Optimal Fiscal Expenditure and Deviation Rules under Political Uncertainty*

Ryo Arawatari† Tetsuo Ono‡
Doshisha University Osaka University

May 8, 2024

Working Paper Series No.61
Faculty of Economics, Doshisha University

Abstract

This study describes optimal fiscal rules within a model with fiscal rule deviations in a two-period political turnover framework. Considering potential power loss, the incumbent political party aims to secure favored expenditures through increased debt issuance. Expenditure and deviation rules are introduced, requiring legislative approval for deviations from the former. Analysis reveals an optimal deviation rule which favors flexible responses to stringent expenditure rules. Larger initial debt balances warrant tighter expenditure rules, while the optimal deviation rule remains unaffected. Finally, political conflict influences the deviation rule's permissiveness, tilting towards the incumbent party's preferences as conflicts escalate.

Key words: Fiscal rules, Government debt, Political turnover.

JEL Classification: D72, D78, H62, H63

---

*Financial support from the Japan Society for the Promotion of Science Grants-in-Aid for Scientific Research (C) (No. 21K01395, Ryo Arawatari; No. 21K01539, Tetsuo Ono) is gratefully acknowledged. The authors declare that they have no known competing financial interests or personal relationships which influenced this work.

†Faculty of Economics, Doshisha University, Karasuma-higashi-iru, Imadegawa-dori, Kamigyo-ku, Kyoto 602-8580, Japan. E-mail: rarawata@mail.doshisha.ac.jp

‡Graduate School of Economics, Osaka University, 1-7, Machikaneyama, Toyonaka, Osaka 560-0043, Japan. Email: tono@econ.osaka-u.ac.jp

1 Introduction

Since the start of the 21st century, addressing excessive budget deficits and continued rise in government debt has emerged as a common policy challenge in advanced countries. To address it, an increasing number of countries have implemented fiscal rules (see Figure 1). Yet, empirical evidence indicates that these rules' adoption has not effectively restrained excessive budget deficits and government debt accumulation. One contributing factor is the lax enforcement of fiscal rules. Legal frameworks and constitutional provisions establishing fiscal rules often incorporate "deviation clauses," permitting deviations from these rules under specific conditions (Eyraud, et al., 2018).

Figure 2 illustrates the prevalence of deviation clauses in fiscal rules across countries. These clauses have primarily been incorporated into the fiscal frameworks of advanced countries since the mid-2000s, with subsequent adoption observed in developing countries. Utilizing international panel data, Davoodi et al. (2022) estimate that the likelihood of compliance with fiscal rules among countries adopting them between 2004 and 2021 is approximately 50 percent.[1] This finding underscores the notable impact of deviation clauses on fiscal rule implementation in recent years.

The deviation clause has both advantages and disadvantages. It facilitates flexible fiscal adjustments during significant external shocks, such as the 2008 financial or 2020 COVID-19 crises, enabling timely responses to economic fluctuations. Conversely, poorly crafted deviation clauses can render fiscal rules ineffective, undermining the ability to adequately manage excessive budget deficits. Hence, deviation clause design plays a crucial role in shaping the effectiveness of fiscal rules.

We theoretically characterize optimal deviation clause design, or *optimal deviation rules*. This study constructs a model that incorporates a deviation from the fiscal rule (Piguillem and Riboni, 2021) within a two-period framework of political turnover, drawing upon the frameworks proposed by Persson and Svensson (1989) and Alesina and Tabellini (1990). The model considers two distinct categories of public expenditure and accounts for two types of voters with differing public expenditure preferences, each represented by a political party. Voters' preferences for public expenditure are assumed to evolve in response to changes in social and economic contexts. Consequently, each party faces the risk of alternation in power due to stochastic shifts in voter composition.

Confronted with the prospect of political turnover, the incumbent party is motivated to augment its current preferred public expenditure through increased public debt issuance. This strategy stems from the concern that its favored public expenditure may face reductions should it lose power in the future. Consequently, political disputes over public expenditure and the potential for changes in power lead to excessive budget deficits. To address these deficits, we supplement the standard model of political turnover with two types of rules: a "expenditure rule" (Piguillem and Riboni, 2021), which imposes a cap on public expenditure, and "deviation

[1] Deviations from budget balance and debt rules are common across countries, even before the pandemic. On average, countries exceeded the deficit and debt limits approximately 50 and 42 percent of the time during 2004-21, respectively (Davoodi et al., 2022).

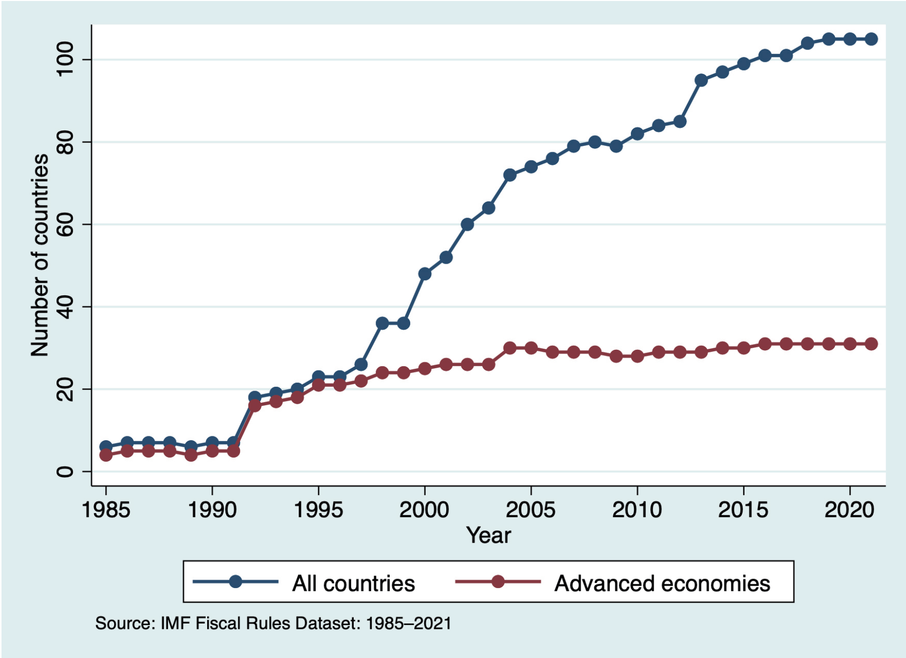

Figure 1: Trends in the number of countries adopting fiscal rules.

(a) (b)

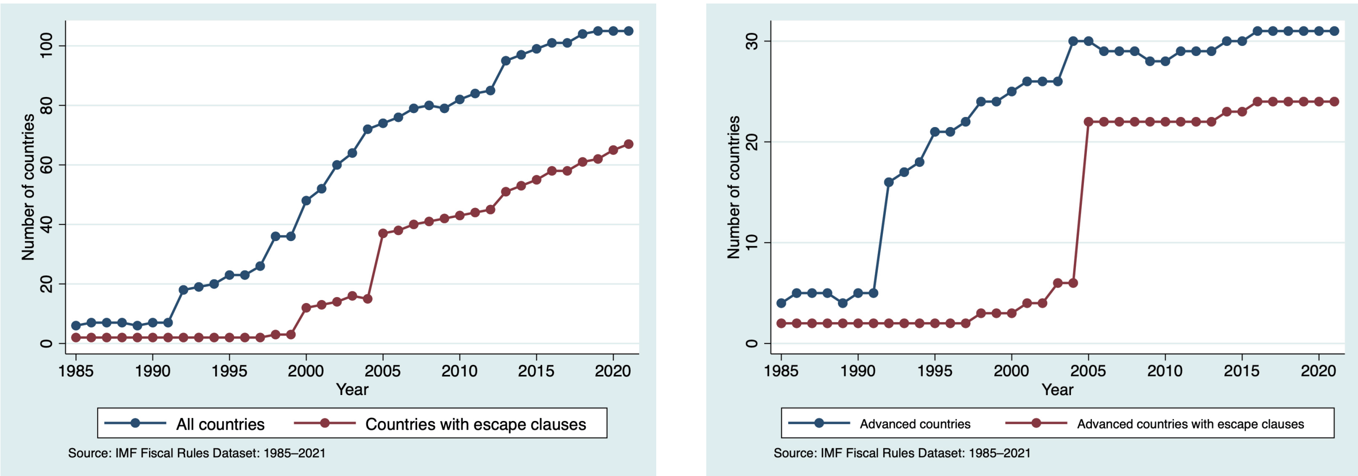

Figure 2: Trends in the number of countries adopting fiscal rule deviation clauses. Panel (a) shows the trends for all countries and panel (b) shows the trends for advanced countries.

rule", which outlines conditions for deviating from the expenditure rule. Specifically, the study examines an environment where deviations from the expenditure rule are permissible if more than δ of the share in legislature members approve the deviation. This feature distinguishes the model used here.

Within this framework, we examine the optimal deviation rule from three perspectives. First, we explore the magnitude of the optimal deviation rule δ under a given expenditure rule. The analysis reveals that the optimal δ for any expenditure rule always remains below 1. This indicates that a rigid expenditure rule requiring strict adherence is undesirable. Furthermore, the stricter the given expenditure rule, the looser the optimal deviation rule. Thus, when fiscal expenditure is tightly restricted by expenditure rules, deviation rules should be preferably designed loosely to allow for flexible responses to changes in economic environments.

Second, we clarify the relationship between the fiscal situation and optimal deviation rule. Specifically, we analyze the optimal deviation rule's characteristics given the initial debt balance. We find that as the initial debt balance increases, a stricter expenditure rule becomes optimal, whereas the optimal deviation rule remains unaffected by the initial debt balance. The following mechanism drives this result: Debt levels influence social welfare solely through public goods expenditure. However, the deviation rule does not impact public goods expenditure because the incumbent party determines it after knowing whether the deviation rule can be applied. Hence, the deviation rule cannot control public goods expenditure based on the initial debt balance. Consequently, we suggest that the deviation rule should maintain consistency over the long-term following the economic structure, rather than changing it in response to changes in the fiscal situation.

Third, we clarify the relationship between the magnitude of political conflict among voters (i.e., between parties) and the optimal deviation rule. Specifically, we investigate how differences in preferences for public goods expenditure among voters impact the optimal deviation rule. We find that as political conflict among voters increases, the optimal deviation rule becomes more permissive. Given more supports of the incumbent party than the opposition party, the optimal deviation rule aligns more with the incumbent's preferences as political conflict evolves. Consequently, a larger disparity in preferences leads to a preference for more permissive deviations and expenditure rules.

The remainder of this article is organized as follows. Section 2 reviews the relevant literature. Section 3 presents the model. Section 4 examines the optimal pair of an expenditure rule and a deviation rule. Section 5 conducts a comparative statics analysis to explore the effects of initial public debt levels and variations in preferences between parties on the optimal fiscal rule. Section 6 provides concluding remarks. The appendix provides the proofs of the lemmas and propositions.

2 Related Literature

This study is closely related to Piguillem and Riboni (2021), who develop a strategic debt model in which preference misalignment between current and future governments creates incentives for the incumbent to overissue public debt (Persson and Svensson, 1989; Alesina and Tabellini, 1990). The authors introduce the possibility for politicians to override fiscal rules with a consensus between the incumbent and opposition. Within this framework, they examine which fiscal rule is the most effective in promoting interparty compromise and reducing debt. We adopt their framework but formulate regulations for deviations from fiscal rules, rather than allowing their negotiation between parties. In this alternative framework, we characterize the optimal deviation rule from the perspective of maximizing social welfare. Our findings provide a basis for evaluating the deviation rules, which are currently being implemented in numerous countries, from a social welfare standpoint.

This study is also related to the political economy literature of fiscal rule violation (Coate and Milton, 2019; Dovis and Kirpalani, 2020; Halac and Yared, 2022; Arawatari and Ono, 2021, 2022). In Coate and Milton's (2019) analysis, two pivotal roles emerge: the constitutional designer, representing citizens, and the politician, who is inclined towards higher taxes. The designer establishes a fiscal limit to curb the politician's bias, yet the politician can override it with citizen approval. The authors focus on determining the optimal fiscal limit and examining its sensitivity to potential overrides. However, the static analytical framework leads to the exclusion of expenditure financing through bond issues and the resulting budget deficits. Consequently, fiscal rules controlling deficits are beyond the study's scope.

Dovis and Kirpalani (2020) introduce instances when fiscal rules were established but lacked ex-post enforcement, as evidenced in the Stability and Growth Pact within the European Union (EU). They present a model populated by local and central governments to elucidate these occurrences. Within their framework, local governments may violate fiscal rules and engage in excessive borrowing; however, the authors do not address the determination of optimal fiscal rules for such cases.

Halac and Yared (2022) examine a government displaying a present bias towards public expenditure while possessing private information about shocks impacting the value of such expenditure. Society selects a fiscal rule, aiming to balance the advantages of committing the government to avoid over-expenditure against the advantages of allowing flexibility to respond to shocks. The authors demonstrate that rule violation occurs under sufficiently high shocks only when the penalties are weak and such shocks are relatively unlikely. However, the authors do not delve into optimal fiscal rule design in the presence of the possibility of fiscal rule violation.

Arawatari and Ono (2021, 2022) also examine a present-biased government's behavior, as in Halac and Yared (2022). The former set of authors utilize the Bisin et al.'s (2015) model, which incorporates time-inconsistent voters and voters with present bias are inclined to increase present consumption by raising public debt issuance. However, this present-biased behavior's influence can be reduced by imposing a debt ceiling. Arawatari and Ono (2021, 2022) extend this analysis by allowing the debt ceiling to be overridden. They illustrate instances of such overrides (Arawatari and Ono, 2021) and present an optimal debt rule under the possibility of overriding (Arawatari and Ono, 2022). However, they do not address deviation rule design. Meanwhile, we aim to demonstrate the optimal deviation rule in a political economy framework, a facet that has not been fully covered in the literature.

3 The Model

Our model is based on a standard two-period framework that incorporates political turnover (Persson and Svensson, 1989; Alesina and Tabellini, 1990). The economic setting involves two distinct public expenditure variables, denoted as g^A and g^B, for two categories of voters, A and B, respectively, who have disparate public expenditure preferences. Each voter type's preferences in each period are specified as follows:

$$u_A(g_A, g_B) = \frac{(g_A)^{1-\sigma}}{1-\sigma} + \theta \cdot \frac{(g_B)^{1-\sigma}}{1-\sigma}, \tag{1}$$

$$u_B(g_A, g_B) = \frac{(g_B)^{1-\sigma}}{1-\sigma} + \theta \cdot \frac{(g_A)^{1-\sigma}}{1-\sigma}, \tag{2}$$

where $\sigma \in (0, 1)$ and $\theta \in [0, 1)$ are parameters shaping the preferences of each type of voters. The functions $u_A(g_A, g_B)$ and $u_B(g_A, g_B)$ represent the preferences of voters favoring public expenditure g^A and g^B, respectively. A smaller value of θ intensifies the political conflict surrounding public expenditure.

The preferences of each voter type are stochastically determined for each period. This stochastic determination can be interpreted as a form of preference shock that reflects changes in voters' preferences for public expenditure in response to shifts in social and economic environments. We consider two political parties exist, A and B, representing voters of types A and B, respectively. Given their symmetric preferences, we denote the incumbent's and opposition's current preferences as follows:

$$u_I(g_I, g_O) = \frac{(g_I)^{1-\sigma}}{1-\sigma} + \theta \cdot \frac{(g_O)^{1-\sigma}}{1-\sigma}, \tag{3}$$

$$u_O(g_I, g_O) = \frac{(g_O)^{1-\sigma}}{1-\sigma} + \theta \cdot \frac{(g_I)^{1-\sigma}}{1-\sigma}, \tag{4}$$

where the indices I and O signify incumbent and opposition, respectively.

Assuming that public expenditure, denoted as $\{g_I, g_O\}$, is funded through tax revenue and public debt issuances, the government's budget constraint for each period is expressed as follows:

$$\tau + b' \geq (1 + r)b + g_I + g_O. \tag{5}$$

Here, $\tau > 0$ represents exogenous tax revenue in each period, b represents the outstanding public debt at the beginning of the period, b' denotes the amount of public debt issued during the period, and r is the interest rate on public debt. We assume that the country functions as a small open economy, engaging in borrowing within foreign asset markets at a given interest rate r.

Political turnover uncertainty arises because of the stochastic variation in the number of voters of each type. With $\theta < 1$, the incumbent party is incentivized to augment its current preferred public expenditure g_I by increasing public debt issuance b', considering the risk that its preferred public expenditure may diminish after becoming the opposition in the future. Essentially, political conflict over public expenditure and the prospect of political turnover contribute to the emergence of excessive budget deficits.

Let p denote the number of seats obtained by the incumbent in the previous period through elections in the current period: it is assumed to follow a uniform distribution within the interval $[0, 1]$. The party that secures the majority of seats holds power. Under these assumptions, the probability of political turnover in each period is $1/2$.

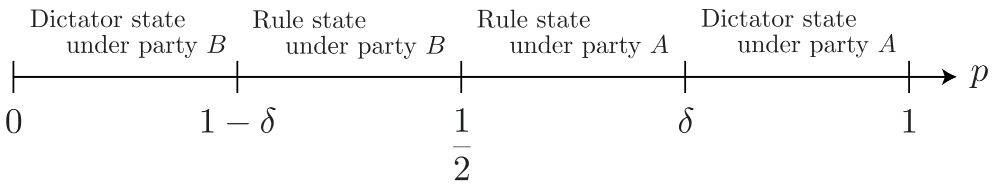

Figure 3: Relationship between the random variable p and incumbent party in the current period if the incumbent in the previous period was party A.

3.1 Expenditure and Deviation Rules

We consider two types of fiscal rules: the expenditure and deviation rules. We follow Piguillem and Riboni (2021), and specify the expenditure rule as follows:

$$g_I + g_O \leq \alpha \cdot (\tau - rb), \quad \alpha \geq 0, \tag{6}$$

where $\alpha \geq 0$ serves as a parameter showing the expenditure rule's stringency. The expenditure rule in (6) dictates that the party in office is restricted to allocate only a certain percentage of tax revenue, net of interest payments on debt. A smaller value of α corresponds to a more rigorous expenditure rule. Specifically, $\alpha = 0$ denotes a shutdown rule, entailing no discretionary expenditure ($g_I + g_O = 0$). Conversely, $\alpha = 1$ denotes adherence to a balanced budget rule, resulting in no public debt balance change ($g_I + g_O + rb = \tau$). When $\alpha = \tau/(\tau - rb)$, a primary balance equilibrium rule exists, ensuring that public expenditure is entirely covered by tax revenues ($g_I + g_O = \tau$).

The pivotal distinction inherent in our model which sets it apart from that of Piguillem and Riboni (2021) is explicitly considering the following deviation rule. We assume that if a political party wins an election and obtains the proportion of seats above $\delta \in [1/2, 1]$, it is entitled to implement fiscal policies without being constrained by the expenditure rule. The parameter δ is an exogenously assigned institutional parameter defining the deviation rule. Similar deviation rules are currently employed in various countries, including Japan, Germany, and Switzerland.

Building upon Piguillem and Riboni (2021), we designate the scenario wherein the incumbent secures more (less) than δ seats as the "dictator (rule) state." Figure 3 illustrates the relationship between the number of seats p held by party A, which held power in the previous period, and applicable expenditure rule for the party in office in the current period. If party A continues to retain power in the current period, it must adhere to the expenditure rule when the probabilistically determined number of seats p falls within the range $1/2 \leq p < \delta$ (rule state). Conversely, if $\delta \leq p$, party A may invoke the deviation rule, bypassing the expenditure rule (dictator state). The equilibrium fiscal policy in the dictator state is denoted as $\{g_I^{d*}, g_O^{d*}, b'^{d*}\}$, while the equilibrium fiscal policy in the rule state is represented as $\{g_I^{r*}, g_O^{r*}, b'^{r*}\}$.

When $\delta = 1/2$, the party in office wins the seats $p \geq 1/2$. Thus, it consistently operates without the expenditure rule, akin to the standard regime change model devoid of fiscal constraints. Conversely, when $\delta = 1$, it is obligated to consistently adhere to the expenditure rule, aligning with the regime change model commonly employed in prior studies on fiscal rules. By specifically examining cases where $1/2 < \delta < 1$, we scrutinize the deviation rule's impact on equilibrium fiscal policy, and explore the optimal interplay between the expenditure and the deviation rules.

3.2 Timing of Events

Policy decisions in a two-period economy begin as follows. In the first period, the randomly assigned variable $p \in [0, 1]$ determines how seats are divided between the political parties A and B. The party with more than half of the seats runs for office, with the authority to shape fiscal policies, including public goods provision and public debt issuance. In the rule state where p falls within the range $(1 - \delta, \delta)$, the party in the office creates and enacts fiscal policies following the expenditure rule. However, in the dictator state where p falls within the range $[\delta, 1]$ or $[0, 1 - \delta]$, the party in the office can decide on fiscal policies without adhering to the expenditure rule.

In the second period, a new random assignment of $p \in [0, 1]$ determines if the incumbent party from the previous period still stays in office. Recall that we assume a uniform distribution for the number of seats p. The incumbent party from the first period may become the opposition in the second period. Consequently, in the first period, it decides fiscal policies considering the possibility. We address the second-period problem before examining the intricacies of the first-period problem.

3.3 Period-2 Incumbent Party's Problem

In period 2, the incumbent party has no opportunity to issue additional debt because the economy ends at the end of period 2. The incumbent party allocates tax revenue to expenditure $\{g_{I,2}, g_{O,2}\}$ and the debt repayment is $(1 + r)b_1$. This renders the expenditure rule irrelevant, eliminating the need to categorize the problem based on whether it is a dictator or rule state. Consequently, the period-2 incumbent party faces the following problem:

$$V_{I,2} = \max_{\{g_{I,2}, g_{O,2}\}} \{u_I(g_{I,2}, g_{O,2})\}, \tag{7}$$

$$\text{s.t. } \tau \geq (1 + r)b_1 + g_{I,2} + g_{O,2}, \tag{8}$$

where $V_{I,2}(b_1)$ is the value function of the incumbent party in period 2.

The supply of public goods chosen by the incumbent party in period 2, $\{g^*_{I,2}, g^*_{O,2}\}$, is given by

$$g^*_{I,2}(b_1) = \frac{\tau - (1 + r)b_1}{1 + \theta^{\frac{1}{\sigma}}}, \quad g^*_{O,2}(b_1) = \theta^{\frac{1}{\sigma}} \cdot \frac{\tau - (1 + r)b_1}{1 + \theta^{\frac{1}{\sigma}}}. \tag{9}$$

From equation (9), it is established that $g^*_{O,2}(b_1)/g^*_{I,2}(b_1) = \theta^{\frac{1}{\sigma}} < 1$. This indicates that as θ decreases, reflecting heightened political conflict, the supply of the public good $g_{O,2}$ preferred by the opposition party and its supporters is relatively small. This also suggests that larger public debt outstanding in period 2, b_1, decreases the supply of public goods. This implies that the incumbent party in period 1 can influence the fiscal policy of the party in office in period 2 through the public debt balance b_1 left to the latter.

By substituting equation (9) into equation (7), we obtain the value function of the incumbent party in period 2 as follows:

$$V_{I,2}(b_1) = u_I\left(g^*_{I,2}(b_1), g^*_{O,2}(b_1)\right) = \frac{1+\theta^{\frac{1}{\sigma}}}{1-\sigma} \cdot \left(\frac{\tau-(1+r)b_1}{1+\theta^{\frac{1}{\sigma}}}\right)^{1-\sigma}. \tag{10}$$

The value function of the opposition party in period 2 is:

$$V_{O,2}(b_1) = u_O\left(g^*_{I,2}(b_1), g^*_{O,2}(b_1)\right) = \frac{\theta+\theta^{\frac{1-\sigma}{\sigma}}}{1-\sigma} \cdot \left(\frac{\tau-(1+r)b_1}{1+\theta^{\frac{1}{\sigma}}}\right)^{1-\sigma}. \tag{11}$$

Comparing the terms $1+\theta^{1/\sigma}$ in equation (10) and $\theta+\theta^{(1-\sigma)/\sigma}$ in equation (11), we get:

$$1+\theta^{\frac{1}{\sigma}} - \left\{\theta+\theta^{\frac{1-\sigma}{\sigma}}\right\} = (1-\theta)\left(1-\theta^{\frac{1-\sigma}{\sigma}}\right) > 0. \tag{12}$$

From the condition in (12), $V_{I,2}(b_1) > V_{O,2}(b_1)$ holds. Thus, the incumbent party's value is higher than that of the opposition party due to political conflict. This is the risk of political turnover for the incumbent party in period 1.

3.4 Period-1 Incumbent Party's Problem

Next, we consider the period-1 incumbent party's problem. Public debt from the outset of period 1, denoted as b_0, is a predetermined variable, and thus, an initial condition for the incumbent party. For this value of b_0, we assume:

Assumption 1 $b_0 < \dfrac{2+r}{(1+r)^2} \cdot \tau \equiv b^{NDL}$.

Note that b^{NDL} denotes the natural debt ceiling. The problem confronting the incumbent party in period 1 hinges on whether its number of seats exceeds δ (dictator state) or falls below δ (rule state). We investigate the period-1 incumbent party's decision-making process for both the dictator and rule state scenarios.

3.4.1 Dictator State Case

Consider the situation where the incumbent party's number of seats in period 1 lies within the range $[\delta, 1]$, thereby realizing the dictator state. Then, the incumbent party possesses the flexibility to implement fiscal policies without being constrained by the expenditure rule. Consequently, the problem faced by the period-1 incumbent party can be defined as follows.

$$V^d_{I,1} = \max_{\{g_{I,1}, g_{O,1}, b_1\}} \left\{u_I(g_{I,1}, g_{O,1}) + \beta W_2(b_1)\right\}, \tag{13}$$

$$\text{s.t. } \tau + b_1 \geq (1+r)b_0 + g_{I,1} + g_{O,1}, \tag{14}$$

where $\beta \in (0, 1)$ is the discount factor and $W_2(b_1)$ is the expected value function of the next period.

The allocation of seats for each party in the subsequent period is determined irrespective of the current seat count. Assuming the uniform distribution of p, the probabilities of being in and out of office in period 2 are equal at 1/2. Consequently, the expected value function for the next period, denoted $W_2(b_1)$, remains identical for both the incumbent and opposition parties, and can be expressed as follows:

$$\begin{aligned} W_2(b_1) &= \frac{1}{2} \cdot V_{I,2}(b_1) + \frac{1}{2} \cdot V_{O,2}(b_1) \\ &= \phi \cdot \frac{1}{1-\sigma} \cdot \left(\frac{\tau - (1+r)b_1}{1+\theta^{\frac{1}{\sigma}}} \right)^{1-\sigma}, \end{aligned} \tag{15}$$

where ϕ is defined as follows:

$$\phi \equiv \frac{1 + \theta^{\frac{1}{\sigma}} + \theta + \theta^{\frac{1-\sigma}{\sigma}}}{2}. \tag{16}$$

To clarify the relationship between expenditure and deviation rules and fiscal policy, we introduce the following assumption.

Assumption 2 $\beta(1 + r) = 1$.

This assumption abstracts from the incumbent party's incentive to save and borrow considering intertemporal optimization. That is, the motivation to issue public debt arises solely from the risk of regime change. This assumption is also a prerequisite for steady-state stability in the infinite-horizon small open economy model.

In the dictator state, the set of fiscal policies selected by the incumbent party is represented as $\{b_1^{d*}(b_0), g_{I,1}^{d*}(b_0), g_{O,1}^{d*}(b_0)\}$. The policies are expressed as:

$$b_1^{d*}(b_0) = \frac{\tau - \left(\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}} \cdot [\tau - (1+r)b_0]}{(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}}, \tag{17}$$

$$g_{I,1}^{d*}(b_0) = \frac{\tau + (1+r)\,[\tau - (1+r)b_0]}{(1+\theta^{\frac{1}{\sigma}})\left[(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right]}, \tag{18}$$

$$g_{O,1}^{d*}(b_0) = \theta^{\frac{1}{\sigma}} \cdot \frac{\tau + (1+r)\,[\tau - (1+r)b_0]}{(1+\theta^{\frac{1}{\sigma}})\left[(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right]}, \tag{19}$$

where $\tau - rb_0 > 0$ holds under Assumption 1.

From equations (18) and (19), the following lemma holds.

Lemma 1 *The greater the conflict between the parties (with a smaller θ), the larger the government expenditure $\left(g_{I,1}^{d*}(b_0) + g_{O,1}^{d*}(b_0)\right)$ in the dictator state.*

Proof. See Appendix A.

The result presented in Lemma 1 implies that the risk of political turnover and conflict between political parties results in excessive fiscal expenditure, similar to Alesina and Tabellini (1990: Proposition 5). The mechanism is as follows. In period 1, the incumbent party faces the risk of political turnover, potentially losing its position in period 2. Since $g^*_{O,2}(b_1)/g^*_{I,2}(b_1) = \theta^{\frac{1}{\sigma}} < 1$ as indicated by equation (9), if the party becomes the opposition in period 2, it lowers preferred public goods provision. Consequently, the incumbent party in period 1 is incentivized to allocate more resources to its preferred public goods while in power. The lower the value of θ and the higher the conflict between parties, the more pronounced these incentives become, increasing public expenditure and public debt issuance.

When no expenditure rule is imposed, public goods provision is determined by equations (18) and (19). With the expenditure rule's introduction, the condition under which the incumbent party is motivated to deviate from the expenditure rule can be expressed as follows.

$$g^{d*}_{I,1}(b_0) + g^{d*}_{O,1}(b_0) > \alpha \cdot (\tau - rb_0) \;\Leftrightarrow\; \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right]}{(\tau - rb_0)\left[(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right]} > \alpha, \tag{20}$$

When the condition in (20) holds, expenditure rule (6) is binding. Then, the incumbent party in period 1 has an incentive to deviate from the expenditure rule. Put differently, in the rule state, the incumbent party must determine its fiscal policy within the confines of both the expenditure and the deviation rules. We introduce the following assumption for this case:

Assumption 3 $\alpha < \dfrac{\tau + (1+r)\left[\tau - (1+r)b_0\right]}{(\tau - rb_0)\left[(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right]} \equiv \bar{\alpha}(b_0),$

$\bar{\alpha}(b_0)$ is a decreasing function of both b_0 and θ. The greater the initial public debt balance (b_0) or the lesser the conflict between parties (resulting in a larger θ), the weaker the incentive for the period-1 incumbent party to provide excessive public goods. Consequently, the expenditure rule does not bind unless it is stronger. In addition, as demonstrated later, the optimal expenditure rule that maximizes social welfare consistently adheres to the condition $\alpha^* < \bar{\alpha}(b_0)$.

Derived from equations (13), (15), (17), and $\bar{\alpha}(b_0)$, the value functions for both the incumbent and opposition parties in the dictator state are respectively as follows.

$$V^d_{I,1}(b_0) = \frac{1}{1-\sigma} \cdot \left[1 + \theta^{\frac{1}{\sigma}} + \beta(1+\theta^{\frac{1}{\sigma}})\left(\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}}\right] \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma}, \tag{21}$$

$$V^d_{O,1}(b_0) = \frac{1}{1-\sigma} \cdot \left[\theta + \theta^{\frac{1-\sigma}{\sigma}} + \beta(1+\theta^{\frac{1}{\sigma}})\left(\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}}\right] \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma}, \tag{22}$$

where $V^d_{I,1}(b_0) > V^d_{O,1}(b_0)$ holds from the condition in (12).

3.4.2 Rule State Case

When the incumbent party has fewer seats than δ in the initial period, the rule state is established. The incumbent party is obligated to formulate fiscal policy in adherence to the expenditure rule. Consequently, the incumbent party's fiscal problem is as follows:

$$V_{I,1}^r = \max_{\{g_{I,1}, g_{O,1}, b_1\}} \{u_I(g_{I,1}, g_{O,1}) + \beta W_2(b_1)\}, \tag{23}$$

$$\text{s.t. } \tau + b_1 \geq (1+r)b_0 + g_{I,1} + g_{O,1}, \tag{24}$$

$$g_{I,1} + g_{O,1} \leq \alpha(\tau - rb_0). \tag{25}$$

The problem faced by the incumbent party in the rule state differs from that in the dictator state in that it involves adhering to the expenditure rule of the form in (25). Furthermore, under Assumption 3, the expenditure rule holds with equality. For this, expression in (25) must hold with an equality sign in the analysis of the ruling state.

Let $\{b_1^{r*}(b_0, \alpha), g_{I,1}^{r*}(b_0, \alpha), g_{O,1}^{r*}(b_0, \alpha)\}$ denote the fiscal policy chosen by the incumbent party in the rule state. Using $g_{O,1}^{r*} = \theta^{\frac{1}{\sigma}} g_{I,1}^{r*}$ derived from the first-order condition, we ascertain that the pair of public expenditure $\{g_{I,1}^{r*}(b_0, \alpha), g_{O,1}^{r*}(b_0 \alpha)\}$ satisfies the following equation:

$$g_{I,1}^{r*} + g_{O,1}^{r*} = (1 + \theta^{\frac{1}{\sigma}}) g_{I,1}^{r*} = \alpha(\tau - rb_0). \tag{26}$$

Utilizing the condition in (26) met by the pair of public expenditure and the budget constraint in (24), we derive the fiscal policy in the rule state as follows:

$$b_1^{r*}(b_0, \alpha) = b_0 + (\alpha - 1)(\tau - rb_0), \tag{27}$$

$$g_{I,1}^{r*}(b_0, \alpha) = \frac{\alpha(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}}, \tag{28}$$

$$g_{O,1}^{r*}(b_0, \alpha) = \theta^{\frac{1}{\sigma}} \cdot \frac{\alpha(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}}. \tag{29}$$

The expressions in equations (27), (28), and (29) indicate that the expenditure rule influences fiscal policy in period-1 of the rule state. Specifically, a stronger expenditure rule (smaller α) leads to more effective control over excessive budget deficits. Essentially, the expenditure rule helps restrain unwarranted fiscal expenditures arising from the risk of political turnover and conflict, as highlighted in Lemma 1.

Substitution of the required fiscal policy, as indicated by equations (27), (28), and (29), in the incumbent party's objective function (23) and expected value of the next period equation (15) yields to the following value functions of the incumbent and opposition parties under the rule state:

$$V_{I,1}^r(b_0, \alpha) = \frac{1}{1-\sigma} \cdot \left\{ \left(1 + \theta^{\frac{1}{\sigma}}\right) \cdot \left[\frac{\alpha(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}}\right]^{1-\sigma} \right.$$

$$+\beta\phi \cdot \left\{ \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}^{1-\sigma} \Bigg\}, \tag{30}$$

$$V_{O,1}^{r}(b_0,\alpha) = \frac{1}{1-\sigma} \cdot \Bigg\{ \left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left[\frac{\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{1-\sigma}$$

$$+\beta\phi \cdot \left\{ \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}^{1-\sigma} \Bigg\}. \tag{31}$$

Equations (30) and (31) demonstrate that the expenditure rule, denoted as α, exerts dual effects on the value functions $V_{I,1}^{r}(b_0,\alpha)$ and $V_{O,1}^{r}(b_0,\alpha)$, respectively. The initial term in equation (30) and (31) functions as an increasing factor with respect to α; a more permissive expenditure rule, indicated by a larger α, results in an augmented provision of public goods $\{g_{I,1}^{r*}(b_0,\alpha), g_{O,1}^{r*}(b_0,\alpha)\}$ during the first period. Conversely, the subsequent term in equation (30) and (31) behaves as a decreasing function of α. A looser expenditure rule corresponds to an increased propensity of the first-period incumbent party to excessively provide public goods and issue public debt. In turn, this diminishes public goods' availability in the second period. Therefore, an increase in α, accompanied by a relaxation of the expenditure rule, yields both positive and negative impacts on the value associated with both the incumbent and opposition parties. The optimal expenditure rule, which is investigated in the next section, represents the point at which these opposing effects are balanced.

In conclusion, we pinpoint the expenditure rule that maximizes the value functions for both the incumbent and opposition parties. By examining equations (30) and (31), we can derive the conditions that determine the optimal rule for the incumbent and opposition parties, respectively, as follows:

$$\frac{\partial V_{I,1}^{r}(b_0,\alpha)}{\partial \alpha} \gtreqless 0 \;\Leftrightarrow\; \alpha \lesseqgtr \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right]}{(\tau - rb_0)\left[(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right]} \equiv \bar{\alpha}(b_0), \tag{32}$$

$$\frac{\partial V_{O,1}^{r}(b_0,\alpha)}{\partial \alpha} \gtreqless 0 \;\Leftrightarrow\; \alpha \lesseqgtr \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right]}{(\tau - rb_0)\left[(1+r) + \left(\phi/\left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]} < \bar{\alpha}(b_0). \tag{33}$$

Expression in (32) suggests that under Assumption 3, the incumbent party favors a relaxed expenditure rule denoted by ($\alpha = \bar{\alpha}(b_0)$). Expression in (33) indicates that the expenditure rule that maximizes the value for the opposition party is an interior solution.

4 Optimal Fiscal Rules

In this section, we derive the optimal expenditure and deviation rules aimed at maximizing social welfare by aggregating voters' utilities and scrutinizing their characteristics.

4.1 Social Welfare Function

Without loss of generality, let p denote the number of seats for party A in period 1. As illustrated in Figure 3, party A becomes the incumbent party when $p \geq \frac{1}{2}$, while party B takes on this role when $p < \frac{1}{2}$. If $p \geq \delta$ or $p \leq 1-\delta$, the state is categorized as the dictator state; otherwise, it is classified as the rule state.

The variable p serves a dual purpose, representing both the number of seats for party A and number of voters of type A. The magnitude of p directly (invesely) correlates with the number of voters for type A (B). Assuming a Benthamite social welfare function, each political party's value should be weighted by the number of seats. Consequently, we introduce the social welfare function $W(b_0; \delta, \alpha)$, defined as follows.

$$\begin{aligned}
W(b_0, \delta, \alpha) =& \mathbb{E}\left[pV_{A,1} + (1-p)V_{B,1}\right] \\
=& \underbrace{\int_{\delta}^{1} \cdot \left[pV_{I,1}^{d}(b_0) + (1-p)V_{O,1}^{d}(b_0)\right] \mathrm{d}p}_{\text{dictator state under party } A} + \underbrace{\int_{1/2}^{\delta} \cdot \left[pV_{I,1}^{r}(b_0, \alpha) + (1-p)V_{O,1}^{r}(b_0, \alpha)\right] \mathrm{d}p}_{\text{rule state under party } A} \\
&+ \underbrace{\int_{1-\delta}^{1/2} \cdot \left[pV_{O,1}^{r}(b_0; \alpha) + (1-p)V_{I,1}^{r}(b_0; \alpha)\right] \mathrm{d}p}_{\text{rule state under party } B} + \underbrace{\int_{0}^{1-\delta} \cdot \left[pV_{O,1}^{d}(b_0) + (1-p)V_{I,1}^{d}(b_0)\right] \mathrm{d}p}_{\text{dictator state under party } B} \\
=& (1-\delta^2)V_{I,1}^{d}(b_0) + (1-\delta)^2 V_{O,1}^{d}(b_0) + \frac{4\delta^2 - 1}{4} \cdot V_{I,1}^{r}(b_0, \alpha) - \frac{4(1-\delta)^2 - 1}{4} \cdot V_{O,1}^{r}(b_0, \alpha),
\end{aligned} \tag{34}$$

where p is uniformly distributed in the $[0, 1]$ interval.

In the following analysis, we initially determine the optimal expenditure rule α corresponding to a given deviation rule δ (in Section 4.2). Then, we identify the optimal δ for a given α (in Section 4.3). Finally, we investigate the optimal combination of α and δ (in Section 4.4).

4.2 Optimal expenditure Rule

Consider the optimal expenditure rule $\alpha^*(b_0, \delta)$ with a given deviation rule δ. If $\delta = 1/2$, the incumbent party acts like a dictator regardless of seat count, rendering the expenditure rule entirely meaningless. Consequently, we concentrate on cases where $\delta > 1/2$ and derive the following lemma.

Lemma 2 *Suppose a deviation rule $\delta \in (1/2, 1]$ is provided. The optimal expenditure rule $\alpha^*(b_0, \delta)$ is then determined by:*

$$\alpha^*(b_0, \delta) = \frac{\tau + (1+r) \cdot [\tau - (1+r)b_0]}{\tau - rb_0} \cdot \frac{\Gamma(\delta)}{1 + (1+r)\Gamma(\delta)} \in (0, \bar{\alpha}(b_0)) \quad \forall \delta \in (1/2, 1], \tag{35}$$

where $\Gamma(\delta)$ *is defined by*

$$\Gamma(\delta) \equiv \left[1 + \frac{(2\delta - 1)\left(1 + \theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right)}{2\left(1 + \theta^{\frac{1}{\sigma}} + \theta + \theta^{\frac{1-\sigma}{\sigma}}\right)}\right]^{\frac{1}{\sigma}} > 1. \tag{36}$$

Proof. See Appendix B.

Lemma 2 indicates that for any deviation rule $\delta \in (1/2, 1]$, the optimal interior solution for the expenditure rule is determined as follows. As observed in equation (34), if a dictator state exists, the expenditure rule does not affect social welfare. The optimal expenditure rule is defined to maximize expected welfare when the rule state occurs. In such cases, the incumbent party prefers the loosest possible expenditure rule ($\alpha = \bar{\alpha}(b_0)$) (see (32) and (33)). Conversely, the opposition party prefers a specific interior solution for the expenditure rule. Analyzing both scenarios shows that the expenditure rule $\alpha^*(b_0, \delta)$ that maximizes social welfare is obtained as an interior solution.

The implication of this result is as follows: Lemma 1's result suggests that the incumbent party has an incentive to overspend with political conflict on the provision of public goods. Stricter expenditure rules can mitigate such excessive expenditure and improve the welfare of the opposition's supporters. However, tighter expenditure rules worsen the incumbent party supporters' welfare. Therefore, the optimal expenditure rule is the one with an interior point that effectively balances these trade-offs.

The result presented in Lemma 2 reveals that the maximum allowable expenditure, denoted as $\alpha^*(b_0, \delta) \cdot (\tau - rb_0)$ under the optimal expenditure rule, is a decreasing function of b_0. Thus, imposing stronger expenditure restrictions is optimal for larger initial public debt balances. Moreover, $\alpha^*(b_0, \delta)$ is an increasing function of δ, indicating that the tighter the deviation rule (demanding greater compliance with the expenditure rule), the more lenient the optimal expenditure rule becomes. Essentially, the strictness of the deviation and expenditure rules are substitutable.

The rationale for this substitutability is as follows. When the deviation rule is stringent, even the incumbent party with many seats is bound by an expenditure rule. In such cases, many voters supporting the incumbent party are constrained by expenditure rules. This implies that the welfare loss associated with enforcing a stringent expenditure rule becomes significant, making a more lenient expenditure rule optimal.

The implications of these findings for real expenditure rules warrant further consideration. Consider Japan, where deviation from the expenditure rule is relatively easy because the government only needs to pass a special bond law in both houses of the Diet. Meanwhile, the expenditure rule is very strict, allowing no deficit government debt except for construction debt aimed at funding public investment, consistent with the relationship outlined in Lemma 2. Consequently, the expenditure rule enforced in Japan may be viewed as suitable for maximizing social welfare.

4.3 Optimal Deviation Rule

Next, given the expenditure rule α, we find the optimal deviation rule $\delta^*(b_0, \alpha)$.

Lemma 3 *For a given expenditure rule $\alpha \in [0, \bar{\alpha}(b_0))$, the optimal deviation rule $\delta^*(b_0, \alpha)$ is:*

$$\delta^*(b_0, \alpha) = \frac{V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0)}{[V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha)] + [V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0)]} < 1. \tag{37}$$

In addition, $\delta^*(b_0, \alpha) > 1/2$ holds if the following condition is satisfied:

$$V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha) < V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0). \tag{38}$$

Proof. See Appendix C.

Lemma 3 shows that, for any expenditure rule $\alpha < \bar{\alpha}(b_0)$, under specific conditions, the optimal deviation rule is determined by the interior point. If δ is small and the deviation rule is loose, the dictator state is likely, rendering the expenditure rule meaningless. This results in over-expenditure by the incumbent party and welfare losses for the opposition. Strengthening the expenditure rule's enforceability involves implementing more stringent deviation rules, curbing excessive fiscal expenditure, and improving opposition supporters' welfare. However, this comes at the cost of worsening the incumbent party supporters' welfare. Therefore, the optimal deviation rule delicately balances these tradeoffs, offering a nuanced solution that mitigates excessive expenditures while considering the welfare of both parties.

Next, consider the condition in (38) for $\delta^*(b_0, \alpha) > 1/2$ to be satisfied. If $\delta = 1/2$, the incumbent party, holding more than half of the seats, can consistently deviate from the expenditure rule. When (38) holds, and therefore, $\delta^*(b_0, \alpha) > 1/2$, the utility gain experienced by the opposition party's supporters during the transition from the dictator to the rule state ($V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0)$) exceeds the utility loss for the incumbent party's supporters ($V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha)$). Given that the incumbent party supporters exceed the opposition party supporters, disregarding the expenditure rule at all times becomes optimal from the social welfare maximization perspective if (38) is violated.

4.4 Optimal Pair of Fiscal Rules

Based on the optimal expenditure rule for a specified deviation rule and optimal deviation rule corresponding to a given expenditure rule derived in Sections 4.2 and 4.3, respectively, here, we seek to identify the optimal combination of expenditure and deviation rules that maximizes social welfare. Formally, we define the optimal combination of the fiscal rules as follows.

Definition 1 *An optimal pair of fiscal rules is represented by a pair $\{\alpha^{opt}(b_0), \delta^{opt}(b_0)\}$ that simultaneously satisfies the following two equations:*

$$\alpha^{opt}(b_0) = \alpha^*(b_0, \delta^{opt}), \quad \delta^{opt}(b_0) = \delta^*(b_0, \alpha^{opt}). \tag{39}$$

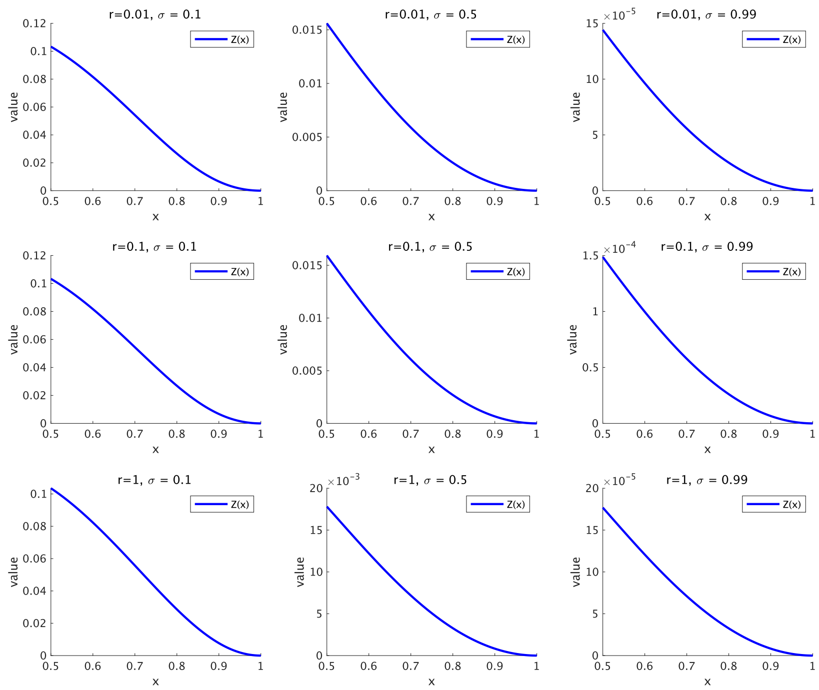

Figure 4: The function $Z(x)$ with $r = \{0.01, 0.1, 1\}$ and $\sigma = \{0.1, 0.5, 0.99\}$.

From Lemmas 2 and 3, we obtain the following proposition.

Proposition 1 *Suppose that the following inequality condition holds:*

$$Z(x) \equiv \left[(1+r) + x^{\frac{1}{\sigma}}\right]^{\sigma} \cdot \left[1 + (1+r)\left(\frac{1}{2} + \frac{1}{2x}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} - (1+r)\left(\frac{1}{2} + \frac{1}{2x}\right)^{\frac{1-\sigma}{\sigma}} - x > 0, \quad (40)$$

$$x \equiv \frac{\phi}{1+\theta^{\frac{1}{\sigma}}} \in \left[\frac{1}{2}, 1\right). \quad (41)$$

There is an interior optimal pair of fiscal rules that satisfy $\alpha^{opt}(b_0) \in (0, \bar{\alpha}(b_0))$ *and* $\delta^{opt}(b_0) \in (1/2, 1)$.

Proof. See Appendix D.

Generally, it is difficult to demonstrate analytically that the condition specified by (40) is universally satisfied. To resolve this difficulty, we conduct a numerical check, confirming that the condition in (40) holds across a broad range of parameters. Figure 4 illustrates the function $Z(x)$ with $r = \{0.01, 0.1, 1\}$ and $\sigma = \{0.1, 0.5, 0.99\}$. Evidently, in all cases, $Z(x) > 0$ holds for any $x \in [1/2, 1)$. Hereafter, we proceed with the analysis assuming that the condition in (40) is satisfied.

Figure 5 illustrates the reaction functions of $\delta^*(b_0, \alpha)$ and $\alpha^*(b_0, \delta)$ for a given initial public debt b_0. The intersection of the two reaction functions is the combination of the optimal

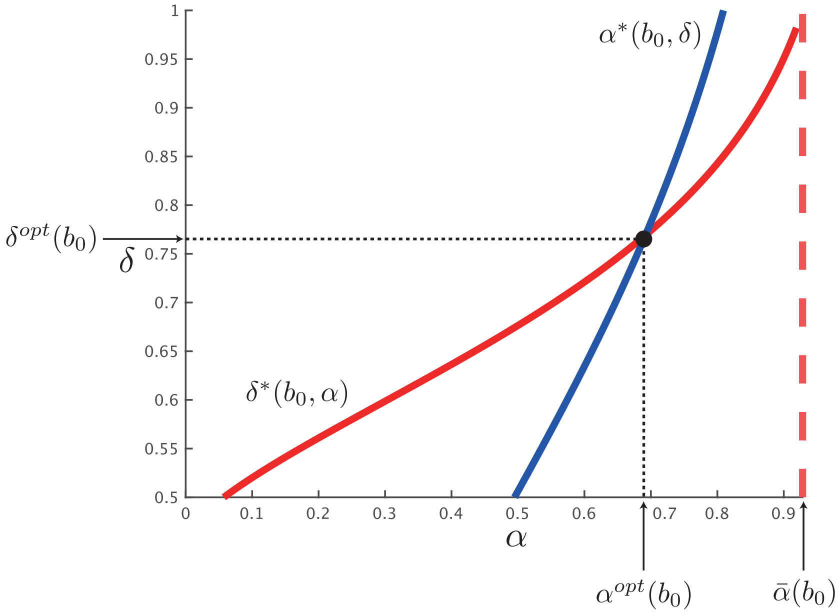

Figure 5: Numerical calculations of the optimal expenditure rule $\alpha^*(b_0,\delta)$ given the deviation rule δ, and optimal deviation rule $\delta^*(b_0,\alpha)$ given the expenditure rule α. Note: $b_0 = 1$, $\sigma = 0.1$, $r = 0.02$ ($\beta = 1/1.02$), $\theta = 0.5$, $\tau = 1$.

fiscal rules $\{\alpha^{opt}(b_0), \delta^{opt}(b_0)\}$ that satisfies Definition 1. In the numerical example depicted in Figure 5, $\alpha^*(b_0,\delta)$ is an increasing function of δ, while $\delta^*(b_0,\alpha)$ is an increasing function of α. This finding highlights that a looser expenditure rule is optimal under a tighter deviation rule, whereas a tighter deviation rule is optimal under a looser expenditure rule. Thus, it represents a substitutable relationship between expenditure and deviation rules.

Figure 5 demonstrates that the optimal pair of fiscal rules at the interior point is attained, as indicated by Proposition 1. Notably, the optimal deviation rule satisfies $\delta^{opt}(b_0) < 1$. This implies that a rule prohibiting any deviation is not optimal. If no deviations are permitted, the incumbent party, which has many supporters, will be obliged to adhere to the expenditure rules. Thus, a rule that causes welfare loss for most members of a society cannot be deemed optimal.

5 Comparative Statics

We have analyzed the optimal fiscal rule considering the initial public debt balance b_0 and degree of political conflict to public expenditure among voters denoted as θ as predetermined. These parameters signify variations in each country's fiscal environment substantially influence the country's optimal fiscal rule. Here, we investigate the impact of the initial public debt balance b_0 and political conflict θ on the formulation of the optimal fiscal rules through comparative statics analysis.

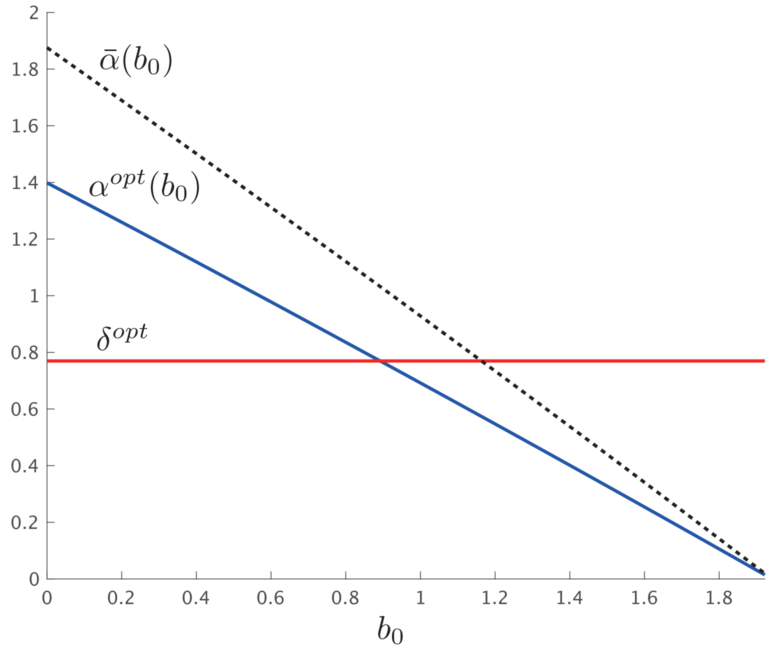

Figure 6: Illustration of $\alpha^{opt}(b_0)$ and δ^{opt} with b_0 on the horizontal axis. For other parameters, $\sigma = 0.1$, $r = 0.02$ ($\beta = 1/1.02$), $\theta = 0.5$ and $\tau = 1$.

5.1 Effects of Initial Public Debt Balance

We examine the impact of the initial public debt balance b_0 on the optimal combination of fiscal rules, $\alpha^{opt}(b_0)$ and $\delta^{opt}(b_0)$. The results of the comparative statics analysis are summarized through the following proposition.

Proposition 2 *The optimal deviation rule δ^{opt} remains independent of b_0, whereas the optimal expenditure rule $\alpha^{opt}(b_0)$ is decreasing in b_0.*

Proof. See Appendix E.

Figure 6 illustrates the relationship between b_0 and $\{\alpha^{opt}(b_0), \delta^{opt}\}$ as presented in Proposition 2, supported by numerical examples. The mechanism underlying Proposition 2's result is as follows: As shown in equations (17)–(19) and (27)–(29), the impact of the initial public debt balance b_0 on social welfare is confined to the provision of public goods. The incumbent party determines the supply of public goods based on whether the dictator or the rule state materializes. Consequently, the deviation rule δ does not influence the supply of public goods. That is, the deviation rule cannot regulate public goods provision based on the initial public debt balance, thus rendering it independent of the latter. Regarding the expenditure rule, $\alpha^{opt}(b_0)$ shows a decreasing trend for b_0, indicating that a more restrictive expenditure rule is optimal for higher initial public debt levels. This trend arises because as b_0 increases, stronger constraints on excessive public expenditure are necessary to meet the terminal condition $b_2 = 0$. Therefore, a higher initial public debt balance requires an optimal expenditure rule with tighter restrictions.[2]

By Proposition 2, the optimal expenditure rule $\alpha^{opt}(b_0)$ is contingent upon the initial public debt balance (b_0), whereas the optimal deviation rule δ^{opt} establishes its optimal level independently of the initial public debt balance. This implies that policymakers should adjust the optimal expenditure rule in response to current fiscal conditions. Once the optimal level of the deviation rule is determined, it ought to be maintained irrespective of the fiscal situation. This result implies that, as observed in Germany, it is socially beneficial to establish a mechanism where the expenditure rule is flexibly adjusted by law, while the deviation rule is constitutionally administered over the long term. This implication derived from the present analysis represents one of the significant conclusions, addressing a dimension not explored in previous studies.

5.2 Effects of Differences in Preferences

The parameter θ represents the disparity in preferences between the incumbent and opposition parties. A smaller value of θ indicates a more substantial difference in preferences, highlighting increased conflict between the incumbent and opposition parties. Since θ functions as a parameter characterizing the shape of the voters' (parties') utility function, its modification not only affects the equilibrium supply of public goods and issuance of public debt but also shapes social welfare through utility. This impact complicates the derivation of the closed-form solution. To address this complexity, we turn to a numerical example to analyze how a change in θ influences the optimal fiscal rules.

Figure 7 illustrates a numerical example depicting the relationship between θ and $\{\alpha^{opt}(b_0), \delta^{opt}\}$. As θ decreases, $\alpha^{opt}(b_0)$ increases and δ^{opt} decreases. That is, stronger conflict between parties corresponds to more relaxed optimal expenditure and deviation rules. The underlying mechanism is as follows: When θ decreases and party conflict intensifies, the change in incumbent party supporters' preferences exerts a stronger influence on social welfare than that in opposition party supporters' preferences. As θ decreases, the optimal fiscal rule that maximizes social welfare becomes more oriented towards the incumbent party. Drawing on the insights from Lemma 1, the heightened conflict between political parties increases public expenditure in the dictator state, where the expenditure rule is irrelevant. Consequently, a smaller θ leads to a more relaxed optimal expenditure rule. Moreover, an optimal looser deviation rule is preferred, because a stricter deviation rule adversely affects the incumbent party supporters' welfare.

6 Conclusion

This study theoretically examines the optimal deviation rule's characteristics based on the expenditure rule, assuming that the latter imposes a public expenditure ceiling. We focus on scenarios where deviations from the expenditure rule are permissible if the incumbent party holds more than δ share of seats in parliament and investigate the determination of the optimal

[2]This implication might not change even if an infinite-period model is considered; even in an infinite-period model, the terminal condition that the outstanding public debt must not exceed the natural debt limit must be satisfied.

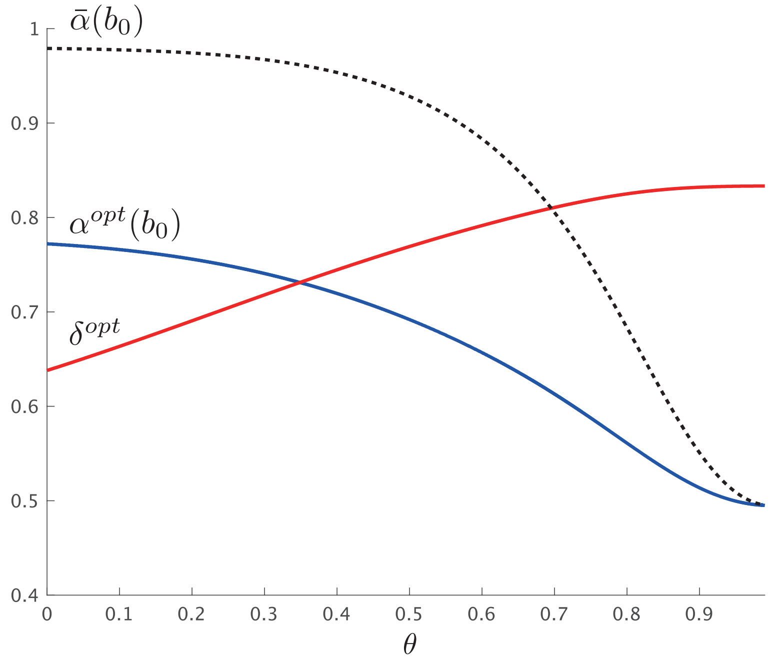

Figure 7: Illustration of $\alpha^{opt}(b_0)$ and δ^{opt} with θ on the horizontal axis. For other parameters, $\sigma = 0.1$, $r = 0.02$ ($\beta = 1/1.02$), $b_0 = 1$, and $\tau = 1$.

δ level that maximizes social welfare. The primary findings are as follows: First, a fiscal rule that disallows any deviation is considered socially undesirable. When skewed voter selection favors the incumbent party with a significant seat majority, temporarily relaxing the expenditure rule may prove beneficial.

Second, the optimal deviation rule remains independent of outstanding public debt. This is because public debt affects social welfare only through public expenditure levels. While the deviation rule dictates adherence to the expenditure rule, public expenditure is regulated by the expenditure rule. Thus, public debt influences the optimal expenditure rule but not the optimal deviation rule. This underscores the necessity of adjusting the expenditure rule in line with current fiscal conditions, while the optimal deviation rule remains constant regardless of the fiscal circumstances once established.

Third, greater disparities in preferences for public goods among voters (or between parties) favor a looser deviation rule. As the incumbent party's supporters outnumber those of the opposition, political conflicts tilt the optimal deviation rule towards the incumbent party's favor, warranting a looser deviance rule.

Several issues warrant further investigation. First, while we examine the optimal deviation rule for the expenditure rule, similar analyses can explore other fiscal rule types. Notably, studying the optimal deviation rule characteristics for rules governing the public debt ceiling in EU countries can be insightful. Second, while we base deviations on the incumbent party's number of seats, in practice, even with a significant seat majority, deviation often requires opposition party agreement. Addressing this by incorporating negotiations between the ruling and opposition parties into the model can be beneficial. Lastly, although we employ a two-period model for analytical tractability, extending it to an infinite-horizon version can facilitate analyzing the relationship between the deviation rule and public debt balance dynamics.

References

[1] Alesina, A., and Tabellini, G., 1990. A positive theory of fiscal deficits and government debt. *Review of Economic Studies*, Vol. 57, No. 3, pp. 403–414.

[2] Arawatari, R., and Ono, T., 2021. Public debt rule breaking by time-inconsistent voters. *European Journal of Political Economy*, https://doi.org/10.1016/j.ejpoleco.2021.102010

[3] Arawatari, R., and Ono, T., 2022. International coordination of debt rules with time-inconsistent voters. *Journal of Public Economic Theory*, Vol. 25, No. 1, pp. 29–60.

[4] Bisin, A., Lizzeri, A., and Yariv, L., 2015. Government policy with time inconsistent voters. *American Economic Review*, Vol. 105, No. 6, pp. 1711–1737.

[5] Coate, S., and Milton, R.T., 2019. Optimal fiscal limits with overrides. *Journal of Public Economics* Vol. 174, pp. 76–92.

[6] Davoodi, H. R., Elger, P., Fotiou, A., Garcia-Macia, D., Han, X., Lagerborg, A., Lam, W. R., and Medas, P., 2022. Fiscal rules and fiscal councils: recent trends and performance during the pandemic. IMF Working Paper No.22/11, International Monetary Fund, Washington, D.C.

[7] Dovis, A., and Kirpalani, R., 2020. Fiscal rules, bailouts, and reputation in federal governments. *American Economic Review*, Vol. 110, No. 3, pp. 860–888.

[8] Eyraud, L., Debrun, M., X., Hodge, A., Lledo, V. D., and Pattillo, C., A., 2018. Second-generation fiscal rules: balancing simplicity, flexibility, and enforceability. International Monetary Fund Staff Discussion Note 18-04.

[9] Halac, M., and Yared, P., 2022. Fiscal rules and discretion under limited enforcement. *Econometrica*, Vol. 90, No. 5, pp. 2093–2127.

[10] Piguillem, F., and Riboni, A., 2021. Fiscal rules as bargaining chips. *Review of Economic Studies*, Vol. 88, No. 5, pp. 2439–2478.

[11] Persson, T., and Svensson, L. E. O., 1989. Why a stubborn conservative would run a deficit: policy with time-inconsistent preferences. *Quarterly Journal of Economics*, Vol. 104, No. 2, pp. 325–345.

Appendix

A Proof of Lemma 1

It is sufficient to show that $\left(g_{I,1}^{d*}(b_0) + g_{O,1}^{d*}(b_0)\right)$ is a decreasing function of θ. From equations(18) and (19), government expenditure in the dictator state is:

$$g_{I,1}^{d*}(b_0) + g_{0,1}^{d*}(b_0) = \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right]}{(1+r) + \left(\phi/(1+\theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}}. \tag{A.1}$$

Here, considering equation (16) and $\sigma \in (0,1), \theta \in [0,1)$, we obtain:

$$\begin{aligned}
\frac{\partial}{\partial \theta}\left(\frac{\phi}{(1+\theta^{\frac{1}{\sigma}}}\right) &= \frac{1 - \frac{1-\sigma}{\sigma} \cdot \theta^{\frac{1}{\sigma}} + \frac{1-\sigma}{\sigma} \cdot \theta^{\frac{1}{\sigma}-2} - \theta^{\frac{2}{\sigma}-2}}{2\left(1+\theta^{\frac{1}{\sigma}}\right)^2} \\
&= \frac{1 + \frac{1-\sigma}{\sigma} \cdot \left(\theta^{\frac{1}{\sigma}-2} - \theta^{\frac{1}{\sigma}}\right) - \theta^{\frac{2}{\sigma}-2}}{2\left(1+\theta^{\frac{1}{\sigma}}\right)^2} \\
&= \frac{1 - \theta^{\frac{2(1-\sigma)}{\sigma}} + \frac{1-\sigma}{\sigma} \cdot \theta^{\frac{1}{\sigma}}\left(\frac{1}{\theta^2} - 1\right)}{2\left(1+\theta^{\frac{1}{\sigma}}\right)^2} \\
&> 0.
\end{aligned}$$

Therefore, $\left(g_{I,1}^{d*}(b_0) + g_{O,1}^{d*}(b_0)\right)$ is a decreasing function of θ.

Q.E.D.

B Proof of Lemma 2

Suppose that $\delta > 1/2$ is given. From Assumption 2 and equation (34), we have:

$$\begin{aligned}
\frac{\partial W(b_0,\delta,\alpha)}{\partial \alpha} &= \left[\frac{4\delta^2 - 1}{4} \cdot \left(1+\theta^{\frac{1}{\sigma}}\right) + \frac{-4\delta^2 + 8\delta - 3}{4} \cdot \left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right)\right] \cdot \alpha^{-\sigma} \cdot \left(\frac{\tau - rb_0}{1+\theta^{\frac{1}{\sigma}}}\right)^{1-\sigma} \\
&\quad + \left(\frac{4\delta^2 - 1}{4} + \frac{-4\delta^2 + 8\delta - 3}{4}\right) \cdot \beta\phi \cdot \left\{\frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right\}^{-\sigma} \\
&\quad \times \frac{-(1+r)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \\
&= (2\delta - 1) \cdot \frac{(2\delta+1)\left(1+\theta^{\frac{1}{\sigma}}\right) + (-2\delta+3)\left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right)}{4} \cdot \alpha^{-\sigma} \cdot \left(\frac{\tau - rb_0}{1+\theta^{\frac{1}{\sigma}}}\right)^{1-\sigma} \\
&\quad + (2\delta - 1)(-1) \cdot \phi \cdot \left\{\frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right\}^{-\sigma} \cdot \frac{\tau - rb_0}{1+\theta^{\frac{1}{\sigma}}} \\
&= (2\delta - 1) \cdot \left\{\frac{(2\delta - 1)\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 2\left(1+\theta^{\frac{1}{\sigma}} + \theta + \theta^{\frac{1-\sigma}{\sigma}}\right)}{4} \cdot \alpha^{-\sigma} \cdot \left(\frac{\tau - rb_0}{1+\theta^{\frac{1}{\sigma}}}\right)^{1-\sigma}\right.
\end{aligned}$$

$$
\begin{aligned}
&\quad -\phi \cdot \left\{ \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}^{-\sigma} \cdot \frac{\tau - rb_0}{1+\theta^{\frac{1}{\sigma}}} \Bigg\} \\
&= \frac{(2\delta-1)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \cdot \left\{ \frac{(2\delta-1)\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi}{4} \cdot \left[\frac{\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{-\sigma} \right. \\
&\quad \left. -\phi \cdot \left\{ \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}^{-\sigma} \right\}. \qquad \text{(B.1)}
\end{aligned}
$$

We verify that the second-order condition is satisfied.

$$
\begin{aligned}
\frac{\partial^2 W(b_0, \delta, \alpha)}{\partial \alpha^2} &= \frac{(2\delta-1)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \cdot (-\sigma) \cdot \left\{ \frac{(2\delta-1)\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi}{4} \cdot \alpha^{-\sigma-1} \cdot \left(\frac{\tau - rb_0}{1+\theta^{\frac{1}{\sigma}}}\right)^{-\sigma} \right. \\
&\quad \left. +\phi \cdot \left\{ \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}^{-\sigma-1} \cdot \frac{(1+r)(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}.
\end{aligned}
$$

From (12), $\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) > 0$ is satisfied; and according to Assumption 1, $(\tau - rb_0 > 0)$ is also true. Furthermore, Assumption 1 implies $b_0 < b^{NDL}$, and Assumption 3 ensures $\alpha < \bar{\alpha}(b_0)$. Therefore, the following expression holds:

$$
\begin{aligned}
\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)(\tau - rb_0)\alpha &> \tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)(\tau - rb_0)\bar{\alpha}(b_0) \\
&= \{\tau + (1+r)\left[\tau - (1+r)b_0\right]\} \cdot \frac{\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}}{(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}} \\
&> \left\{\tau + (1+r)\left[\tau - (1+r)b^{NDL}\right]\right\} \cdot \frac{\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}}{(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}} \\
&= 0. \qquad \text{(B.2)}
\end{aligned}
$$

Given that we are currently examining the case where $\delta > 1/2$, $(\partial^2 W(b_0; \delta, \alpha))/(\partial \alpha^2) < 0$ is affirmed based on the aforementioned formula. Consequently, the second-order condition is satisfied.

Next, we explore the first-order condition of optimization. We represent the optimal level of the expenditure rule given δ as α^*. Note that we are addressing the scenario where $\delta > 1/2$, and utilizing equation (B.1), we derive the following:

$$
\begin{aligned}
&\frac{\partial W(b_0, \delta, \alpha^*)}{\partial \alpha} = 0 \\
\Leftrightarrow\ &\frac{(2\delta-1)\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi}{4} \cdot \left[\frac{\alpha^*(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}\right]^{-\sigma}
\end{aligned}
$$

$$= \phi \cdot \left\{ \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha^*(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}} \right\}^{-\sigma}$$

$$\Leftrightarrow \left[\frac{(2\delta - 1)\left(1 + \theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi}{4\phi} \right]^{\frac{1}{\sigma}} \cdot \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha^*(\tau - rb_0)}{1+\theta^{\frac{1}{\sigma}}}$$

$$= \alpha^* \cdot \left[\frac{\tau - rb_0}{1+\theta^{\frac{1}{\sigma}}} \right].$$

We define $\Gamma(\delta)$ as follows:

$$\Gamma(\delta) \equiv \left[\frac{(2\delta - 1)\left(1 + \theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi}{4\phi} \right]^{\frac{1}{\sigma}} \geq 1. \tag{B.3}$$

We then obtain the following expression. [3]

$$\alpha^*(b_0, \delta) = \frac{\tau + (1+r) \cdot \left[\tau - (1+r)b_0\right]}{\tau - rb_0} \cdot \frac{\Gamma(\delta)}{1 + (1+r)\Gamma(\delta)}. \tag{B.4}$$

From Assumption 1, the following two conditions hold:

$$\tau + (1+r) \cdot \left[\tau - (1+r)b_0\right] > \tau + (1+r) \cdot \left[\tau - (1+r)b^{NDL}\right] = 0, \tag{B.5}$$

$$\tau - rb_0 > \tau - rb^{NDL} = \frac{1}{(1+r)^2} > 0, \tag{B.6}$$

These two conditions ensure that $\alpha^*(b_0, \delta) > 0$. Additionally, based on Assumption 3 and the conditions in (B.5) and (B.6), the following conditions are fulfilled:

$$\begin{aligned}
\alpha^*(b_0, \delta) < \bar{\alpha}(b_0) \;\; &\Leftrightarrow \;\; \Gamma(\delta) \cdot \left(\phi / \left(1 + \theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}} < 1 \\
&\Leftrightarrow \;\; (2\delta - 1)\left(1 + \theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) + 4\phi < 4 \cdot \left(1 + \theta^{\frac{1}{\sigma}}\right) \\
&\Leftrightarrow \;\; \delta < \frac{3}{2}.
\end{aligned}$$

From the above, for any $\delta \in (1/2, 1]$, $\alpha^*(b_0, \delta) < \bar{\alpha}(b_0)$ must hold.

Q.E.D.

C Proof of Lemma 3

Suppose $\alpha < \bar{\alpha}(b_0)$ holds. From equation (34), the first-order condition is:

$$\frac{\partial W(b_0, \delta^*, \alpha)}{\partial \delta} = -2 \cdot \left[\delta^* V^d_{I,1}(b_0) + (1-\delta^*)V^d_{O,1}(b_0) - \delta^* V^r_{I,1}(b_0, \alpha) - (1-\delta^*)V^r_{O,1}(b_0, \alpha)\right]. \tag{C.1}$$

[3] Note that we are considering the range $\delta > 1/2$ and that $\Gamma(\delta) \geq 1$ always holds from (12).

The second derivative of $W(b_0, \delta, \alpha)$ with respect to δ is:

$$\frac{\partial^2 W(b_0, \delta, \alpha)}{\partial \delta^2} = -2 \cdot \left[V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha) + V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0) \right]. \tag{C.2}$$

From Assumption 3, $\alpha < \bar{\alpha}(b_0)$ holds; and from (12), $1 + \theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}} > 0$ holds. Given these two conditions with equations (21), (22), (30), and (31), we obtain:

$$\begin{aligned}
&V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha) + V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0) \\
&\quad = \frac{1}{1-\sigma} \cdot \left(1 + \theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}} \right]^{1-\sigma} \\
&\qquad - \frac{1}{1-\sigma} \cdot \left(1 + \theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left[\frac{\alpha(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}} \right]^{1-\sigma} \\
&\quad = \frac{\left(1 + \theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right)(\tau - rb_0)^{1-\sigma}}{(1-\sigma)\left(1 + \theta^{\frac{1}{\sigma}}\right)^{1-\sigma}} \cdot \left[\bar{\alpha}(b_0)^{1-\sigma} - \alpha^{1-\sigma} \right] \\
&\quad > 0.
\end{aligned} \tag{C.3}$$

Expressions in (C.2) and (C.3) ensure that the second derivative in (C.2) is negative, implying that the second-order condition holds. Thus, from equation (C.1), the optimal δ is:

$$\delta^*(b_0, \alpha) = \frac{V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0)}{[V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha)] + [V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0)]}. \tag{C.4}$$

Next, we examine the condition for $\delta^*(b_0, \alpha) < 1$ to be met. Considering condition in (C.3), we derive the following equation:

$$\delta^*(b_0, \alpha) < 1 \Leftrightarrow V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha) > 0. \tag{C.5}$$

Considering the expression in (32), $V^r_{I,1}(b_0; \alpha)$ is monotonically increasing in α within the parameter range where Assumption 3 holds. Considering the definition of $\bar{\alpha}(b_0)$ as stipulated by Assumption 3, we get:

$$\begin{aligned}
V^r_{I,1}(b_0, \bar{\alpha}(b_0)) &= \frac{1}{1-\sigma} \cdot \left\{ \left(1 + \theta^{\frac{1}{\sigma}}\right) \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}} \right]^{1-\sigma} \right. \\
&\qquad \left. + \beta\phi \cdot \left\{ \frac{\bar{\alpha}(b_0)(\tau - rb_0)\left[(1+r) + \left(\phi/(1 + \theta^{\frac{1}{\sigma}})\right)^{\frac{1}{\sigma}}\right] - (1+r)\bar{\alpha}(b_0)(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}} \right\}^{1-\sigma} \right\} \\
&= \frac{1}{1-\sigma} \cdot \left[1 + \theta^{\frac{1}{\sigma}} + \beta(1 + \theta^{\frac{1}{\sigma}}) \left(\frac{\phi}{1 + \theta^{\frac{1}{\sigma}}} \right)^{\frac{1}{\sigma}} \right] \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}} \right]^{1-\sigma} \\
&= V^d_{I,1}(b_0).
\end{aligned} \tag{C.6}$$

Hence, for any $\alpha < \bar{\alpha}(b_0)$, it holds that $0 < V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha)$. Put differently, as long as Assumption 3 is met, the condition $\delta^*(b_0, \alpha) < 1$ remains satisfied.

Finally, expression in (C.3) leads to the following condition.

$$\delta^*(b_0, \alpha) > \frac{1}{2} \Leftrightarrow V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha) < V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0).$$

Q.E.D.

D Proof of Proposition 1

From equations (35) and (37), incorporating $\bar{\alpha}(b_0)$ from Assumption 3, the optimal pair of fiscal rules, denoted as $\alpha^{opt}, \delta^{opt}$, is determined by solving the following simultaneous equations.

$$\delta = \frac{V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0)}{[V^d_{I,1}(b_0) - V^r_{I,1}(b_0, \alpha)] + [V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0)]}, \tag{D.1}$$

$$\alpha = \bar{\alpha}(b_0) \cdot \left[(1+r) + \left(\phi / \left(1 + \theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right] \cdot \frac{\Gamma(\delta)}{1 + (1+r)\Gamma(\delta)}. \tag{D.2}$$

We reformulate both the numerator and denominator of equation (D.1). Initially, by considering (22), (31), and (D.2), the numerator is reformulated as follows:

$$\begin{aligned}
&V^r_{O,1}(b_0, \alpha) - V^d_{O,1}(b_0) \\
&= \frac{1}{1-\sigma} \cdot \left\{ \left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left[\frac{\alpha(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}}\right]^{1-\sigma} \right. \\
&\quad \left. + \beta\phi \cdot \left\{ \frac{\tau + (1+r)\left[\tau - (1+r)b_0\right] - (1+r)\alpha(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}} \right\}^{1-\sigma} \right\} \\
&\quad - \frac{1}{1-\sigma} \cdot \left[\theta + \theta^{\frac{1-\sigma}{\sigma}} + \beta(1 + \theta^{\frac{1}{\sigma}}) \left(\frac{\phi}{1 + \theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}}\right] \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}}\right]^{1-\sigma} \\
&= \frac{1}{1-\sigma} \cdot \left\{ \left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left[\bar{\alpha}(b_0) \cdot \left[(1+r) + \left(\phi / \left(1 + \theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right] \cdot \frac{\Gamma(\delta)}{1 + (1+r)\Gamma(\delta)} \cdot \frac{\tau - rb_0}{1 + \theta^{\frac{1}{\sigma}}}\right]^{1-\sigma} \right. \\
&\quad \left. + \beta\phi \cdot \left\{ \frac{\bar{\alpha}(b_0) \cdot (\tau - rb_0) \cdot \left[(1+r) + \left(\phi / \left(1 + \theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]}{(1 + \theta^{\frac{1}{\sigma}})\left[1 + (1+r)\Gamma(\delta)\right]} \right\}^{1-\sigma} \right\} \\
&\quad - \frac{1}{1-\sigma} \cdot \left[\theta + \theta^{\frac{1-\sigma}{\sigma}} + \beta(1 + \theta^{\frac{1}{\sigma}}) \left(\frac{\phi}{1 + \theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}}\right] \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{1 + \theta^{\frac{1}{\sigma}}}\right]^{1-\sigma} \\
&= \frac{1}{1-\sigma} \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{(1 + \theta^{\frac{1}{\sigma}})\left[1 + (1+r)\Gamma(\delta)\right]}\right]^{1-\sigma} \\
&\quad \times \left\{ \left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left[(1+r) + \left(\phi / \left(1 + \theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \cdot \Gamma(\delta)^{1-\sigma} \right.
\end{aligned}$$

$$
+ \beta\phi \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}
$$

$$
- [1+(1+r)\Gamma(\delta)]^{1-\sigma} \cdot \left[\theta + \theta^{\frac{1-\sigma}{\sigma}} + \beta(1+\theta^{\frac{1}{\sigma}})\left(\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right)^{\frac{1}{\sigma}}\right]\Bigg\}. \qquad \text{(D.3)}
$$

Next, from (C.3) and (D.2), the denominator can be rewritten as follows:

$$
\begin{aligned}
&[V_{I,1}^d(b_0) - V_{I,1}^r(b_0,\alpha)] + [V_{O,1}^r(b_0,\alpha) - V_{O,1}^d(b_0)] \\
&= \frac{\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right)(\tau - rb_0)^{1-\sigma}}{(1-\sigma)\left(1+\theta^{\frac{1}{\sigma}}\right)^{1-\sigma}} \cdot \left[\bar{\alpha}(b_0)^{1-\sigma} - (\alpha)^{1-\sigma}\right] \\
&= \frac{\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right)(\tau - rb_0)^{1-\sigma}}{(1-\sigma)\left(1+\theta^{\frac{1}{\sigma}}\right)^{1-\sigma}} \\
&\quad \times \left[\bar{\alpha}(b_0)^{1-\sigma} - \left\{\bar{\alpha}(b_0) \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right] \cdot \frac{\Gamma(\delta)}{1+(1+r)\Gamma(\delta)}\right\}^{1-\sigma}\right] \\
&= \frac{1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}}{1-\sigma} \cdot \left[\frac{\bar{\alpha}(b_0)(\tau - rb_0)}{(1+\theta^{\frac{1}{\sigma}})\left[1+(1+r)\Gamma(\delta)\right]}\right]^{1-\sigma} \\
&\quad \times \left\{[1+(1+r)\Gamma(\delta)]^{1-\sigma} - \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \cdot \Gamma(\delta)^{1-\sigma}\right\}. \qquad \text{(D.4)}
\end{aligned}
$$

Finally, upon substituting equations (D.3) and (D.4) into equation (D.1), we derive the condition necessary for determining the optimal value of δ.

$$
\delta = \frac{\left[\begin{aligned}&\left(\theta + \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \cdot \Gamma(\delta)^{1-\sigma} \\ &\qquad + \beta\phi \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \\ &\qquad\qquad - [1+(1+r)\Gamma(\delta)]^{1-\sigma} \cdot \left[\theta + \theta^{\frac{1-\sigma}{\sigma}} + \beta(1+\theta^{\frac{1}{\sigma}})\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]\end{aligned}\right]}{\left(1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}\right) \cdot \left\{[1+(1+r)\Gamma(\delta)]^{1-\sigma} - \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \cdot \Gamma(\delta)^{1-\sigma}\right\}}
$$

$$
\equiv RHS(\delta). \qquad \text{(D.5)}
$$

Then, the optimal deviation rule δ^{opt} is defined as the value of δ that satisfies $\delta = \text{RHS}(\delta)$.

Next, we establish the condition for the existence of a $\delta \in (1/2, 1)$ that fulfills equation (D.5). First, we demonstrate the positivity of the denominator in equation (D.5). Using the condition in (12), the requisite condition for the denominator to be positive is:

$$
[1+(1+r)\Gamma(\delta)]^{1-\sigma} - \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \cdot \Gamma(\delta)^{1-\sigma} > 0
$$

$$\Leftrightarrow\ 1+(1+r)\Gamma(\delta) > \left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]\cdot\Gamma(\delta)$$

$$\Leftrightarrow\ 1 > \left[\frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\cdot\frac{2(1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}})+(2\delta-1)(1+\theta^{\frac{1}{\sigma}}-\theta-\theta^{\frac{1-\sigma}{\sigma}})}{2(1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}})}\right]^{\frac{1}{\sigma}}$$

$$\Leftrightarrow\ 1 > \frac{2(1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}})+(2\delta-1)(1+\theta^{\frac{1}{\sigma}}-\theta-\theta^{\frac{1-\sigma}{\sigma}})}{4(1+\theta^{\frac{1}{\sigma}})}$$

$$\Leftrightarrow\ (3-2\delta)(1+\theta^{\frac{1}{\sigma}})+(2\delta-1)(\theta+\theta^{\frac{1-\sigma}{\sigma}})>0.$$

As $\delta\in[1/2,1]$, the inequality in the above equation must be satisfied. Consequently, the denominator in equation (D.5) is always positive.

If the following two conditions are satisfied, then there exists at least one $\delta\in(1/2,1)$ such that equation (D.5) holds.

$$RHS\left(\frac{1}{2}\right)>\frac{1}{2},\quad RHS(1)<1.$$

Initially, we demonstrate the consistent satisfaction of $RHS\left(\frac{1}{2}\right)>\frac{1}{2}$. To establish this, note that $\Gamma\left(\frac{1}{2}\right)=1$ (as indicated in equation (36)) and that $\beta(1+r)=1$ (according to Assumption 2). Moreover, considering the definition of ϕ provided in equation (16), we derive the following conditions from equation (D.5).

$$\begin{aligned}
&RHS\left(\frac{1}{2}\right)>\frac{1}{2}\\
&\Leftrightarrow\ \left[2\left(\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)+2\beta\phi+\left(1+\theta^{\frac{1}{\sigma}}-\theta-\theta^{\frac{1-\sigma}{\sigma}}\right)\right]\cdot\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}\\
&\qquad>(2+r)^{1-\sigma}\cdot\left[\left(1+\theta^{\frac{1}{\sigma}}-\theta-\theta^{\frac{1-\sigma}{\sigma}}\right)+2\left(\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)+2\beta\left(1+\theta^{\frac{1}{\sigma}}\right)\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]\\
&\Leftrightarrow\ \left[\left(1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)+2\beta\phi\right]\cdot\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}\\
&\qquad>(2+r)^{1-\sigma}\cdot\left[\left(1+\theta^{\frac{1}{\sigma}}+\theta+\theta^{\frac{1-\sigma}{\sigma}}\right)+2\beta\left(1+\theta^{\frac{1}{\sigma}}\right)\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]\\
&\Leftrightarrow\ 2\phi(1+\beta)\cdot\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}\\
&\qquad>2\phi\beta(2+r)^{1-\sigma}\cdot\left[(1+r)+\frac{1+\theta^{\frac{1}{\sigma}}}{\phi}\cdot\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)^{\frac{1}{\sigma}}\right)\right]\\
&\Leftrightarrow\ (2+r)^{\sigma}\cdot\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma}-\left[(1+r)+\left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}-1}\right]>0. \quad \text{(D.6)}
\end{aligned}$$

We define x as $x\equiv\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)$ and denote the left-hand side of the above equation as $W(x)$. The expression in (D.6) can be reformulated as follows.

$$RHS\left(\frac{1}{2}\right)>\frac{1}{2}\ \Leftrightarrow\ W(x)=(2+r)^{\sigma}\cdot\left[(1+r)+x^{\frac{1}{\sigma}}\right]^{1-\sigma}-\left[(1+r)+x^{\frac{1}{\sigma}-1}\right]>0. \qquad \text{(D.7)}$$

The differentiation of $W(x)$ with respect to x yields:

$$\begin{aligned}\frac{\partial W(x)}{\partial x} &= (1-\sigma)(2+r)^{\sigma} \cdot \left[(1+r) + x^{\frac{1}{\sigma}}\right]^{-\sigma} \cdot \frac{1}{\sigma} \cdot x^{\frac{1}{\sigma}-1} - \frac{1-\sigma}{\sigma} \cdot x^{\frac{1}{\sigma}-2} \\ &= \frac{1-\sigma}{\sigma} \cdot x^{\frac{1}{\sigma}-2} \cdot \left\{ \left[\frac{2+r}{(1+r) + x^{\frac{1}{\sigma}}}\right]^{\sigma} \cdot x - 1 \right\}.\end{aligned}$$

The condition for $\partial W(x)/\partial x$ to be negative is as follows.

$$\begin{aligned}\frac{\partial W(x)}{\partial x} < 0 \ &\Leftrightarrow \ \left[\frac{2+r}{(1+r) + x^{\frac{1}{\sigma}}}\right]^{\sigma} \cdot x < 1 \\ &\Leftrightarrow \ \frac{2+r}{(1+r) + x^{\frac{1}{\sigma}}} \cdot x^{\frac{1}{\sigma}} < 1 \\ &\Leftrightarrow \ (1+r)x^{\frac{1}{\sigma}} < (1+r) \\ &\Leftrightarrow \ x < 1. \end{aligned} \tag{D.8}$$

From (12) and (16), we have

$$x \equiv \frac{\phi}{1+\theta^{\frac{1}{\sigma}}} = \frac{1+\theta^{\frac{1}{\sigma}} + \theta + \theta^{\frac{1-\sigma}{\sigma}}}{2\left(1+\theta^{\frac{1}{\sigma}}\right)} = \frac{1}{2} + \frac{\theta + \theta^{\frac{1-\sigma}{\sigma}}}{2\left(1+\theta^{\frac{1}{\sigma}}\right)} < \frac{1}{2} + \frac{1}{2} = 1, \tag{D.9}$$

By demonstrating that the condition in (D.8) consistently holds, we establish that $W(x)$ is a monotonically decreasing function of x. Given that $x < 1$, we derive the following result:

$$\begin{aligned}W(x) > W(1) &= (2+r)^{\sigma} \cdot [(1+r)+1]^{1-\sigma} - [(1+r)+1] \\ &= (2+r) - (2+r) \\ &= 0. \end{aligned} \tag{D.10}$$

As $W(x) > 0$ consistently holds, (D.7) guarantees that $RHS\left(\frac{1}{2}\right) > \frac{1}{2}$ is always satisfied.

Next, we derive the condition for $RHS\,(1) < 1$ to be satisfied.

$$\begin{aligned}&RHS\,(1) < 1 \\ &\Leftrightarrow \ \left[\left(1+\theta^{\frac{1}{\sigma}}\right)\Gamma(1)^{1-\sigma} + \beta\phi\right] \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \\ &\qquad < \left[\left(1+\theta^{\frac{1}{\sigma}}\right) + \beta\left(1+\theta^{\frac{1}{\sigma}}\right) \cdot \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right] \cdot [1 + (1+r)\Gamma(1)]^{1-\sigma} \\ &\Leftrightarrow \ \beta\left(1+\theta^{\frac{1}{\sigma}}\right)\left[(1+r)\Gamma(1)^{1-\sigma} + \frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right] \cdot \left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \\ &\qquad < \beta\left(1+\theta^{\frac{1}{\sigma}}\right)\left[(1+r) + \left(\phi/\left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right] \cdot [1 + (1+r)\Gamma(1)]^{1-\sigma}\end{aligned}$$

$$\Leftrightarrow \; 0 < \left[(1+r) + \left(\phi / \left(1+\theta^{\frac{1}{\sigma}}\right)\right)^{\frac{1}{\sigma}}\right]^{\sigma} \cdot [1+(1+r)\Gamma(1)]^{1-\sigma} - \left[(1+r)\Gamma(1)^{1-\sigma} + \frac{\phi}{1+\theta^{\frac{1}{\sigma}}}\right]. \tag{D.11}$$

We denote the right-hand side of (D.11) as $Z(x)$. Utilizing the definition of $\Gamma(\delta)$ provided in Lemma 2, we derive the following.

$$\Gamma(1) = \left[1 + \frac{1+\theta^{\frac{1}{\sigma}} - \theta - \theta^{\frac{1-\sigma}{\sigma}}}{2\left(1+\theta^{\frac{1}{\sigma}} + \theta + \theta^{\frac{1-\sigma}{\sigma}}\right)}\right]^{\frac{1}{\sigma}} = \left(\frac{1}{2} + \frac{1}{2x}\right)^{\frac{1}{\sigma}}, \tag{D.12}$$

Using this expression, (D.11) is rewritten as follows:

$$\begin{aligned} RHS\,(1) < 1 \;\Leftrightarrow\; Z(x) = \left[(1+r) + x^{\frac{1}{\sigma}}\right]^{\sigma} \cdot \left[1 + (1+r)\left(\frac{1}{2} + \frac{1}{2x}\right)^{\frac{1}{\sigma}}\right]^{1-\sigma} \\ - (1+r)\left(\frac{1}{2} + \frac{1}{2x}\right)^{\frac{1-\sigma}{\sigma}} - x > 0. \end{aligned} \tag{D.13}$$

If the inequality in (D.13) is met, then the condition $RHS(1) < 1$ is also fulfilled, indicating the existence of at least one $\delta \in (1/2, 1)$ that satisfies the equality in (D.5).

Lastly, as per Lemma 2, if the condition $\delta^{opt} \in (1/2, 1)$ is met, then it follows that $\alpha^{opt}(b_0) = \alpha^*(b_0, \delta^{opt}) \in (0, \bar{\alpha}(b_0))$ is also satisfied.

Q.E.D.

E Proof of Prpopsition 2

We initially establish that δ^{opt} is independent of b_0. The optimal deviation rule, denoted as δ^{opt}, is the value of δ that satisfies equation (D.5). Importantly, equation (D.5) does not include b_0. Consequently, δ^{opt} is independent of b_0.

We then demonstrate that α^{opt} is a decreasing function of b_0. Notably, δ^{opt} is independent of b_0. Moreover, considering the expression in (35), we have:

$$\frac{\partial \alpha^{opt}}{\partial b_0} = \frac{\Gamma(\delta^{opt})}{1+(1+r)\Gamma(\delta^{opt})} \cdot \frac{\partial}{\partial b_0}\left(\frac{\tau + (1+r)\left[\tau - (1+r)b_0\right]}{\tau - rb_0}\right) \tag{E.1}$$

$$= \frac{\Gamma(\delta^{opt})}{1+(1+r)\Gamma(\delta^{opt})} \cdot \frac{-\tau}{(\tau - rb_0)^2} \tag{E.2}$$

$$< 0, \tag{E.3}$$

showing that α^{opt} is a decreasing function of b_0.

Q.E.D.