}{a_i}$, and thus $x = \sigma\eta Q$, $s = (1-\sigma)\eta Q$, and $C_t = A_{t-2}\sigma^{\beta}(1-\sigma)^{1-\beta}\eta Q$ there. When $\sigma \leq \beta$ — that is, when less than $\beta\eta Q$ of general knowledge accumulation is permitted — unique interior solution $\varepsilon_x = \beta\frac{\eta Q}{a_i}$ and $\varepsilon_s = (1-\beta)\frac{\eta Q}{a_i}$ is obtained, and thus $x = \beta\eta Q$, $s = (1-\beta)\eta Q$, and $C_t = A_{t-2}\beta^{\beta}(1-\beta)^{1-\beta}\eta Q$ there. These indicate that unlike in the case of high-level schooling, employing old technology A_{t-2} might be preferable depending on the value of σ.

To see this, Figure 1 illustrates $x^{\alpha}s^{1-\alpha}$ and $x^{\beta}s^{1-\beta}$ under $s = \eta Q - x$. They are inverted U-shaped and equalize at $x = \frac{1}{2}\eta Q$. $x^{\alpha}(\eta Q - x)^{1-\alpha}$ reaches its maximum $\alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta Q$ at $x = \alpha\eta Q$, whereas $x^{\beta}(\eta Q - x)^{1-\beta}$ is maximized to $\beta^{\beta}(1-\beta)^{1-\beta}\eta Q$ at $x = \beta\eta Q$. As discussed above, $x = \alpha\eta Q$ is chosen if employing A_{t-1}. If employing A_{t-2}, individuals choose x maximizing $x^{\beta}(\eta Q - x)^{1-\beta}$ under $x \geq \sigma\eta Q$. Then, $x = \sigma\eta Q$ is chosen when $\sigma\eta Q > \beta\eta Q$, while $x = \beta\eta Q$ is chosen when $\sigma\eta Q \leq \beta\eta Q$. Noting here that for $0 < \chi < 1$, $\chi^{\chi}(1-\chi)^{1-\chi}$ is convex, symmetric, and minimized to $\frac{1}{2}$ at $\chi = \frac{1}{2}$, $\alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta Q$ is smaller than $\beta^{\beta}(1-\beta)^{1-\beta}\eta Q$ if

$$\alpha < 1 - \beta$$

as depicted there. In this situation, the unique value of x, $\hat{x}$, which satisfies $\hat{x}^{\beta}(\eta Q - \hat{x})^{1-\beta} = \alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta Q$ and $\hat{x} > \beta\eta Q$ exists uniquely in $\beta\eta Q < x < \frac{1}{2}\eta Q$. Equivalently, denoting $\frac{\hat{x}}{\eta Q}$ by $\hat{\sigma}$, the unique value of $\frac{x}{\eta Q}$, $\hat{\sigma}$, is defied by:

$$\hat{\sigma}^{\beta}(1-\hat{\sigma})^{1-\beta} = \alpha^{\alpha}(1-\alpha)^{1-\alpha} \qquad \text{and} \qquad \hat{\sigma} > \beta, \tag{5}$$

exists in $\beta < \frac{x}{\eta Q} < \frac{1}{2}$. As far as $\sigma \geq \hat{\sigma}$, employing A_{t-1} is necessarily preferable since $\sigma^{\beta}(1-\sigma)^{1-\beta}\eta Q \leq \alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta Q$ holds. When $\sigma < \hat{\sigma}$, however, depending on the rate of technological progress in period $t-1$ or, equivalently, on the size of M_{t-1}^{R}, employing A_{t-2} could be preferable; that is, $A_{t-2}\sigma^{\beta}(1-\sigma)^{1-\beta}\eta Q > A_{t-1}\alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta Q$ is possible since $\sigma^{\beta}(1-\sigma)^{1-\beta}\eta Q > \alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta Q$ holds.

Lemma 1 Suppose α and β satisfy

$$0 < \beta < \frac{1}{2} < \alpha < 1 - \beta < 1. \tag{6}$$

Then, employing A_{t-1} in period t is always preferable for individuals with low-level schooling when $\hat{\sigma} \leq \sigma < \frac{1}{2}$. When $0 < \sigma < \hat{\sigma}$, however, there exists a positive value of M_{t-1}^R, $\hat{M}^R$, defined below, and employing A_{t-2} becomes preferable if $0 \leq M_{t-1}^R < \hat{M}^R$, although employing A_{t-1} is still preferable if $M_{t-1}^R \geq \hat{M}^R$. Particularly in $\sigma \leq \beta$, all σ in (7) are replaced by β:

$$1 + \phi \hat{M}^{R^\theta} = \frac{\sigma^\beta (1-\sigma)^{1-\beta}}{\alpha^\alpha (1-\alpha)^{1-\alpha}}. \tag{7}$$

Proof. The result directly follows from the above discussion when $\hat{\sigma} \leq \sigma < \frac{1}{2}$. $\phi \hat{M}^{R^\theta}$ is the rate of technological progress in period $t-1$ at which employing A_{t-2} and employing A_{t-1} become indifferent. Specifically, when $\beta < \sigma < \hat{\sigma}$, $\hat{M}^R$ is defined by the value satisfying $A_{t-2}\sigma^\beta(1-\sigma)^{1-\beta}\eta Q = (1+\phi\hat{M}^{R^\theta})A_{t-2}\alpha^\alpha(1-\alpha)^{1-\alpha}\eta Q$ from (2). $\hat{M}^R > 0$ is assured from $\sigma^\beta(1-\sigma)^{1-\beta}\eta Q > \alpha^\alpha(1-\alpha)^{1-\alpha}\eta Q$, and $A_{t-2}\sigma^\beta(1-\sigma)^{1-\beta}\eta Q > (1+\phi M_{t-1}^{R^\theta})A_{t-2}\alpha^\alpha(1-\alpha)^{1-\alpha}\eta Q$ holds if $M_{t-1}^R < \hat{M}^R$, and vice versa. The same discussion applies when $0 < \sigma \leq \beta$, although $\hat{M}^R$ is redefined by $\phi\hat{M}^{R^\theta} = \frac{\beta^\beta(1-\beta)^{1-\beta}}{\alpha^\alpha(1-\alpha)^{1-\alpha}} - 1$ instead of (7). □

(6) means not only that old technologies are specific knowledge-intensive whereas new technologies are general knowledge-intensive, but also that the degree of intensiveness is larger in the former than in the latter. In this situation, when low-level schooling imposes less than $\hat{\sigma}\eta Q$ of accumulation of general knowledge, labor productivity in operating A_{t-2} can be larger than those in operating A_{t-1} by accumulating as much specific knowledge as possible. Therefore, if gains from superior technology cannot overcome losses from this disadvantage of labor productivity, that is, if the entrepreneurship of generation $t-1$ is not sufficient to induce a rate of technological progress greater than $\phi\hat{M}^{R^\theta}$, employing old technology would be the better choice.

Consequently, when $\hat{\sigma} \leq \sigma < \frac{1}{2}$, A_{t-2}-adjusted lifetime utility in the case of finishing low-level schooling, u^L, is simply given as follows for all $0 \leq M_{t-1}^R \leq M$:

$$\bar{u}^L(a_i, M_{t-1}^R) = \left(1 - \frac{\eta Q}{a_i}\right)^\gamma \left[(1+\phi M_{t-1}^{R^\theta})\alpha^\alpha(1-\alpha)^{1-\alpha}\eta Q\right]^{1-\gamma}. \tag{8}$$

When $0 < \sigma < \hat{\sigma}$, although $\bar{u}^L(a_i, M_{t-1}^R)$ still governs u^L as long as $\hat{M}^R \leq M_{t-1}^R$, it is replaced by

$$\underline{u}^L(a_i) = \left(1 - \frac{\eta Q}{a_i}\right)^\gamma \left[\sigma^\beta(1-\sigma)^{1-\beta}\eta Q\right]^{1-\gamma} \tag{9}$$

if $0 \leq M_{t-1}^R < \hat{M}^R$, where all σ in $\underline{u}^L(a_i)$ and in $\hat{M}^R$ are replaced by β when $0 < \sigma \leq \beta$.

The last is the case where individuals receive no schooling in period $t-1$. In this case, they must work as manual laborers in period t. We simply assume that irrespective of ability for educational attainment, individuals with no schooling have access only to technologies developed before birth, $\{A_0, .., A_{t-2}\}$, with productivity μ ($0 < \mu < \frac{1}{2}\eta Q$). The A_{t-2}-adjusted lifetime utility corresponding to this choice is simply given by

$$u^N = \mu^{1-\gamma}. \tag{10}$$

Following the previous discussion, considering the situation in which σ falls below β provides no additional insights since it is merely an extreme case of $\sigma < \frac{1}{2}$. Thus, in the following, this analysis omits this situation and focuses on the situation in which $\beta < \sigma < \frac{1}{2}$.

[Figure1 around here]

2.4 Entrepreneurship and Individuals' Maximization Problem

In addition to the three choices presented in the previous section, individuals of generation t have the option of being entrepreneurs who engage in R&D in order to obtain monopoly profits from the patent on the new technology A_t. Because research and starting a business rely heavily on individuals' ability, it is assumed that only individuals completing high-level schooling when young or, equivalently, possessing Q aggregate knowledge, have an opportunity to become entrepreneurs when old.[5] So, in period t, individuals of generation t who complete high-level schooling in period $t-1$ face a decision of whether to produce consumption goods or to engage in research, where we assume this decision is irreversible and must be made at the beginning of period t.

After a research period λ, the patent for the new technology A_t is assumed to be allocated randomly to one entrepreneur of generation t. Hence, all entrepreneurs have the same probability $\frac{1}{M_t^R}$ for this event. During the remainder of period t, $1-\lambda$, the entrepreneur successful in obtaining the patent employs individuals as workers and manages A_t-based production to maximize monopoly profits, Π_t. Because of the inaccessibility of non-educated individuals to new technologies, individuals who engage in consumption goods production after finishing more than low-level schooling are eligible for the workforce. Their labor productivity under A_t, as well as their number, would be critical for Π_t. In this sense, whether σ exceeds $\hat{\sigma}$ has great significance for the entrepreneurship of individuals with high-level schooling.

It is supposed that the entrepreneur can make a "take it or leave it"offer to consumption goods producers. Thus, the entrepreneur proposes wages equal to their outside option. When $\hat{\sigma} \leq \sigma < \frac{1}{2}$, it corresponds to $A_{t-1}(1-\lambda)\alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta Q$ for individuals with low-level schooling, whereas $A_{t-1}(1-\lambda)\alpha^{\alpha}(1-\alpha)^{1-\alpha}Q$ of wages are paid to individuals who become consumption goods producers after finishing high-level schooling. Since their labor productivity is maintained entirely under A_t in this situation, denoting the number of individuals of generation t with low-level schooling by M_t^L and those with high-level schooling who become producers of consumption goods by M_t^H, Π_t is given by

$$\begin{aligned}\overline{\Pi}_t &= A_t(1-\lambda)\left[\alpha^{\alpha}(1-\alpha)^{1-\alpha}QM_t^H + \alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta QM_t^L\right] \\ &\quad - \left[A_{t-1}(1-\lambda)\alpha^{\alpha}(1-\alpha)^{1-\alpha}QM_t^H + A_{t-1}(1-\lambda)\alpha^{\alpha}(1-\alpha)^{1-\alpha}\eta QM_t^L\right]\end{aligned}$$

[5] That is, we simply assume that individuals' research production function is given by (2) only if they have more than Q knowledge altogether, and $A_t = A_{t-1}$ otherwise.

$$= A_{t-1}\phi M_t^{R^\theta}(1-\lambda)\alpha^\alpha(1-\alpha)^{1-\alpha}Q\left[M_t^H+\eta M_t^L\right].$$

Thus, the A_{t-2}-adjusted expected utility of a member i of M_t^R, u^R, is represented as follows[6]:

$$\bar{u}^R(a_i, M_{t-1}^R) = \frac{1}{M_t^R}\left(1-\tfrac{Q}{a_i}\right)^\gamma\left[\frac{\overline{\Pi}_t}{A_{t-2}}\right]^{1-\gamma} = u^H(a_i, M_{t-1}^R)\frac{1}{M_t^R}\left[\phi M_t^{R^\theta}(1-\lambda)(M_t^H+\eta M_t^L)\right]^{1-\gamma}. \quad (11)$$

Even when $\beta < \sigma < \hat{\sigma}$, the result is the same if $M_{t-1}^R \geq \hat{M}^R$. If $0 \leq M_{t-1}^R < \hat{M}^R$, however, employing members of M_t^L is not always profitable for the successful entrepreneur. As shown in Lemma 1, members of M_t^L accumulate specific knowledge as much as possible in this situation, i.e., $x = \sigma\eta Q$ and $s = (1-\sigma)\eta Q$, because using old technology A_{t-2} is optimal for them. This means their labor productivity drops from $\sigma^\beta(1-\sigma)^{1-\beta}\eta Q$ to $\sigma^\alpha(1-\sigma)^{1-\alpha}\eta Q$ according to the technology shift to A_t, nevertheless, $A_{t-2}(1-\lambda)\sigma^\beta(1-\sigma)^{1-\beta}\eta Q$ of wages must still be guaranteed to employ them. Thus, employing them is profitable only if $A_t(1-\lambda)\sigma^\alpha(1-\sigma)^{1-\alpha}\eta Q \geq A_{t-2}(1-\lambda)\sigma^\beta(1-\sigma)^{1-\beta}\eta Q$. This, together with (7), proposes that even when $\hat{M}^R$ of individuals of generation $t-1$ are engaged in research, employing M_t^L cannot be profitable as far as

$$1+\phi M_t^{R^\theta} < \frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\sigma^\alpha(1-\sigma)^{1-\alpha}}.$$

Therefore, conditional on

$$\frac{\sigma^\beta(1-\sigma)^{1-\beta}}{\alpha^\alpha(1-\alpha)^{1-\alpha}} < 1+\phi M^\theta < \frac{\alpha^\alpha(1-\alpha)^{1-\alpha}}{\sigma^\alpha(1-\sigma)^{1-\alpha}}, \quad (12)$$

not only is the interior $\hat{M}^R$ acquired, but not employing M_t^L is always optimal for the successful entrepreneur. (12) is likely to hold for a large α and β since $\alpha^\alpha(1-\alpha)^{1-\alpha}$ increases with α, whereas $\sigma^\alpha(1-\sigma)^{1-\alpha}$ decreases with α and $\sigma^\beta(1-\sigma)^{1-\beta}$ decreases with β. In this situation, Π_t is given by

$$\underline{\Pi}_t = A_{t-1}\phi M_t^{R^\theta}(1-\lambda)\alpha^\alpha(1-\alpha)^{1-\alpha}QM_t^H$$

and the A_{t-2}-adjusted expected utility of a member i of M_t^R is represented by [7]

$$\underline{u}^R(a_i, M_{t-1}^R) = \frac{1}{M_t^R}\left(1-\tfrac{Q}{a_i}\right)^\gamma\left[\frac{\underline{\Pi}_t}{A_{t-2}}\right]^{1-\gamma} = u^H(a_i, M_{t-1}^R)\frac{1}{M_t^R}\left[\phi M_t^{R^\theta}(1-\lambda)M_t^H\right]^{1-\gamma}. \quad (13)$$

[6]The assumption that the decision concerning R&D is irreversible means $C_t = 0$ for entrepreneurs failing to obtain a patent for the new technology. It might be more appropriate to suppose that entrepreneurs can go back to a consumption goods producer after research period λ if they fail to acquire a patent. Although it slightly changes Π_t and u^R and thus makes analysis somewhat complicated, all the main findings derived below could be maintained under this modification.

[7]Though analysis mainly concentrates on the situation in which (12) is satisfied, it is not essential for our main findings. As formally examined below, the fact that $\underline{\Pi}_t$ becomes smaller than $\overline{\Pi}_t$ when $M_{t-1}^R = \hat{M}^R$ is critical for our analysis. If the latter part of (12) is violated, $A_t = (1+\phi M_{t-1}^{R^\theta})(1+\phi M_t^{R^\theta})A_{t-2}$ might satisfy $A_t(1-\lambda)\sigma^\alpha(1-\sigma)^{1-\alpha}\eta Q \geq A_{t-2}(1-\lambda)\sigma^\beta(1-\sigma)^{1-\beta}\eta Q$ under sufficiently large M_{t-1}^R and M_t^R. In this situation, $\underline{\Pi}_t$ is given by

$$\underline{\Pi}_t = A_{t-1}\phi M_t^{R^\theta}(1-\lambda)\alpha^\alpha(1-\alpha)^{1-\alpha}QM_t^H + (1-\lambda)\left[A_t\sigma^\alpha(1-\sigma)^{1-\alpha} - A_{t-2}\sigma^\beta(1-\sigma)^{1-\beta}\right]\eta Q M_t^L,$$

where $A_t(1-\lambda)\alpha^\alpha(1-\alpha)^{1-\alpha}\eta Q > A_t(1-\lambda)\sigma^\alpha(1-\sigma)^{1-\alpha}\eta Q$ and $A_{t-1}(1-\lambda)\alpha^\alpha(1-\alpha)^{1-\alpha}\eta Q < A_{t-2}(1-\lambda)\sigma^\beta(1-\sigma)^{1-\beta}\eta Q$. From this and the fact that, when $M_{t-1}^R = \hat{M}^R$, the utmost numbers of M_t^L and M_t^H are the same when $\hat{\sigma} \leq \sigma < \frac{1}{2}$ and $\beta < \sigma < \hat{\sigma}$, $\underline{\Pi}_t < \overline{\Pi}_t$ is maintained for a given M_t^R when $M_{t-1}^R = \hat{M}^R$, although formally considering this situation makes analysis somewhat troublesome.

It is noted that in terms of C_t, whether to be employed by the successful entrepreneur does not matter for consumption goods producers. Therefore, the maximization problem of an individual i of generation t is to attain the largest utility from among the following four choices:

Choice N**:** receive no schooling in period $t-1$ and obtain u^N given by (10).

Choice L**:** receive low level of schooling in period $t-1$ and obtain $u^L(a_i, M_{t-1}^R)$ given by (9) when $\beta < \sigma < \hat{\sigma}$ and $0 \leq M_{t-1}^R < \hat{M}^R$, and (8) otherwise.

Choice H**:** become a consumption goods producer in period t after finishing high-level schooling in period $t-1$ and obtain $u^H(a_i, M_{t-1}^R)$ given by (4).

Choice R**:** become an entrepreneur in period t after finishing high-level schooling in period $t-1$ and obtain $u^R(a_i, M_{t-1}^R)$ given by (13) when $\beta < \sigma < \hat{\sigma}$ and $0 \leq M_{t-1}^R < \hat{M}^R$, and (11) otherwise.

Letting M_t^N denote the number of individuals who receive no schooling, period t equilibrium is characterized by the distribution of individuals of that generation into M_t^N, M_t^L, M_t^H, and M_t^R ($M_t^N + M_t^L + M_t^H + M_t^R = M$), which is determined according to the result of all individuals' optimal choice.

3 General Equilibrium

3.1 Sub-problem

As a step to solve the individuals' maximization problem, this section considers the following sub-problem:

$$\max \left[u^N, u^L(a_i, M_{t-1}^R), u^H(a_i, M_{t-1}^R)\right].$$

Examining the above reduces the individuals' maximization problem to the alternative of the solution of the sub-problem and choice R, and hence helps to derive the equilibrium distribution of individuals.

Figure 2 illustrates u^N, $u^L(a_i, M_{t-1}^R)$, and $u^H(a_i, M_{t-1}^R)$ when $\hat{\sigma} \leq \sigma < \frac{1}{2}$, where $u^L(a_i, M_{t-1}^R)$ is given by $\bar{u}^L(a_i, M_{t-1}^R)$ for all $0 \leq M_{t-1}^R \leq M$. $\bar{a}^H$ is the ability at which $u^H(a_i, M_{t-1}^R)$ and $\bar{u}^L(a_i, M_{t-1}^R)$ are equalized. Since $u^H(a_1, M_{t-1}^R) < \bar{u}^L(a_1, M_{t-1}^R)$ and $\frac{\partial}{\partial a_i} u^H(a_i, M_{t-1}^R) > \frac{\partial}{\partial a_i} \bar{u}^L(a_i, M_{t-1}^R)$ for $a_1 \leq a_i$ holds from (4), (8), and $a_1 = Q$, a unique $\bar{a}^H$, defined by

$$\bar{a}^H = \frac{1 - \eta^{\frac{1}{\gamma}}}{1 - \eta^{\frac{1-\gamma}{\gamma}}} Q, \tag{14}$$

is guaranteed in $a_1 < a_i$ and $u^H(a_i, M_{t-1}^R) >$ (resp. $\leq$) $\bar{u}^L(a_i, M_{t-1}^R)$ holds for $a_i >$ (resp. $\leq$) $\bar{a}^H$. As in (14), $\bar{a}^H$ is independent from M_{t-1}^R because a rise in the rate of technological progress in period $t-1$ increases both $u^H(a_i, M_{t-1}^R)$ and $\bar{u}^L(a_i, M_{t-1}^R)$ by the same proportion.

On the other hand, $\bar{a}^L(M_{t-1}^R)$ is the ability at which $\bar{u}^L(a_i, M_{t-1}^R)$ equalizes to u^N. Since $\frac{\partial}{\partial a_i} \bar{u}^L(a_i, M_{t-1}^R) > 0$ holds for all $a_1 \leq a_i$, if $\bar{u}^L(a_1, M_{t-1}^R) < u^N < \bar{u}^L(a_M, M_{t-1}^R)$, it is uniquely obtained in $a_1 < a_i < a_M$

and $\bar{u}^L(a_i, M_{t-1}^R) >$ (resp. $\leq$) u^N holds for $a_i >$ (resp. $\leq$) $\bar{a}^L(M_{t-1}^R)$. From (8) and (10), $\bar{a}^L(M_{t-1}^R)$ is defined by

$$\bar{a}^L(M_{t-1}^R) = \frac{\eta Q}{1 - \left[\frac{\mu}{(1+\phi M_{t-1}^{R^\theta})\alpha^\alpha(1-\alpha)^{1-\alpha}\eta Q}\right]^{\frac{1-\gamma}{\gamma}}}, \tag{15}$$

and, unlike $\bar{a}^H$, $\bar{a}^L(M_{t-1}^R)$ negatively depends on M_{t-1}^R as differentiating (15) yields

$$\bar{a}^{L'}(M_{t-1}^R) = -\frac{1-\gamma}{\gamma}\frac{\bar{a}^L(M_{t-1}^R)^2}{\eta Q}\frac{\phi\theta M_{t-1}^{R^{\theta-1}}}{1+\phi M_{t-1}^{R^\theta}}\left[1 - \frac{\eta Q}{\bar{a}^L(M_{t-1}^R)}\right] < 0.$$

This is because through a rise in the rate of technological progress in period $t-1$, an increase in M_{t-1}^R makes employing the new technology A_{t-1} relatively more attractive. Therefore, if

$$\frac{\eta}{1 - \left[\frac{\mu}{\alpha^\alpha(1-\alpha)^{1-\alpha}\eta Q}\right]^{\frac{1-\gamma}{\gamma}}} < \frac{1-\eta^{\frac{1}{\gamma}}}{1-\eta^{\frac{1-\gamma}{\gamma}}}, \tag{16}$$

then $\bar{a}^L(0) < \bar{a}^H$ is assured and the solution of the sub-problem is a classified boundary in $\bar{a}^L(M_{t-1}^R)$ and $\bar{a}^H$ as follows.

Lemma 2 Suppose (16) is satisfied. Then, if $a_1 < \bar{a}^L(0)$ and $\bar{a}^H < a_M$, the solution of the sub-problem in the circumstance of $\hat{\sigma} \leq \sigma < \frac{1}{2}$ is given by

$$\begin{cases} u^N & \text{for } a_1 \leq a_i \leq \bar{a}^L(M_{t-1}^R) \\ u^L(a_i, M_{t-1}^R) & \text{for } \bar{a}^L(M_{t-1}^R) < a_i \leq \bar{a}^H \\ u^H(a_i, M_{t-1}^R) & \text{for } \bar{a}^H < a_i \leq a_M \end{cases},$$

although $u^L(a_i, M_{t-1}^R)$ is the best for $a_1 \leq a_i \leq \bar{a}^H$ under a large M_{t-1}^R such that $\bar{a}^L(M_{t-1}^R) \leq a_1$.

Proof. Follows from the above discussion.[8] □

In the circumstance of $\beta < \sigma < \hat{\sigma}$, some modifications are required when $0 \leq M_{t-1}^R < \hat{M}^R$ because $\underline{u}^L(a_i)$ rather than $\bar{u}^L(a_i, M_{t-1}^R)$ governs $u^L(a_i, M_{t-1}^R)$, as in (9). Figure 3 depicts this situation. $\underline{u}^L(a_i)$ lies above $\bar{u}^L(a_i, M_{t-1}^R)$ as long as $M_{t-1}^R < \hat{M}^R$, although $\bar{u}^L(a_i, M_{t-1}^R)$ shifts upward with a rise in M_{t-1}^R and overtakes $\underline{u}^L(a_i)$ when $M_{t-1}^R = \hat{M}^R$. Thus, in this situation, $\underline{a}^L$ and $\underline{a}^H(M_{t-1}^R)$ are the abilities classifying the solution of the sub-problem. $\underline{a}^L$ is the ability at which u^N and $\underline{u}^L(a_i)$ are equalized, and is defined by

$$\underline{a}^L = \frac{\eta Q}{1 - \left[\frac{\mu}{\sigma^\beta(1-\sigma)^{1-\beta}\eta Q}\right]^{\frac{1-\gamma}{\gamma}}} \tag{17}$$

from (9) and (10). $\underline{a}^L$ is independently determined by M_{t-1}^R because neither u^N nor $\underline{u}^L(a_i)$ rest on the new technology A_{t-1}. $\underline{a}^L < \bar{a}^L(0)$ from $\underline{u}^L(a_i) = \bar{u}^L(a_i, \hat{M}^R) > \bar{u}^L(a_i, 0)$.

[8] $\bar{a}^L(M) > a_1$ is assured if $(1-\eta)^{\frac{\gamma}{1-\gamma}}(1+\phi M^\theta)\alpha^\alpha(1-\alpha)^{1-\alpha}\eta Q < \mu$.

$\underline{a}^H(M^R_{t-1})$ is the ability at which $\underline{u}^L(a_i)$ equalizes to $u^H(a_i, M^R_{t-1})$. From $\underline{u}^L(a_1) > u^H(a_1, M^R_{t-1})$, the existence of $\underline{a}^H(M^R_{t-1})$ in $a_1 < a_i < a_M$ is assured for all $0 \leq M^R_{t-1} \leq \hat{M}^R$ if $u^H(a_M, M^R_{t-1})$ is larger than $\underline{u}^L(a_M)$ when $M^R_{t-1} = 0$, i.e., if the following inequality is satisfied:

$$\left[\frac{\sigma^\beta(1-\sigma)^{1-\beta}\eta}{\alpha^\alpha(1-\alpha)^{1-\alpha}}\right]^{\frac{1-\gamma}{\gamma}} < \frac{a_M - Q}{a_M - \eta Q}. \tag{18}$$

Furthermore, since $\frac{\partial}{\partial a_i} u^H(a_i, M^R_{t-1}) > \frac{\partial}{\partial a_i}\underline{u}^L(a_i)$ holds for all $0 \leq M^R_{t-1} < \hat{M}^R$ under (18), the uniqueness of $\underline{a}^H(M^R_{t-1})$ is also assured.[9] (4) and (9) bring about

$$\underline{a}^H(M^R_{t-1}) = \frac{1 - \Psi(M^R_{t-1})\eta^{\frac{1}{\gamma}}}{1 - \Psi(M^R_{t-1})\eta^{\frac{1-\gamma}{\gamma}}} Q \quad , \quad \text{where} \quad \Psi(M^R_{t-1}) = \left[\frac{\sigma^\beta(1-\sigma)^{1-\beta}}{(1+\phi M^{R^\theta}_{t-1})\alpha^\alpha(1-\alpha)^{1-\alpha}}\right]^{\frac{1-\gamma}{\gamma}}. \tag{19}$$

Since, in this situation, a rise in the rate of technological progress in period $t-1$ benefits only individuals with high-level schooling, $\underline{a}^H(M^R_{t-1})$ negatively depends on M^R_{t-1}, as indicated by

$$\underline{a}^{H'}(M^R_{t-1}) = -\frac{1-\gamma}{\gamma} \frac{\phi\theta M^{R^{\theta-1}}_{t-1}}{1+\phi M^{R^\theta}_{t-1}} \frac{(1-\eta)\Psi\eta^{\frac{1-\gamma}{\gamma}}}{(1-\Psi\eta^{\frac{1-\gamma}{\gamma}})^2} Q < 0.$$

Specifically, it corresponds to $\bar{a}^H$ when $M^R_{t-1} = \hat{M}^R$ since $\Psi(\hat{M}^R) = 1$ follows from (7).

When $\hat{M}^R \leq M^R_{t-1} \leq M$, the same argument as in the circumstance of $\hat{\sigma} \leq \sigma < \frac{1}{2}$ applies because $\bar{u}^L(a_i, M^R_{t-1})$ dominates $\underline{u}^L(a_i)$ as depicted in Figure 4. Consequently, the solution of the sub-problem in the circumstance of $\beta < \sigma < \hat{\sigma}$ is summarized as follows.

Lemma 3 Suppose (12), (16), and (18) are satisfied. Then, if $a_1 < \underline{a}^L$, the solution of the sub-problem in the circumstance of $\beta < \sigma < \hat{\sigma}$ is given by

$$\begin{cases} u^N & \text{for} \quad a_1 \leq a_i \leq \underline{a}^L \\ u^L(a_i, M^R_{t-1}) & \text{for} \quad \underline{a}^L < a_i \leq \underline{a}^H(M^R_{t-1}) \\ u^H(a_i, M^R_{t-1}) & \text{for} \quad \underline{a}^H(M^R_{t-1}) < a_i \leq a_M \end{cases}$$

when $0 \leq M^R_{t-1} < \hat{M}^R$; otherwise, it is the same as in the circumstance of $\hat{\sigma} \leq \sigma < \frac{1}{2}$.

Proof. Follows from the above discussion. □

Under (18), $\bar{a}^H < a_M$, which is one premise for Lemma 2, is satisfied since $\bar{a}^H < \underline{a}^H(0)$. Similarly, if $a_1 < \underline{a}^L$, that is, if

$$(1-\eta)^{\frac{\gamma}{1-\gamma}} < \frac{\mu}{\sigma^\beta(1-\sigma)^{1-\beta}\eta Q}, \tag{20}$$

$a_1 < \bar{a}^L(0)$, which is the other premise for Lemma 2, is satisfied since $\underline{a}^L < \bar{a}^L(0)$. This condition and (12), (16), and (18) are likely to hold for large α, β, σ, and a_M and intermediate γ, μ, and Q. Hereafter,

[9] From (4) and (9), if $(1+\phi M^{R^\theta}_{t-1})\alpha^\alpha(1-\alpha)^{1-\alpha} > \sigma^\beta(1-\sigma)^{1-\beta}\eta$, then $\frac{\partial}{\partial a_i}u^H(a_i, M^R_{t-1}) > \frac{\partial}{\partial a_i}\underline{u}^L(a_i)$. Thus, if $\frac{\sigma^\beta(1-\sigma)^{1-\beta}\eta}{\alpha^\alpha(1-\alpha)^{1-\alpha}} < 1$, $\frac{\partial}{\partial a_i}u^H(a_i, M^R_{t-1})$ always exceeds $\frac{\partial}{\partial a_i}\underline{u}^L(a_i)$. This condition is certainly satisfied under (18) since $\frac{a_M - Q}{a_M - \eta Q} < 1$.

we mainly highlight the situation where parameters satisfy these conditions for simplicity, which is not essential for our analysis.

[Figure 2, Figure 3, and Figure 4 around here]

3.2 Equilibrium Distribution of Individuals

From Lemmas 2 and 3, individuals' maximization problem is reduced to the alternative depending on their ability of educational attainment. In the circumstance of $\hat{\sigma} \leq \sigma < \frac{1}{2}$, it is simplified to

$$\begin{aligned}
&\max \left[u^N, \bar{u}^R(a_i, M_{t-1}^R)\right] \quad \text{for} \;\; a_1 \leq a_i \leq \bar{a}^L(M_{t-1}^R) \\
&\max \left[\bar{u}^L(a_i, M_{t-1}^R), \bar{u}^R(a_i, M_{t-1}^R)\right] \quad \text{for} \;\; \bar{a}^L(M_{t-1}^R) < a_i \leq \bar{a}^H \\
&\max \left[u^H(a_i, M_{t-1}^R), \bar{u}^R(a_i, M_{t-1}^R)\right] \quad \text{for} \;\; \bar{a}^H < a_i \leq a_M
\end{aligned}$$

from Lemma 2. Here, decision timing should be noted. Individuals whose ability is above $\bar{a}^H$ inevitably receive high-level schooling in period $t-1$. The only substantial problem they face is deciding whether to be an entrepreneur, i.e., whether they belong to M_t^R or M_t^H, at the beginning of period t, where M_t^N, M_t^L, and $M_t^H + M_t^R$ are known because the schooling choice of all individuals has been made in period $t-1$. Noting that the entry of an individual with more than $\bar{a}^H$ of ability into R&D coincides with his exit from M_t^H, it causes

$$\begin{aligned}
\frac{\partial}{\partial M_t^R}\bar{u}^R(a_i, M_{t-1}^R) \;\; = \;\; & u^H(a_i, M_{t-1}^R)\Bigg\{-\frac{1}{M_t^{R^2}}\left[\phi M_t^{R^\theta}(1-\lambda)(M_t^H + \eta M_t^L)\right]^{1-\gamma} \\
& + \frac{1}{M_t^{R^2}}\theta(1-\gamma)\left[\phi M_t^{R^\theta}(1-\lambda)(M_t^H + \eta M_t^L)\right]^{1-\gamma} \\
& - \frac{1}{M_t^R}(1-\gamma)\phi M_t^{R^\theta}(1-\lambda)\left[\phi M_t^{R^\theta}(1-\lambda)(M_t^H + \eta M_t^L)\right]^{-\gamma}\Bigg\}
\end{aligned}$$

of change in $\bar{u}^R(a_i, M_{t-1}^R)$. The first term in the brace on the LHS represents a negative effect due to falls in the probability of obtaining the patent, whereas the second term reflects the positive effect on $\overline{\Pi}_t$ through raising the quality of A_t. From $\theta(1-\gamma) < 1$, the latter is dominated by the former. In addition, since the negative effect of decreases in M_t^H on $\overline{\Pi}_t$, represented by the third term, is added, $\bar{u}^R(a_i, M_{t-1}^R)$ falls with the entry of individuals with more than $\bar{a}^H$ of ability into R&D. This and free entry to R&D indicate that under given M_t^N, M_t^L, and $M_t^H + M_t^R$, entering R&D from $a_i > \bar{a}^H$ of individuals carries on until $\bar{u}^R(a_i, M_{t-1}^R)$ falls to $u^H(a_i, M_{t-1}^R)$. They are then distributed to M_t^R and M_t^H so that $\bar{u}^R(a_i, M_{t-1}^R) = u^H(a_i, M_{t-1}^R)$; that is, from (4) and (11),

$$\frac{1}{M_t^R}\left[\phi M_t^{R^\theta}(1-\lambda)(M_t^H + \eta M_t^L)\right]^{1-\gamma} = 1, \tag{21}$$

which could be interpreted as a research arbitrage condition, is satisfied.

On the other hand, individuals whose ability is below $\bar{a}^H$ must solve the above problem at the beginning of period $t-1$; that is, they decide whether to enter R&D when making the schooling choice, taking the above behavior of individuals with more than $\bar{a}^H$ of ability as given because they never receive high-level schooling without being an entrepreneur. Since M_t^N, M_t^L, and $M_t^H + M_t^R$, on which $\bar{u}^R(a_i, M_{t-1}^R)$ depends, are determined as the result of their decision, their symmetric expectation for these values must be self-fulfilling in the period-t rational expectations equilibrium.

Proposition 1 Suppose $u^H(a_i, M_{t-1}^R)$ exceeds $\bar{u}^R(a_i, M_{t-1}^R)$ under $M_t^N = \frac{\bar{a}^L(M)-a_1}{m} + 1$, $M_t^L = \frac{\bar{a}^H - \bar{a}^L(M)}{m}$, $M_t^H = 0$, and $M_t^R = \frac{a_M - \bar{a}^H}{m}$; namely,

$$\frac{\left[\phi(1-\lambda)\eta\frac{\bar{a}^H-\bar{a}^L(M)}{m}\right]^{1-\gamma}}{\left[\frac{a_M-\bar{a}^H}{m}\right]^{1-\theta(1-\gamma)}} \leq 1. \tag{22}$$

Then, individuals with less than $\bar{a}^H$ of ability never enter R&D in equilibrium. Hence, the period-t equilibrium distribution of individuals in the circumstance of $\hat{\sigma} \leq \sigma < \frac{1}{2}$ is uniquely given by

$$\begin{cases} M_t^N(M_{t-1}^R) & = \frac{\bar{a}^L(M_{t-1}^R)-a_1}{m} + 1 \\ M_t^L(M_{t-1}^R) & = \frac{\bar{a}^H - \bar{a}^L(M_{t-1}^R)}{m} \\ M_t^H(M_{t-1}^R) & = \frac{a_M-\bar{a}^H}{m} - \overline{M}_t^R(M_{t-1}^R) \\ M_t^R(M_{t-1}^R) & = \overline{M}_t^R(M_{t-1}^R) \end{cases}, \tag{23}$$

where $\overline{M}_t^R(M_{t-1}^R)$ is defined by the value of M_t^R that satisfies (21) under $M_t^L = \frac{\bar{a}^H-\bar{a}^L(M_{t-1}^R)}{m}$ and $M_t^H + M_t^R = \frac{a_M-\bar{a}^H}{m}$:

$$\overline{M}_t^R(M_{t-1}^R)\ :\ \frac{\left[\phi(1-\lambda)\left\{\frac{a_M-\bar{a}^H}{m} - \overline{M}_t^R + \eta\frac{\bar{a}^H-\bar{a}^L(M_{t-1}^R)}{m}\right\}\right]^{1-\gamma}}{\overline{M}_t^{R\,1-\theta(1-\gamma)}} = 1. \tag{24}$$

Proof. From (11), an increase in M_t^R by an entry of $a_i \leq \bar{a}^H$ of individuals into R&D decreases $\bar{u}^R(a_i, M_{t-1}^R)$. (22) ensures that for all $0 \leq M_{t-1}^R \leq M$, $\bar{u}^R(a_i, M_{t-1}^R)$ is lowered to $u^H(a_i, M_{t-1}^R)$ as the result of $a_i > \bar{a}^H$ of individuals' occupational choice even when no one of $a_i \leq \bar{a}^H$ of individuals engages in R&D. This means that entering R&D cannot be optimal for $a_i \leq \bar{a}^H$ of individuals under any expectations of $M_t^N \leq \frac{\bar{a}^L(M_{t-1}^R)-a_1}{m} + 1$ and $M_t^L \leq \frac{\bar{a}^H-\bar{a}^L(M_{t-1}^R)}{m}$; thus, the only expectation of $M_t^N = \frac{\bar{a}^L(M_{t-1}^R)-a_1}{m} + 1$ and $M_t^L = \frac{\bar{a}^H-\bar{a}^L(M_{t-1}^R)}{m}$ (thus $M_t^H + M_t^R = \frac{a_M-\bar{a}^H}{m}$) is self-fulfilling. $a_i > \bar{a}^H$ of individuals are distributed to M_t^R and M_t^H according to (24) if

$$\phi(1-\lambda)\left[\frac{a_M-\bar{a}^H}{m} - 1 + \eta\frac{\bar{a}^H-\bar{a}^L(M_{t-1}^R)}{m}\right] \geq 1,$$

while M_t^R bounds to 0 otherwise. □

Characteristic of this equilibrium is that individuals with low-level schooling contribute to the profit of the successful entrepreneur, and thus to technological progress indirectly. Because of the restriction that more than $\hat{\sigma}\eta Q$ of general knowledge must be accumulated in low-level schooling, low-level schooling is demanded purely for learning technological adaptability; that is, individuals enrolled there accumulate two types of knowledge so that their labor productivity is maximized under new technologies. This encourages individuals completing high-level schooling to enter R&D through raising the profit of the successful entrepreneur, and, as the result, a $\phi \overline{M}_t^R (M_{t-1}^R)^\theta$ rate of technological progress is achieved.

A rise in the rate of technological progress in period $t-1$, or, equivalently, an increase in M_{t-1}^R, stimulates the entrepreneurship of generation t, as indicated by

$$\overline{M}_t^{R'}(M_{t-1}^R) = \frac{-\frac{\eta}{m}\bar{a}^{L'}(M_{t-1}^R)}{\frac{1-\theta(1-\gamma)}{(1-\gamma)\phi(1-\lambda)}\overline{M}_t^R(M_{t-1}^R)^{\frac{\gamma}{1-\gamma}-\theta}+1} > 0$$

from (24) and $\bar{a}^{L'}(M_{t-1}^R) < 0$. This is also due to the contribution of members of $M_t^L(M_{t-1}^R)$ on $\overline{\Pi}_t$. A rise in the rate of technological progress in period $t-1$ makes employing new technology A_{t-1} more advantageous than employing A_{t-2}. Thus, more individuals choose to receive low-level schooling in order to acquire knowledge adaptable for new technologies, i.e., $\bar{a}^L(M_{t-1}^R)$ falls. This attracts individuals with high-level schooling to engage in R&D since the labor force that is profitable for the successful entrepreneur increases. Consequently, $\overline{M}_t^R(M_{t-1}^R)$ increases and the quality of A_t rises via R&D spillovers.

In the circumstance of $\beta < \sigma < \hat{\sigma}$, the statement of Proposition 1 apparently applies when $\hat{M}^R \leq M_{t-1}^R \leq M$. When $0 \leq M_{t-1}^R < \hat{M}^R$, the reduced form of the individuals' maximization problem is given by

$$\begin{aligned}
&\max\ \left[u^N, \underline{u}^R(a_i, M_{t-1}^R)\right] \quad \text{for}\ \ a_1 \leq a_i \leq \underline{a}^L \\
&\max\ \left[\underline{u}^L(a_i), \underline{u}^R(a_i, M_{t-1}^R)\right] \quad \text{for}\ \ \underline{a}^L < a_i \leq \underline{a}^H(M_{t-1}^R) \\
&\max\ \left[u^H(a_i, M_{t-1}^R), \underline{u}^R(a_i, M_{t-1}^R)\right] \quad \text{for}\ \ \underline{a}^H(M_{t-1}^R) < a_i \leq a_M
\end{aligned}$$

from Lemma 3. In this situation, both individuals always receiving high-level schooling and those receiving it only if entering R&D make decisions in the same manner as in the circumstance of $\hat{\sigma} \leq \sigma < \frac{1}{2}$.

Individuals with more than $\underline{a}^H(M_{t-1}^R)$ of ability decide whether to be an entrepreneur at the beginning of period t after observing M_t^N, M_t^L, and $M_t^H + M_t^R$. Because $\underline{u}^R(a_i, M_{t-1}^R)$ decreases with their inflows to M_t^R from M_t^H from (13), they are distributed to M_t^R and M_t^H so that $\underline{u}^R(a_i, M_{t-1}^R)$ equalizes to $u^H(a_i, M_{t-1}^R)$; that is,

$$\frac{1}{M_t^R}\left[\phi M_t^{R^\theta}(1-\lambda)M_t^H\right]^{1-\gamma} = 1 \tag{25}$$

is satisfied. Taking this into consideration, individuals with less than $\underline{a}^H(M_{t-1}^R)$ of ability make their decision on R&D in period $t-1$ under the expectedly given M_t^N, M_t^L, and $M_t^H + M_t^R$. The rational expectations equilibrium in this situation is given as follows.

Proposition 2 Suppose $\underline{u}^R(a_i, M_{t-1}^R)$ exceeds $u^H(a_i, M_{t-1}^R)$ under $M_t^N = \frac{\underline{a}^L - a_1}{m}+1$, $M_t^L = \frac{\underline{a}^H(0)-\underline{a}^L}{m}$,

$M_t^H = \frac{a_M - \underline{a}^H(0)}{m} - 1$, and $M_t^R = 1$; namely,

$$\phi(1-\lambda)\left[\frac{a_M - \underline{a}^H(0)}{m} - 1\right] \geq 1. \tag{26}$$

Then, period t equilibrium in the circumstance of $\beta < \sigma < \hat{\sigma}$ and $0 \leq M_{t-1}^R < \hat{M}^R$ is uniquely characterized by the following distribution of individuals:

$$\begin{cases} M_t^N(M_{t-1}^R) & = \frac{\underline{a}^L - a_1}{m} + 1 \\ M_t^L(M_{t-1}^R) & = \frac{\underline{a}^H(M_{t-1}^R) - \underline{a}^L}{m} \\ M_t^H(M_{t-1}^R) & = \frac{a_M - \underline{a}^H(M_{t-1}^R)}{m} - \underline{M}_t^R(M_{t-1}^R) \\ M_t^R(M_{t-1}^R) & = \underline{M}_t^R(M_{t-1}^R) \end{cases}. \tag{27}$$

$\underline{M}_t^R(M_{t-1}^R)$ is the value of M_t^R satisfying (25) under $M_t^L = \frac{\underline{a}^H(M_{t-1}^R) - \underline{a}^L}{m}$ and $M_t^H + M_t^R = \frac{a_M - \underline{a}^H(M_{t-1}^R)}{m}$, and it is acquired in $1 \leq M_t^R(M_{t-1}^R) < \frac{a_M - \underline{a}^H(M_{t-1}^R)}{m}$ for all $0 \leq M_{t-1}^R < \hat{M}^R$:

$$\underline{M}_t^R(M_{t-1}^R) \;:\; \frac{\left[\phi(1-\lambda)\left\{\frac{a_M - \underline{a}^H(M_{t-1}^R)}{m} - \underline{M}_t^R\right\}\right]^{1-\gamma}}{\underline{M}_t^{R^{1-\theta(1-\gamma)}}} = 1. \tag{28}$$

Proof. From (13), regardless of $0 \leq a_i \leq \underline{a}^H(M_{t-1}^R)$ of individuals' behavior, $\underline{u}^R(a_i, M_{t-1}^R)$ certainly falls below $u_t^H(a_i, M_{t-1}^R)$ if all of $\underline{a}^H(M_{t-1}^R) < a_i \leq a_M$ of individuals enter R&D since it means that $M_t^H = 0$. Thus, not being an entrepreneur is always optimal for $0 \leq a_i \leq \underline{a}^H(M_{t-1}^R)$ of individuals. $\underline{M}_t^R(M_{t-1}^R) < \frac{a_M - \underline{a}^H(M_{t-1}^R)}{m}$ is easily seen from (28), and (26) ensures $1 \leq \underline{M}_t^R(M_{t-1}^R)$ for all $0 \leq M_{t-1}^R < \hat{M}^R$. □

This proposition states that unlike in the circumstance of $\hat{\sigma} \leq \sigma < \frac{1}{2}$, entrepreneurship of individuals with high-level schooling originates only from themselves in this situation. Because low-level schooling imposes less than $\hat{\sigma}\eta Q$ of general knowledge accumulation, it can be demanded as a device to improve labor productivity for old technology when $A_{t-1} < (1 + \phi \hat{M}^{R^\theta})A_{t-2}$. Because of this secondary use of low-level schooling, demand for high-level schooling becomes smaller than that in the circumstance of $\hat{\sigma} \leq \sigma < \frac{1}{2}$, i.e., $\underline{a}^H(M_{t-1}^R) > \bar{a}^H$. Moreover, individuals with low-level schooling are valueless in the workforce for the successful entrepreneur because they accumulate specific knowledge rather than general knowledge in school and thus fail to learn technological adaptability. For these reasons, the profit of the successful entrepreneur is diminished to $\underline{\Pi}_t$, and thus the number of entrepreneurs remains at $\underline{M}_t^R(M_{t-1}^R)$, where $\underline{M}_t^R(M_{t-1}^R) < \overline{M}_t^R(M_{t-1}^R)$ from (24) and (28).

A rise in the quality of A_{t-1} increases demand for high-level schooling, i.e., it lowers $\underline{a}^H(M_{t-1}^R)$ because employing A_{t-1} becomes relatively preferable. Since this increase in individuals with high-level schooling leads to enter R&D, $\underline{M}_t^R(M_{t-1}^R)$ increases with M_{t-1}^R as (28) and $\underline{a}^{H'}(M_{t-1}^R) < 0$ yield

$$\underline{M}_t^{R'}(M_{t-1}^R) = \frac{-\frac{1}{m}\underline{a}^{H'}(M_{t-1}^R)}{\frac{1-\theta(1-\gamma)}{(1-\gamma)\phi(1-\lambda)}\underline{M}_t^R(M_{t-1}^R)^{\frac{\gamma}{1-\gamma}-\theta} + 1} > 0.$$

4 Long-run Growth

4.1 Steady State Analysis

Under (22) and (26), the dynamic system of the economy is simply given by

$$M_t^R(M_{t-1}^R) = \overline{M}_t^R(M_{t-1}^R) \tag{29}$$

in the circumstance of $\hat{\sigma} \leq \sigma < \frac{1}{2}$. In the circumstance of $\beta < \sigma < \hat{\sigma}$, it is given by

$$M_t^R(M_{t-1}^R) = \begin{cases} \overline{M}_t^R(M_{t-1}^R) & \text{if} \quad \hat{M}^R \leq M_{t-1}^R \leq M \\ \underline{M}_t^R(M_{t-1}^R) & \text{if} \quad 0 \leq M_{t-1}^R < \hat{M}^R \end{cases} \tag{30}$$

since $\underline{M}_t^R(M_{t-1}^R)$ instead of $\overline{M}_t^R(M_{t-1}^R)$ governs the equilibrium number of entrepreneurs as long as $M_{t-1}^R < \hat{M}^R$.[10] Figure 5 is an example of a phase diagram for (29). Although $\overline{M}_t^{R''}(M_{t-1}^R)$ is ambiguous, the $\overline{M}_t^R(M_{t-1}^R)$ curve cuts the diagonal from above at least at once in $1 < M_{t-1}^R < \frac{a_M - \bar{a}^H}{m}$ from $\overline{M}_t^{R'}(M_{t-1}^R) > 0$, $1 \leq \overline{M}_t^R(0) < \overline{M}_t^R(1)$, and $\overline{M}_t^R(\frac{a_M - \bar{a}^H}{m}) < \overline{M}_t^R(M) \leq \frac{a_M - \bar{a}^H}{m}$. Hence, at least one locally stable steady state, M^{R*}, defined by $\overline{M}_t^R(M^{R*}) = M^{R*}$, exists in this area. On the other hand, an example of a phase diagram for (30) is presented in Figure 6. From $\underline{M}_t^R(M_{t-1}^R) < \overline{M}_t^R(M_{t-1}^R)$, discontinuity of $M_t^R(M_{t-1}^R)$ arises at $M_{t-1}^R = \hat{M}^R$. If $\underline{M}_t^R(\hat{M}^R) \leq \hat{M}^R$, at least one locally stable steady state, M^{R**}, defined by $\underline{M}_t^R(M^{R**}) = M^{R**}$, exists in $1 < M_{t-1}^R \leq \hat{M}^R$ since $\underline{M}_t^{R'}(M_{t-1}^R) > 0$ and $1 \leq \underline{M}_t^R(0) < \underline{M}_t^R(1)$, although $\underline{M}_t^{R''}(M_{t-1}^R)$ is ambiguous. Additionally, if $\hat{M}^R < \overline{M}_t^R(\hat{M}^R)$ is also satisfied, two types of locally stable steady state, M^{R*} and M^{R**}, where $M^{R**} < M^{R*}$, coexist as illustrated there.

Here the growth rate of aggregate output in each steady state should be confirmed. In an equilibrium characterized by $M_t^R = \overline{M}_t^R(M_{t-1}^R)$, members of M_t^H and M_t^L produce consumption goods using A_{t-1} during part (λ) of period t and thereafter work under the monopolist of A_t, whereas members of M_t^N use A_{t-2} throughout period t. From this and (2), (3), and (23), period-t aggregate output is given by

$$\begin{aligned} Y_t \;=\; & A_{t-2}\Big\{\Big[\tfrac{\bar{a}^L(M_{t-1}^R) - a_1}{m} + 1\Big]\mu \\ & + (1 + \phi M_{t-1}^{R^\theta})\Big[\tfrac{a_M - \bar{a}^H}{m} - M_t^R + \eta\tfrac{\bar{a}^H - \bar{a}^L(M_{t-1}^R)}{m}\Big]\lambda\alpha^\alpha(1-\alpha)^{1-\alpha}Q \\ & + (1 + \phi M_{t-1}^{R^\theta})(1 + \phi M_t^{R^\theta})\Big[\tfrac{a_M - \bar{a}^H}{m} - M_t^R + \eta\tfrac{\bar{a}^H - \bar{a}^L(M_{t-1}^R)}{m}\Big](1-\lambda)\alpha^\alpha(1-\alpha)^{1-\alpha}Q\Big\}. \end{aligned}$$

The term within the large bracket on the RHS is constant over time in a steady state since $M_j^R = M^{R*}$ for all j there. Hence, the growth rate of aggregate output corresponds to that of technology in M^{R*}, i.e., ϕM^{R*^θ}. The result is the same if the economy is in the equilibrium characterized by $M_t^R = \underline{M}_t^R(M_{t-1}^R)$. Noting that both members of M_t^N and M_t^L remain in A_{t-2} throughout period t in this situation, Y_t is

[10] From $\phi(1-\lambda)\left[\frac{a_M - \bar{a}^H}{m} - 1 + \eta\frac{\bar{a}^H - \bar{a}^L(0)}{m}\right] > \phi(1-\lambda)\left[\frac{a_M - \underline{a}^H(0)}{m} - 1\right]$, $1 \leq \overline{M}_t^R(0)$ is also assured under (26).

expressed as

$$
\begin{aligned}
Y_t \quad = \quad & A_{t-2}\left\{\left[\frac{\underline{a}^L - a_1}{m} + 1\right]\mu + \frac{\underline{a}^H(M^R_{t-1}) - \underline{a}^L}{m}\sigma^\beta(1-\sigma)^{1-\beta}\eta Q\right. \\
& + (1 + \phi M^{R^\theta}_{t-1})\left[\frac{a_M - \underline{a}^H(M^R_{t-1})}{m} - M^R_t\right]\lambda\alpha^\alpha(1-\alpha)^{1-\alpha}Q \\
& \left. + (1 + \phi M^{R^\theta}_{t-1})(1 + \phi M^{R^\theta}_t)\left[\frac{a_M - \underline{a}^H(M^R_{t-1})}{m} - M^R_t\right](1-\lambda)\alpha^\alpha(1-\alpha)^{1-\alpha}Q\right\}
\end{aligned}
$$

from (2), (3), and (27). This and $M^R_j = M^{R**}$ for all j in a steady state mean that aggregate output and technology grow at the same rate, ϕM^{R**^θ}, in M^{R**}. Therefore, M^{R*} and M^{R**} correspond to the high-growth and low-growth steady state, respectively.

Proposition 3 Suppose (22), (26), and $\underline{M}^R_t(\hat{M}^R) \leq \hat{M}^R < \overline{M}^R_t(\hat{M}^R)$ hold and each type of steady state is uniquely acquired. Then, the economy settles in the high-growth steady state M^{R*} in the long run and achieves a ϕM^{R*^θ} rate of output growth if $\hat{\sigma} \leq \sigma$. If $\sigma < \hat{\sigma}$, however, the economy may be caught in the development trap; that is, the economy that starts with $M^R_0 < \hat{M}^R$ converges to the low-growth steady state M^{R**} where output growth remains at the low rate ϕM^{R**^θ}.

Proof. Follows from the above discussion. □

[Figure 5 and Figure 6 around here]

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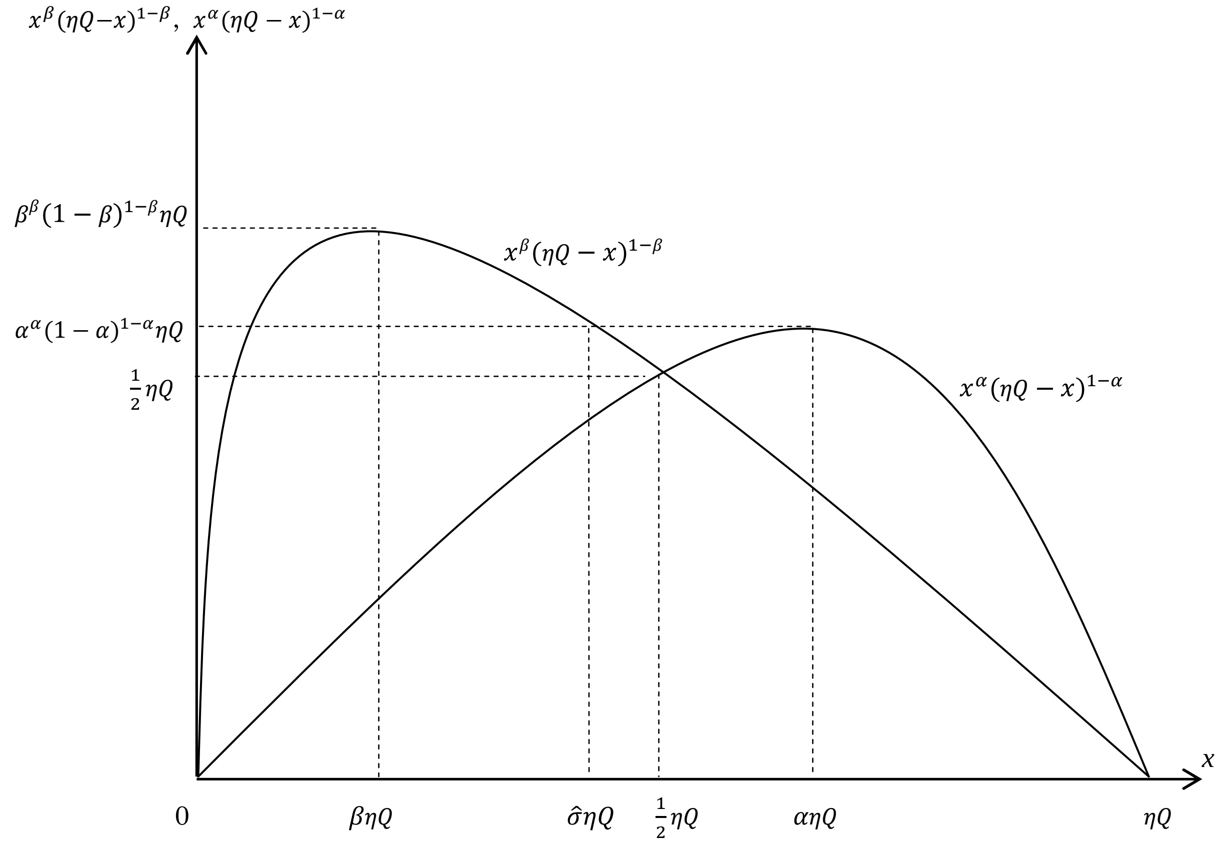
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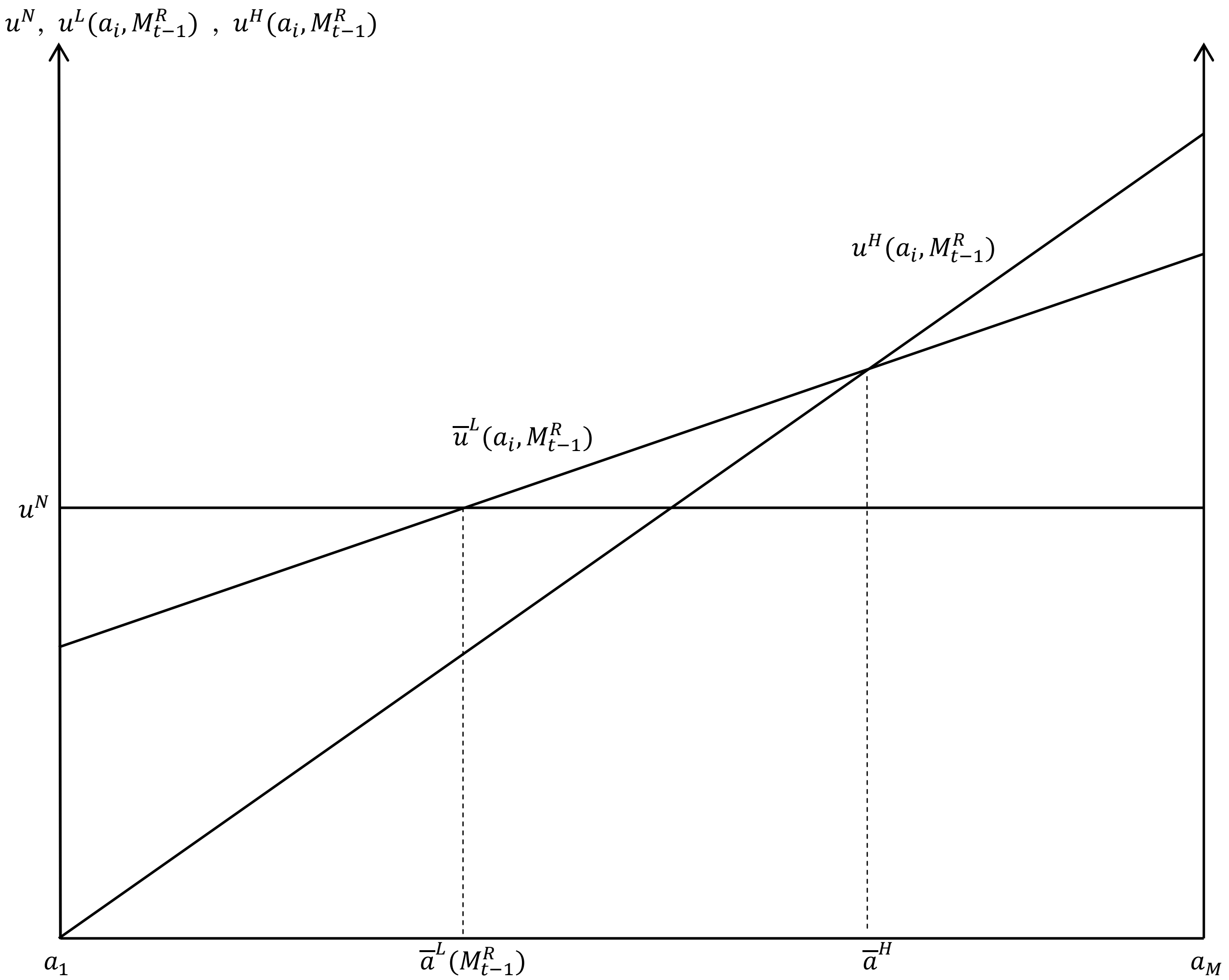
Figure 1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$\overline{u}^L(a_i, M_{t-1}^R)$
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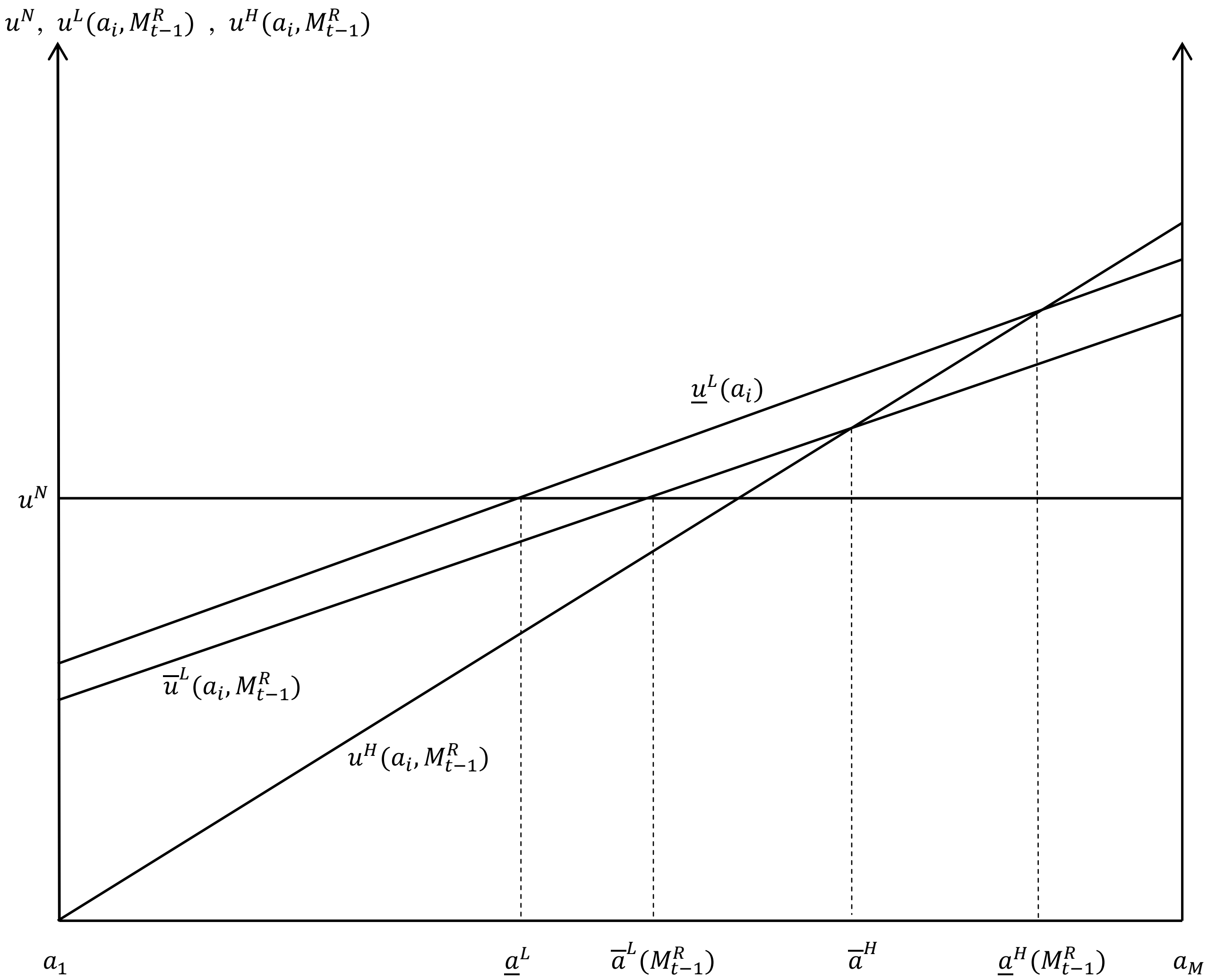
a_1
$\overline{a}^L(M_{t-1}^R)$
$\overline{a}^H$
a_M

Figure 2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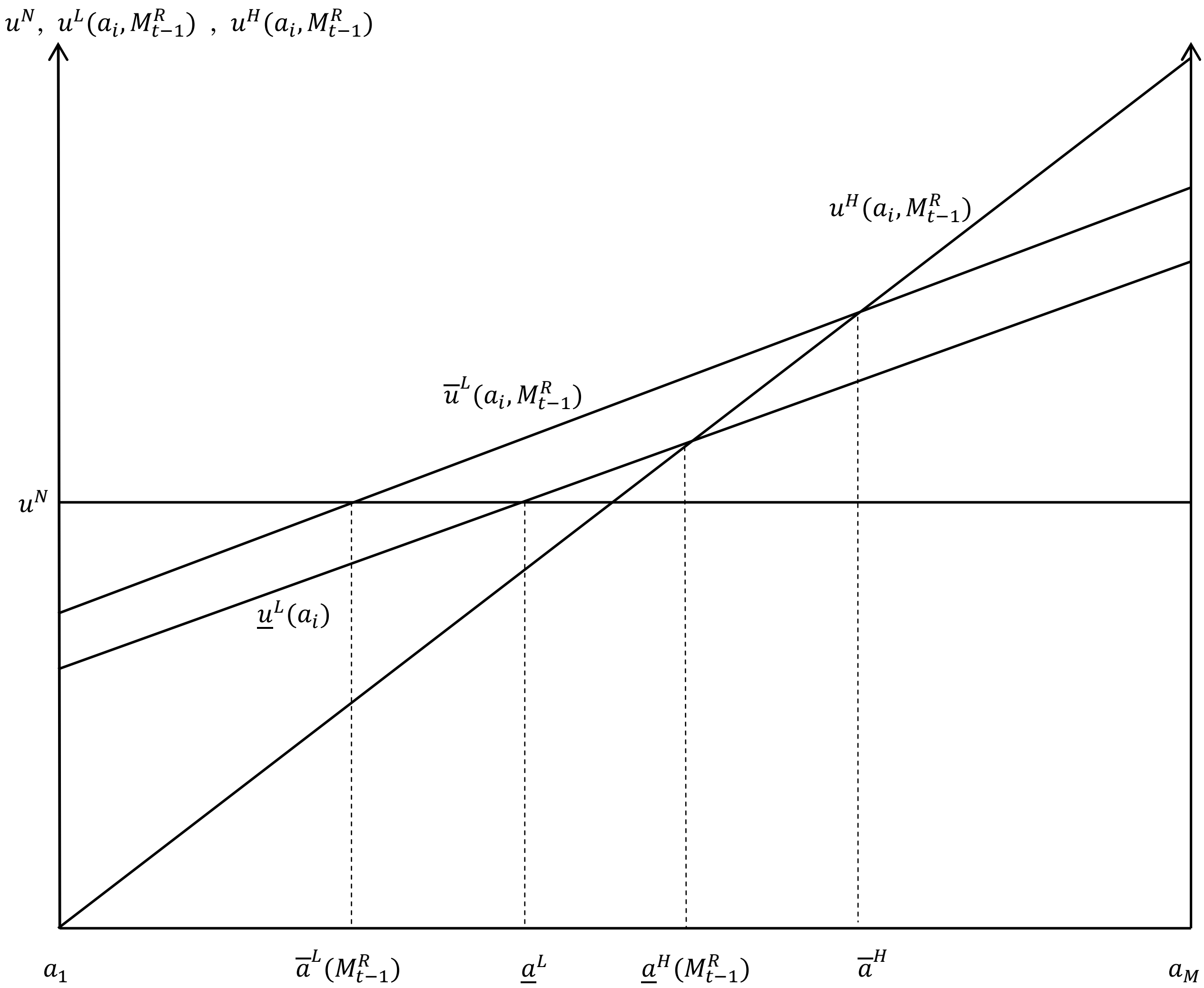
$\overline{u}^L(a_i, M^R_{t-1})$
$u^H(a_i, M^R_{t-1})$
a_1
$\underline{a}^L$
$\overline{a}^L(M^R_{t-1})$
$\overline{a}^H$
$\underline{a}^H(M^R_{t-1})$
a_M

Figure 3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$\overline{u}^L(a_i, M_{t-1}^R)$
u^N
$\underline{u}^L(a_i)$
a_1
$\overline{a}^L(M_{t-1}^R)$
$\underline{a}^L$
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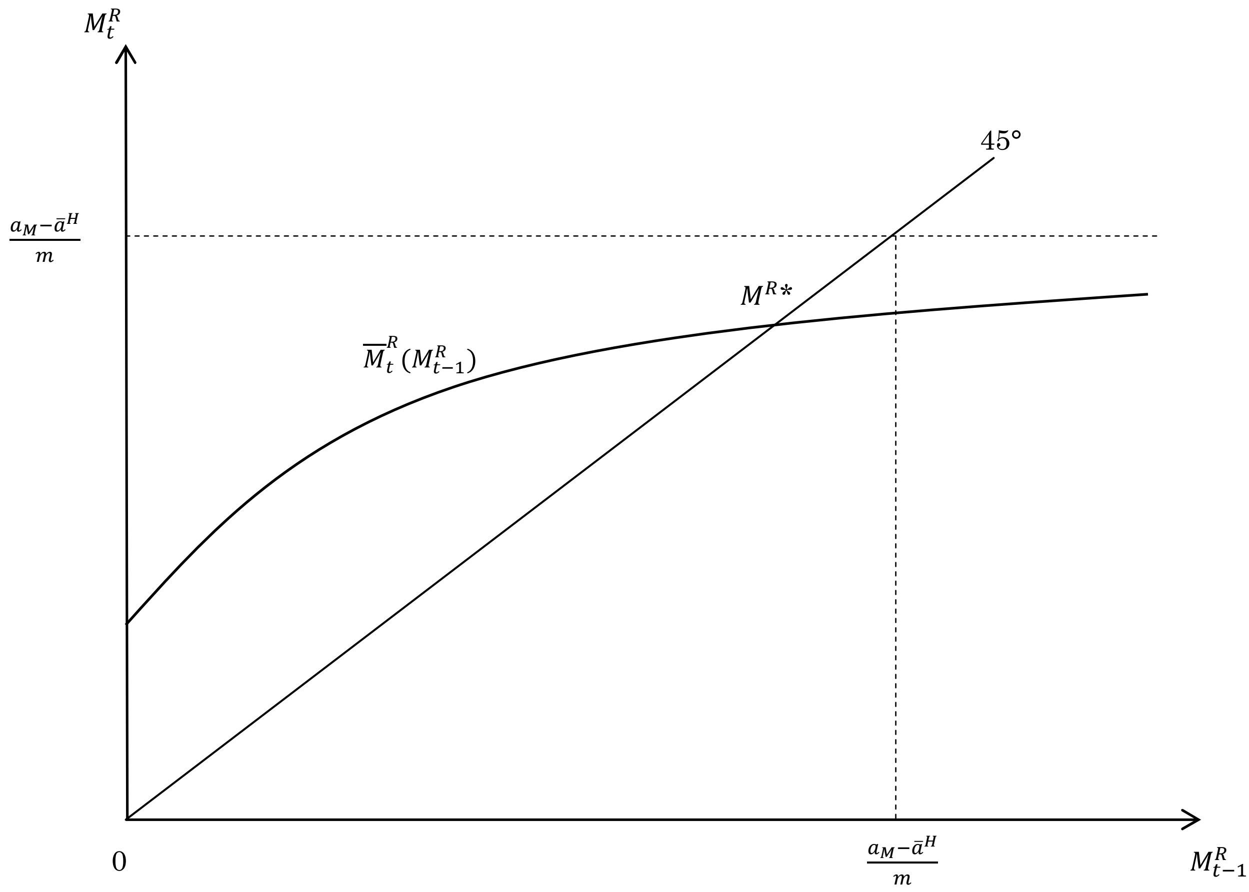
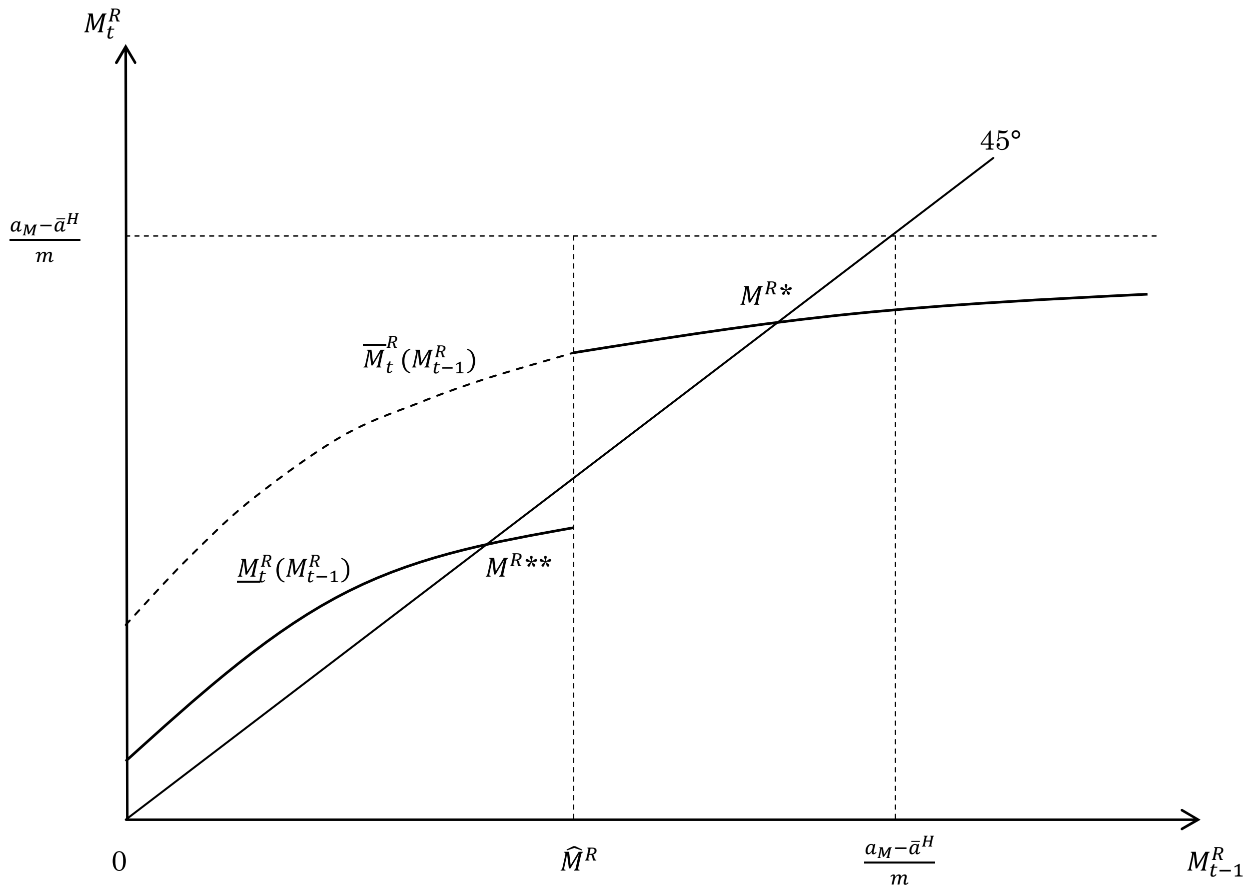
$\overline{a}^H$
a_M

Figure 4



M_t^R
$\frac{a_M - \bar{a}^H}{m}$
45°
M^R*
$\overline{M}_t^R(M_{t-1}^R)$
0
$\frac{a_M - \bar{a}^H}{m}$
M_{t-1}^R

Figure 5



M_t^R
45°
$\frac{a_M - \bar{a}^H}{m}$
M^{R*}
$\overline{M}_t^R(M_{t-1}^R)$
$\underline{M}_t^R(M_{t-1}^R)$
M^{R**}
0
$\widehat{M}^R$
$\frac{a_M - \bar{a}^H}{m}$
M_{t-1}^R

Figure 6