Delegation in Teams*

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Abstract

This paper considers delegation of a decision within a team of informed agents. It shows under what conditions formal decision-making authority should be delegated and identifies the individuals to whom such authority should be conferred. This determination hinges on a trade-off between 1) enhanced communication when the delegated agent's preferences are closely aligned with those of the other agents, and 2) the costs incurred from distorted decisions arising due to biased preferences of the delegated agent.

Keywords: Delegation, communication.

JEL Classification: D23, D82, L23

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1 Introduction

To invest in a project or idea in the most optimal manner, access to a wide array of dispersed information within an organization and across various individuals or functions is essential. For instance, to successfully design and launch a product, one would require comprehensive knowledge about marketing, technical specifications, consumer group compatibility, the market environment, and pricing, among other dimensions. Similarly, the process of evaluating and selecting optimal investment options necessitates not only financial expertise but also industry-specific knowledge and managerial acumen.

In this paper, we develop a model of communication and information flow within organizations. Our starting point is the assumption that different individuals within an organization possess diverse pieces of information, all pertinent to a specific decision. Therefore, an individual making a decision must collect all of this information to enhance the quality and precision of their decision-making. Specifically, we concentrate on situations where the uninformed principal must choose between retaining decision-making authority or delegating it to one of her informed agents, and we explore the circumstances under which delegation is optimal and, if so, to whom this authority should be delegated.

The primary driving force in our model is the misaligned preferences among different individuals. While they all agree on the project's overall direction, such as launching a product, they may differ on specific project details and funding size. Individuals possess decision-relevant information, for example, the optimal investment size. The disparity in preferences is evident in communication efficiency when the agent with decision-making authority gathers information from others, placing the question of who excels at information collection at the core of the analysis.

Our model facilitates the decomposition of organizational decision-making into information gathering and ultimate decision-making. The trade-off elucidated in our framework centers on the dynamics between these processes: the cost associated with delegating decision-making authority to an individual whose preferences may misalign with those of the principal, versus the advantage of leveraging superior information. In a two-agent case, where each agent possesses distinct yet equally significant pieces of relevant information, we demonstrate that if delegation is optimal it should be to the agent whose preference more closely align with that of the principal. Interestingly, this result may not hold when more agents have decision-relevant information. This is because delegating authority to a more biased agent may enhance communication with other agents, and the advantages of improved information resulting from such delegation outweigh the drawbacks of deviating from the decision-maker's preferred outcome.

Our model captures the relatively intuitive notion that the agent most proficient in communicating with others is one who has a central position in terms of bias. When the biases of agents are small, the risk of losing control is negligible, prompting the principal to delegate decision-making authority to such an agent. On the contrary, when the biases of agents are large, the risk of losing control becomes significant, making the principal prefer to delegate to an agent whose preferences are more aligned with hers. Regarding whether to delegate decision-making authority in the first place, we demonstrate that as the number of agents increases, the resulting loss of control becomes less pronounced, leading the principal to increasingly favor delegating decision-making to one of the agents.

As an extension, we also consider an alternative information structure wherein the principal groups agents and designates a subset of them as information collectors (group leaders) within their specific group of agents, while retaining the ultimate decision-making authority. The leaders subsequently relay their gathered information to the principal. We demonstrate that such a grouping is effective when each agent displays a significant bias and is partitioned into subgroups that exhibit substantial preference disparities with one another.

Our model is most closely related to Crawford and Sobel (1982), Dessein (2002), and Harris and Raviv (2005). In their seminal paper on cheap talk, Crawford and Sobel (1982) developed a model that analyzes the quantity of information that will be communicated by an informed agent, assuming that the principal makes the decision. Dessein (2002) builds upon this model to answer the question of whether it is optimal to delegate the decision to the agent rather than rely upon cheap talk à la Crawford and Sobel (1982). Harris and Raviv (2005) further extend the model in Dessein (2002) by incorporating the principal's relevant private information, demonstrating that the likelihood of delegation rises with the increasing significance of the agent's private information relative to that of the principal's. We extend these literature on communication and delegation to environments where decision-relevant information is dispersed across several agents (senders) and show that although the effects in the previous literature is remain present, new trade-offs also arise.

In examining the issue of identifying the optimal decision-maker in the realm of strategic communication and organizational decision-making, this study is related to Deimen (2024). Her model explores a two-divisional organizational structure, where each division holds distinct pieces of information. The information may pertain to either common or private interests, with one division possessing information about both interests. Our study, in contrast, assumes that there is only one decision to make and thus all agents have information solely of common interest,[1] concentrating on determining which agent is proficient at aggregating information among many agents.

Starting from Holmstrom (1977), the literature on delegation has shown that, in an

[1]In this regard, this study differs from Alonso, Dessein, and Maouschek (2008) and Rantakari (2008), which examine the tradeoff between adaptation and coordination within a multi-divisional organization.

environment with partially aligned preferences and no state-contingent payments, various forms of delegation of decision rights may improve upon communication with only soft information (referred to as cheap talk in the literature).[2] Our paper is closely related to the literature that studies how the allocation of authority among misaligned parties shapes communication and decisions in organizations (see, for instance, Alonso and Matouschek, 2008, and Chakraborty and Yilmaz, 2017). The closest among them to ours is Li and Suen (2004), who consider a model of delegation with a team of experts. In their model, delegated experts face a problem of strategic information aggregation through voting games, while in our model, the decision is delegated to a single expert who then plays cheap talk communication games with the other experts.

The last part of our analysis is close to Migrow (2021), who examines the optimal communication hierarchy, specifying the reporting relationships and their sequence. We differ in several ways. For instance, in his model, agents receive binary private signals, whereas in our model, signals are continuous. This leads us to derive a distinct rationale for the principal to implement two-layer information structures, thereby altering the relative magnitude of bias and the quantity of information in Crawford and Sobel's (1982) sender-receiver game. Additionally, related work by Hori (2006) employs a similar framework to ours, comparing the effectiveness of hierarchical versus horizontal communication, but without accounting for the possibility of delegation.

The remainder of the paper is organized as follows. The model and benchmark results are presented in Section 2. Results for the two-agent case is presented in Section 3. Section 4 extends the analysis to the three-agent case while general insights for many agents are presented in Section 5. Section 6 discusses grouping of agents. A brief conclusion is provided in Section 7.

2 The Model

Our model extends those of Dessein (2002) and Harris and Raviv (2005) to a multi-agent framework in which there are $n \geq 1$ agents and decision-relevant information is dispersed among these agents. Each agent i possesses a piece of information that we denote by θ_i. It is common knowledge θ_i is independently and uniformly distributed on the support $[-\frac{L_i}{2}, \frac{L_i}{2}]$.[3] Only agent i observes the realization of the random variable θ_i. The set of agents is denoted by $\mathcal{N}$.

The principal may give formal decision-making rights to one of the n agent who then

[2] See, for instance, Holmstrom (1984), Melumad and Shibano (1991), and Alonso and Matouschek (2008).

[3] Hence θ_i has variance of $\frac{L_i^2}{12}$.

makes a decision. The quality of the decision depends upon the decision $y \in R^+$ and the n pieces of random variables θ_i for $i \in \mathcal{N}$.

We assume that the preferences of all players are represented by quadratic loss functions. This implies that the principal's ex post payoff is $-\left(\sum_i^n \theta_i - y\right)^2$. This means that the principal's ideal decision is $y = \sum_i^n \theta_i$. Agent i's payoff is $-(\sum_j^n \theta_j + b_i - y)^2$ and his ideal decision is $y = \sum_j^n \theta_j + b_i$. Thus the misaligned preferences between the principal's payoff and agent i's payoff is modeled by b_i that captures the difference between the principal and agent i's preferred decision. This difference, b_i, is commonly referred to as agent i's bias in the literature. In fact, all players agree on the direction of the decision. Higher values of θ_i should be associated with higher decisions, but they disagree on the exact level of the decision. When $b_i > 0$ ($b_i < 0$) agent i would like a higher (lower) decision than the principal. We may interpret L_i as a measure of the quantity of (private) information possessed by agent i. If $L_i = 0$, agent i's information is irrelevant. Denoting by $\mathbf{b}$ and $\mathbf{L}$ the vectors of biases and the quantity of agents' information $(b_1, b_2, ..., b_n)$ and $(L_1, L_2, ..., L_n)$, respectively, the decision-making environment for the principal is represented by $\{\mathcal{N}, \mathbf{b}, \mathbf{L}\}$.

If the principal delegates decision-making authority to an agent, the designated agent plays a cheap-talk game with each of the other agents, serving as the information recipient. Conversely, if the principal retains decision-making authority, the principal engages in a cheap-talk game with each of the agent, wherein the principal is the information receiver. This case will sometimes be referred to as power retention (as opposed to delegation of power in the previous case). In this paper we focus on cheap talk equilibria where the principal's expected payoff is maximized, i.e., the equilibrium that yields the highest number of partitions (see next subsection for more details).

2.1 Benchmarks

We first provde a benchmark result of what would be the result of one-to-one communication between the principal and each of the agents in this setting. Following Crawford and Sobel (1982), the agent conveys an interval within which the true state of the world resides, and these intervals form non-overlapping partitions that encompass the entire state space. Informative communication is feasible (in the sense that the state space is divided into at least two intervals) when the sender's bias is sufficiently small.

Proposition 1 *(Crawford and Sobel, 1982): The equilibria of the communication game between a sender and a receiver whose difference in preference biases is b and the sender's information has support $[-\frac{L}{2}, \frac{L}{2}]$ are partition equilibria in which the following is true:*

1. The maximum partition number $N(b, L)$ is the largest positive integer j such that $2j(j-1)|b| < L$.

2. The residual variance of θ in the equilibrium with the maximum number of partition, $\sigma^2(b, L)$, is expressed as:

$$\sigma^2(b, L) = \frac{L^2}{12N(b, L)^2} + \frac{b^2(N(b, L)^2 - 1)}{3}.$$

Throughout this paper, we focus on equilibria where the principal's expected payoff is maximized. i.e., the equilibrium that yields the highest number of partitions. The following corollary addresses the characteristics of the residual variance in communication, which is used throughout our analysis as a measure of the information loss and thus efficiency in the communication game.

Corollary 1 *1. Fixing L, $\sigma^2(b, L)$ is continuous and strictly increasing in $b \in (0, \frac{L}{4})$. Also, fixing b, $\sigma^2(b, L)$ is continuous and strictly increasing in $L \in (4b, \infty)$.*

2. Fixing L, for all $b \leq \frac{L}{4}$ such that $\frac{\partial \sigma^2(b,L)}{\partial b}$ exists, $\frac{\partial \sigma^2(b,L)}{\partial b} \in (\frac{L}{6}, \frac{L}{2})$. Furthermore $\sigma^2(b, L) \leq \frac{|b|L}{3}$ for all b, $\sigma^2(b, L) = \frac{|b|L}{3}$ for all b that changes $N(b, L)$, and strictly concave in b and also in L in each region in which $N(b, L)$ is constant.

3. For all b' and b'' such that $\frac{L}{4} > b'' > b' > 0$, it holds that $\sigma^2(b'', L) - \sigma^2(b', L) \geq (b'')^2 - (b')^2$.

Most importantly, the first statement says that the information loss $\sigma^2(b, L)$ increases with the divergence of players' preferences. The same result from a change in L follows from the fact that messages of higher realized values become noisier in equilibrium (the information partition expands for larger states). When L increases, communication becomes noisier.

The second and the third statements pertain to the quantitative characterizations of $\sigma^2(b, L)$. The fact that $\frac{\partial \sigma^2(b,L)}{\partial b} \in (\frac{L}{6}, \frac{L}{2})$ provides the lower and upper bounds on the first-order effect of a decrease in b on communication efficiency, which will be used for the local characterizations on whether to delegate or not around the point where the principal is indifferent (Proposition 6). The local concavity of $\sigma^2(b, L)$ stems from the concavity of the principal's payoff. In $b \in [0, \frac{L}{4}]$, the concave closure of $\sigma^2(b, L)$ is the linear line $\frac{|b|L}{3}$ and $\sigma^2(b, L)$ touches the linear line at points where the partition number in the equilibrium changes. These imply that communication efficiency may vary relative to the magnitude of the bias; communication is inefficient relative to b when $\sigma^2(b, L)$ approaches $\frac{|b|L}{3}$. The third statement provides a lower bound on the impact of changes in b on communication efficiency. Figure 1 illustrates $\sigma^2(b, L)$ as a function of b, with L held constant.

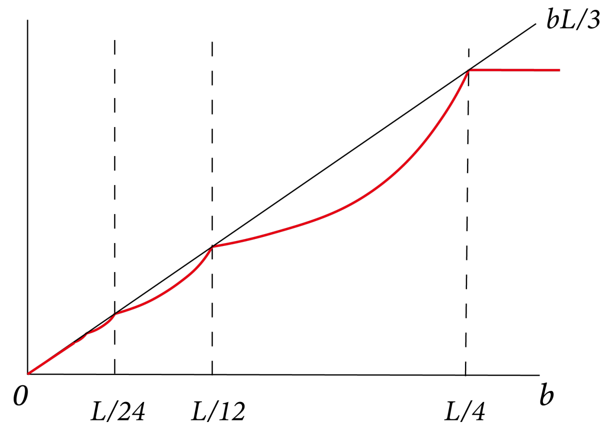

Figure 1: Partitions and $\sigma^2(b, L)$ for a constant L.

When the individual pieces of private information are independently and identically distributed, Crawford and Sobel (1982)'s result can easily be generalized to n-senders. The following lemma gives the principal's expected payoff both in case of power retention and delegation, which will be compared in the subsequent analysis.

Proposition 2 *If the principal delegates to decision-making to agent i, her expected payoff $W_i(\mathcal{N}, \mathbf{b}, \mathbf{L})$ is*

$$-b_i^2 - \sum_{j \neq i, j \in \mathcal{N}} \sigma^2(b_j - b_i, L_j).$$

If the principal retains authority, her expected payoff $W_p(\mathcal{N}, \mathbf{b}, \mathbf{L})$ is

$$-\sum_{i \in \mathcal{N}} \sigma^2(b_i, L_i).$$

The first term of the principal's payoff from delegation, $-b_i^2$, is the cost of the *loss of control*. This stems from the delegated agent's biased decision relative to the the principal's preferred decision. The second term, $-\sum_{j \neq i} \sigma^2(b_j - b_i, L_j)$, is the cost of the overall *loss of information*, which is the sum of information loss incurred in the (one-to-one) communication with each of the other agents, which is due to the imprecise information transmitted from another agent j to agent i. If, on the other hand, the principal retains authority, the cost from the loss of control is zero, and thus her payoff equals the cumulative cost of information loss incurred during communication with each of the n agents.

In the single agent setting of our model, there is no choice of who to delegate to: The principal simply decides whether to delegate or to retain power. Dessein (2002) demonstrates that, in this case, the principal delegates the decision making rights unless the agent is sufficiently biased.

Proposition 3 *(Dessein, 2002): In the one-agent case* $\mathcal{N} = \{1\}$, *for all* $b_1 < \frac{L_1}{2\sqrt{3}}$, *the principal delegates authority to the agent;*

$$W_1(\mathcal{N}, \mathbf{b}, \mathbf{L}) = -\left(b_1\right)^2 > -\sigma(b_1, L_1)^2 = W_p(\mathcal{N}, \mathbf{b}, \mathbf{L}).$$

This result states that the principal chooses to retain decision-making right if and only if the loss incurred from imprecise information in the communication game outweighs the cost of relinquishing control due to the biased decision made by the agent. In the opposite case, the principal prefers to delegate the decision to the agent.

3 Two Agents

In this section, we analyze the two-agents model in which $\mathcal{N}$ =$\{1, 2\}$. Initially, we defer the question of whether to delegate and instead offer a general insight into whom to delegate to, should the principal choose to delegate.

Proposition 4 *1. Suppose that* $L_1 = L_2$ *and denote* $|b_i| = \min\{b_1, b_2\}$. *We have* $W_i(\mathcal{N}, \mathbf{b}, \mathbf{L}) > W_{\neq i}(\mathcal{N}, \mathbf{b}, \mathbf{L})$.

2. Suppose that $b_1 = -b_2$ *and denote* $|L_i| = \max\{L_1, L_2\}$. *We have* $W_i(\mathcal{N}, \mathbf{b}, \mathbf{L}) > W_{\neq i}(\mathcal{N}, \mathbf{b}, \mathbf{L})$.

3. Suppose $L_2 > 0$ *and* $\mathbf{b}$ *such that* $|b_1| > |b_2|$. *Then there is* $L > L_2$ *such that* $W_1(\mathcal{N}, \mathbf{b}, \mathbf{L}) > W_2(\mathcal{N}, \mathbf{b}, \mathbf{L})$ *iff* $L_1 > L$.

Proposition 4-1 states that when the agents possesses information of equivalent importance, then that the principal should delegate authority to the agent whose preferences are closer to her own. To see this, observe that the amount of loss of information remains constant regardless of the sender and receiver roles. This arises because the disagreement regarding the optimal decision hinges solely on the absolute difference between their biases. Formally, this can be seen by $\sigma^2(b_1 - b_2, L_1) = \sigma^2(b_2 - b_1, L_2)$.

Consequently, the cost of delegation—resulting from a distorted decision—is minimized when decision-making authority is conferred to the agent with the smaller bias. From Proposition 2 the principal's prefers to delegate to agent i over agent j when

$$-b_i^2 - \sigma^2(b_j - b_i, L_j) > -b_j^2 - \sigma^2(b_i - b_j, L_i) \Leftrightarrow |b_j| > |b_i|\,.$$

Importantly, as will be discussed later, this conclusion cannot be generalized to cases with more than two agents. With more than two agents, the agent with the smallest bias will not necessarily be best positioned to communicate effectively with the other agents.

Proposition 4-2 states that when the agents have opposing biases, but that are at the same distance (as measured by the absolute value of the bias), the quantity of information possessed by the agents is also crucial in determining whom to delegate to. In fact, in this case, the principal should delegate to the agent with larger quantity of information. This is due to the fact that the information of the whom the decision is delegated to will be fully utilized. This observation echoes the findings of Harris and Raviv (2005), who demonstrate that the principal's motivation to delegate intensifies with the relative increase in the agent's information compared to that of the principal.

Proposition 4-3 illustrates that a trade-off may emerge between selecting an agent with larger quantity of information and an agent with a smaller bias. Although one of the agents exhibits a greater bias, the principal prefers to delegate to him if he holds a substantially larger quantity of information compared to the other one.

With these insights into whom to delegate to, we now turn to the question of whether the principal should delegate authority in the first place. To simplify the analysis, we assume $L_1 = L_2$. Consequently, based on previous results, if the principal chooses to delegate, she should do so to the agent with the smaller bias (in absolute terms) in both the case where the two biases are of the same sign (denoted the unidirectional case) and the case where biases are of opposing signs (opposing biases).

To determine whether delegation is preferable, three factors must be considered: 1) the loss of control, 2) the informational gain from the complete utilization of the delegated agent's information, and 3) the informational gain from communication with the one without the decision-making authority. The third factor becomes negative when agents have opposing biases because the difference between the two agents' preferences is greater than the difference between one agent's preferences and that of the principal.

The next proposition demonstrates that in the case of unidirectional biases, the third factor implies a generalization of the result of Dessein (2002) summarized in Proposition 3.

Proposition 5 *Suppose that* $0 < b_1 < b_2$ *and* $L_1 = L_2 = L > 0$. *Then:*

1. *If* $b_1 \leq \frac{L}{2\sqrt{3}}$, *then* $W_1(\mathcal{N}, \mathbf{b}, \mathbf{L}) > W_P(\mathcal{N}, \mathbf{b}, \mathbf{L})$.
2. *If* $b_1 \in (\frac{L}{2\sqrt{3}}, \frac{L}{\sqrt{6}})$, *then* $W_1(\mathcal{N}, \mathbf{b}, \mathbf{L}) > W_P(\mathcal{N}, \mathbf{b}, \mathbf{L})$ *iff* $b_2 < p(b_1)$. *The function* $p(b_1)$ *is a decreasing function such that* $p(\frac{L}{2\sqrt{3}}) = \frac{L}{\sqrt{3}}$ *and* $p(\frac{L}{\sqrt{6}}) = \frac{L}{\sqrt{6}}$.
3. *If* $b_1 > \frac{L}{\sqrt{6}}$, *then* $W_P(\mathcal{N}, \mathbf{b}, \mathbf{L}) > W_1(\mathcal{N}, \mathbf{b}, \mathbf{L})$.

To understand the intuition of this Proposition, notice first that from Proposition 4 we know that if the principal delegates her decision, she prefers to delegate to agent 1.

However even if agent 1 is sufficiently biased ($b_1 > \frac{L}{2\sqrt{3}}$) so that in the absence of agent 2, the principal prefers not to delegate, the principal may want to delegate to him when a second agent is introduced in order to facilitate communication with the other agent, thereby allowing her to leverage the information from both agents. In this sense, increasing the number of informed agents gives the principal more incentives to delegate.

When the two agents are biased in opposing directions, the decision to delegate or retain authority is more complex. Indeed, the following example demonstrates that even in the simplest case where $L_1 = L_2$ and $b_1 = -b_2 = b$, $W_p(\mathcal{N}, \mathbf{b}, \mathbf{L}) - W_1(\mathcal{N}, \mathbf{b}, \mathbf{L})$ is not monotone in b.

Example 1 *Let $L_1 = L_2 = L$, $b_1 = -b_2 = b$. Then:*

1. *If $b = \frac{L}{24}$, $W_p(\mathcal{N}, \mathbf{b}, \mathbf{L}) > W_1(\mathcal{N}, \mathbf{b}, \mathbf{L})$.*
2. *If $b = \frac{L}{12}$, $W_1(\mathcal{N}, \mathbf{b}, \mathbf{L}) > W_p(\mathcal{N}, \mathbf{b}, \mathbf{L})$.*
3. *If $b = \frac{L}{8}$, $W_p(\mathcal{N}, \mathbf{b}, \mathbf{L}) > W_1(\mathcal{N}, \mathbf{b}, \mathbf{L})$.*

In the first part of the example where $b = \frac{L}{24}$, the overall loss of information in delegation is $\sigma^2(b_1 - b_2, L) = \frac{L}{12}$, while that from retaining authority is also $\sigma^2(b_1, L) + \sigma^2(b_2, L) = \frac{L}{12}$. Therefore, the latter is preferable as it avoids the cost associated with the loss of control, $-b^2$.

The second part of the example where $b = \frac{L}{12}$ is seen in Figure 2. According to Corollary 1-2, the information loss in communication never exceeds $\frac{bL}{3}$ but approaches this value when players' preferences are close to the thresholds that alter the partition number in communication, such as when $b = \frac{L}{12}$. Hence, combined with local concavity of $\sigma^2(b, L)$ in b, the overall information loss from retaining authority, which is $\sigma^2(-\frac{L}{12}, L) + \sigma^2(\frac{L}{12}, L)$ (in the figure, this is twice of the height of a), gets larger than the overall information loss from delegation, which is $\sigma^2(\frac{L}{6}, L)$ (in the figure, this is the height of b). In this example, the loss of control is negligible, making delegation the better option.

In the third part of the example where $b = \frac{L}{8}$, if the principal delegates to one of the agents, the communication between agents is uninformative as their preferences differ by $L/4$. The principal is better positioned to communicate with both agents. Therefore, the retention of authority results in superior information flow. It is worth noticing that, in this part of the example as well as in the first part of the example, the agents' biases are small, $b_1 < \frac{L_1}{2\sqrt{3}}$ and $b_2 < \frac{L_2}{2\sqrt{3}}$, yet the principal prefers to retain authority. This finding is in stark contract to the two-agent result in which the agents possess biases of the same sign.

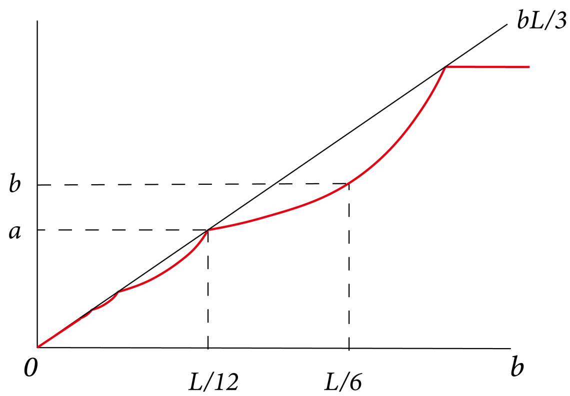

Figure 2: Example 1.

4 Three Agents

In this section we examine the three-agent case, $\mathcal{N} = \{1, 2, 3\}$. From this section, we focus on the difference in biases and thus consider environments $\{\mathcal{N}, \mathbf{b}, \mathbf{L}\}$ such that $\mathbf{L} = \mathbf{1}$. Accordingly, we simplify the notation the $W_i(\mathcal{N}, \mathbf{b}, \mathbf{L})$ to $W_i(\mathcal{N}, \mathbf{b})$.

We order the three agents according to the direction of biases: $b_1 < b_2 < b_3$. The following proposition demonstrates that when the direction of agents' biases are not all of the same sign, if the principal delegates authority she should assign it to agent 2, whose bias is centrally located.

Proposition 6 *If $b_1 \leq 0 \leq b_2 \leq b_3$ and $b_3 - b_1 < \frac{1}{4}$, then*

$$W_2(\mathcal{N}, \mathbf{b}) > \max\{W_1(\mathcal{N}, \mathbf{b}), W_3(\mathcal{N}, \mathbf{b})\}.$$

Furthermore, if $\max\{|b_1|, b_3\} < \frac{1}{4}$, there is $\varepsilon > 0$ such that $W_2(\mathcal{N}, \mathbf{b}) > W_p(\mathcal{N}, \mathbf{b})$ for $b_2 < \varepsilon$.

This proposition conveys the idea that the principal should delegate to the agent most capable of communicating effectively with others, specifically the agent whose preference is positioned in the middle. The condition $b_3 - b_1 < \frac{1}{4}$ ensures that no agent's bias is excessively large so that communication collapses completely and becomes uninformative, which would otherwise revert the analysis to earlier sections.

Concerning whether the principal should delegate the decision or not, it is essential to consider the relative importance of the loss of control from delegation against the loss of information from retaining power. When agents exhibit smaller biases, the principal opts to delegate in order to fully leverage the delegated agent's information. To formally see this, when $b_2 = 0$ the principal is indifferent between delegating to agent 2 and retaining authority. Now, consider the effect of increasing b_2 on the efficacy of delegation to agent 2 versus retaining control. Regarding agent 2's information, this results in a net positive impact that outweighs the newly incurred cost from the loss of control (Proposition 1). With respect to information from the other two agents, communication with agent 3 improves while communication with agent 1 deteriorates. When the agents' biases are small, these two effects are comparable (first-order effects are close to $\frac{1}{3}$) and effectively cancel each other out. Consequently, this makes delegation to agent 2 more advantageous.

When all agents exhibit positive biases, $b_i > 0$ for all $i \in \{1, 2, 3\}$, the principal opts to delegate to an agent unless all agents are sufficiently biased. This is formally proven in a more general n-agent analysis in the next section (Propositions 8 and 9). Before turning to that more general case, the next proposition illustrates the trade-off between the loss of information and the loss of control in determining whom to delegate to, which is not captured in the two-agent case.

Proposition 7 *Suppose that $b_3 > b_2 > b_1 > 0$ and $b_3 - b_1 < \frac{1}{4}$. Then:*

1. If $b_1 > \frac{1}{4}$, then $W_1(\mathcal{N}, \mathbf{b}) > \max\{W_2(\mathcal{N}, \mathbf{b}), W_3(\mathcal{N}, \mathbf{b})\}$.

2. If $b_1 < \frac{1}{12}$, then $W_2(\mathcal{N}, \mathbf{b}) > \max\{W_1(\mathcal{N}, \mathbf{b}), W_3(\mathcal{N}, \mathbf{b})\}$ for all $b_2 \in (b_1, b_1 + k)$ for some $k > 0$.

3. If $b_3 > b_2 + b_1$, then $W_2(\mathcal{N}, \mathbf{b}) > \max\{W_1(\mathcal{N}, \mathbf{b}), W_3(\mathcal{N}, \mathbf{b})\}$.

This proposition highlights the critical importance of the magnitude of agents' biases in shaping the principal's delegation decision. When agents are significantly biased ($b_i > 1/4$ for all i), the principal's main concern shifts to the loss of control, prompting her to delegate to the most like-minded agent (agent 1). Conversely, when biases are small, the cost of losing control diminishes, and communication efficiency becomes the principal's primary objective, leading her to delegate to the intermediary agent (agent 2). The reasoning underlying the third statement is similar; a substantial bias in agent 3 necessitates delegation to agent 2 in order to facilitate effective communication with agent 3.

5 Many Agents

We now examine the general case involving more than three agents, under the assumption that all agents exhibit positive biases, i.e., $b_i > 0$, and $L_i = L$ for all $i \in \mathcal{N}$. The main insight of this section is that as the number of agents increases, the loss of control becomes increasingly less significant.

The simplest case arises when $\forall i \in \mathcal{N}$, $b_i = b$. In this case, the principal's expected payoff from retaining authority is $-n\sigma^2(b, 1)$. Conversely, if she delegates to one of the agents, all other agents will fully disclose their information to this agent, resulting in an expected payoff of $-b^2$ for the principal. This leads to the following generalization of Proposition 3.

Proposition 8 *If all agents have the same bias, i.e., $b_i = b$, $W_1(\mathcal{N}, \mathbf{b}) > W_P(\mathcal{N}, \mathbf{b})$ iff*

$$|b| < \frac{1}{2\sqrt{3}}\sqrt{n}.$$

From this proposition it can be seen that the set of biases for which delegation is optimal increases with the number of agents n. This is because with homogeneous agents, delegating to agent i not only provides better information about θ_i but also fully reveals information from the remaining $n-1$ agents without additional cost. Of course, in this case the principal is indifferent regarding whom to delegate to.

For cases with heterogeneous agents, we have the following results.

Proposition 9 *Suppose that $b_i \geq 0$ for all $i \in \mathcal{N}$, b_i is increasing in i. Then the following holds;*

1. If $\min_{j \in \mathcal{N}} b_j < \frac{1}{2\sqrt{3}}$, the principal delegates to an agent.

2. If $n \geq 2$ and $b_n > b_1$, then $n \notin \arg\max_i W_i(\mathcal{N}, \mathbf{b})$.

3. If $|b_n - b_1| < \frac{1}{4}$ and $\sum_{i \neq j.j+1} b_i - \frac{(n-1)(b_{j+1}+b_j)}{2} > 0$ for $j < n$, then $W_{j+1}(\mathcal{N}, \mathbf{b}) > W_j(\mathcal{N}, \mathbf{b})$.

The first part of this proposition represents a further generalization of Proposition 3, extending it to cases involving multiple agents. The existence of at least one agent with a small bias compels the principal to delegate her authority. The second part extends insights from the two- and three-agent cases and asserts that delegating authority to the agent with the largest bias among all of the agents is never optimal. This agent harbors the most extreme preferences and, as a result, is among the least effective communicators. Moreover, he makes the worst decisions for the principal, thereby offering no rationale for delegation to this agent.

The third part of Proposition 9 generalizes Proposition 7-3. It compares the effectiveness of delegation between two consecutive agents, $j+1$ and j. The condition $\sum_{i \neq j.j+1} b_i -$

$\frac{(n-1)(b_{j+1}+b_j)}{2} > 0$ indicates that when the aggregate bias of agents is substantial, i.e., $\sum_{i\neq j.j+1} b_i$ is large, but the bias of agent $j+1$ is relatively small, it is more advantageous to delegate to agent $j+1$ rather than agent j. A corollary of this is that when $\sum_{i\geq 3} b_i - \frac{(n-1)(b_2+b_1)}{2} > 0$, specifically n is large or when there is an agent with a significantly higher bias relative to agent 2, delegating to agent 2 is more favorable than delegating to agent 1. Consequently, in such cases, it is not optimal to delegate to the agent whose preferences are most closely aligned with the principal.

When agents' preferences are aligned and their biases are large, delegating to the most aligned agent may become optimal. For instance, consider the example in which $b_i = k$ for all $i \geq 2$ and $k > \frac{n-2}{6}$. Then there exists ε such that it is better to delegate to agent 1 if $b_1 \in (k-\varepsilon, k)$.[4] In this example, agents possess similar yet large biases, rendering the information loss negligible irrespective of the agent whom the principal has delegated decision-amking rights to and making the potential loss of control the principal's primary concern. Consequently, she delegates authority to the agent whose preferences are the most aligned with her own.

6 Grouping

Up to this point, we have exclusively analyzed single-layer information structures, wherein the retention of authority necessitates that the principal personally gathers information from others. In this section, following Migrow (2021), we study two-layer information structures in which the principal selects a subset of agents as information collectors (team leaders) for other agents, while still retaining decision-making authority. The team leaders subsequently communicate their collected information to the principal. We refer to this information structure as "grouping."

It should be noted that grouping implies the principal retains decision-making authority. However, throughout this section, we refer to the information structure we have considered in which the principal retains authority without grouping simply as "retaining authority", which involves a different information structure from grouping.

Formally, a grouping is represented by (P, l) where $P = \{P_1, P_2, .., P_g\}$ is a partition of $\mathcal{N}$, and $l = \{l_1, l_2, , , l_g\}$ is the list of leaders such that $l_i \in P_i$.[5] Denote the principal's

[4] This is seen by noting that $W_1(b_1, k, .., k) = W_2(b_1, k, .., k)$ at $b_1 = k$ and;

$$\frac{\partial\{W_1(b_1, k, .., k) - W_2(b_1, k, .., k)\}}{\partial b_1} = \frac{1}{3}(N-2) - 2k < 0 \text{ at } b_1 = k.$$

[5] If $|P_j| = 1$ for all j, then (P, l) is identical to retaining authority that we have considered and thus $G(P, l) = W_p(\mathcal{N}, \mathbf{b})$.

expected payoff from grouping (P, l) by $G(P, l)$.

Consider first the case where $b_i = b$ for all $i \in \mathcal{N}$, meaning the agents' preferences are perfectly aligned. This makes communication between team members and the team leader perfectly informative. Communication between each leader and the principal follows from Proposition 1, with the modification that the leader' quantity of information has increased.

Proposition 10 *Suppose that $b_i = b$ for all $i \in \mathcal{N}$. If $b < \frac{\sum_{j\in\mathcal{N}} L_j}{2\sqrt{3}}$, it holds that*

$$W_i(\mathcal{N}, \mathbf{b}) > G(P, l) = -\sum_{i=1}^{g} \sigma^2(b, \sum_{j\in P_i} L_j) \text{ for all } (P, l) \in G.$$

This result mirrors Proposition 3. When the team leader communicates the gathered information to the principal, the information loss exceeds b^2. Consequently, rather than incurring this information cost, it is preferable to delegate decision-making authority to one of the agents. Grouping, therefore, is not preferable for $b < \frac{\sum_{j\in\mathcal{N}} L_j}{2\sqrt{3}}$.

We now proceed to more general cases. Unfortunately, however, it is not feasible to analyze this problem in general as characterizing the hierarchical communication equilibrium (e.g., from agent 2 to agent 1 to the principal) is complex. The difficulty arises because, in the communication game between the principal and a team leader, the leader is not fully informed of θ_j within his team members, rendering the equilibrium characterization in Proposition 1 inapplicable (see Ivanov, 2010). Therefore, we concentrate on a type of grouping where the groups consist solely of agents who possess identical preferences; we examine the groupings delineated by a partition $P = \{P_1, P_2, .., P_g\}$ of $\mathcal{N}$ such that $b_i = b_j$ if $i, j \in P_k$. Agents with identical biases form a team, and one member becomes the team leader; thus, the principal's payoff from grouping (P, l) is $-\sum_{i=1}^{g} \sigma^2(b, \sum_{j\in P_i} L_j)$.

The following example provides valuable insight into the benefits and drawbacks of grouping agents. In this example, we consider grouping in which agents 1 and 2, who share identical biases, form one team, while agents 3 and 4, who also share identical biases, constitute another. Depending on the extent of their biases, grouping may be more advantageous than delegation and retaining authority.

Example 2 *1. Let $n = 4$, $\mathbf{L} = \mathbf{1}$, and $b_1 = b_2 = -\frac{1}{6}$ and $b_3 = b_4 = \frac{1}{6}$. Then;*

$$W_p(\mathcal{N}, \mathbf{b}) > G(\{(1,2),(3,4)\},\{1,3\}) > W_i(\mathcal{N}, \mathbf{b}) \text{ for all } i \in \mathcal{N}.$$

2. Let $n = 4$, $\mathbf{L} = \mathbf{1}$, and $b_1 = b_2 = -\frac{1}{4}$ and $b_3 = b_4 = \frac{1}{4}$. Then;

$$G(\{(1,2),(3,4)\},\{1,3\}) > W_p(\mathcal{N}, \mathbf{b}) > W_i(\mathcal{N}, \mathbf{b}) \text{ for all } i \in \mathcal{N}.$$

In both examples, delegation is the least advantageous alternative for the principal, as the preferences of the agent with the decision-making right and those of the agents in the different preference group differ significantly and leads to a relatively large loss of control. Which of the two remaining information structures, retaining authority and grouping, is preferable depends solely on their relative performance in gathering information.

In the first example, grouping results in greater information loss than retaining authority. This is because, due to grouping, team leaders must convey a large quantity of information to the principal, which increases the information loss. Owing to the non-linearity of $\sigma^2(b, L)$ in L, there are cases in which $\sigma^2(b, 2L)$, the information loss from communicating with a team leader, exceeds $\sigma^2(b, L) + \sigma^2(b, L)$, the total information loss from communicating with each agent in the group. In this specific example, grouping induces communication equilibrium between the principal and the team leader played at an inefficient point ($b = \frac{L}{12}$) and thus induces larger information loss.

In the second example, grouping results in less information loss. Here, the bias of each agent is so substantial relative to their respetive quantity of information ($b_i \geq \frac{L_i}{4}$) that if the principal attempts to communicate with each agent directly, she fails to obtain any information. However, organizing agents into groups enhances communication efficiency by ensuring that team leaders have more information, mitigating the effects of individual biases and facilitating effective information exchange between the principal and team leaders.

The aforementioned results elucidate the conditions under which grouping may be beneficial. Grouping proves effective when each agent possesses a large bias that renders informative communication with the principal impossible, and when the preference disparities between groups are substantial enough to hinder inter-group communication. The next example further illustrates that an optimal group size exists and that clustering all agents with identical biases into the same group is not necessarily the optimal method of grouping.

Example 3 *1. Let* $n = 8$, $\mathbf{L} = \mathbf{1}$, *and* $b_j = -\frac{1}{3}$ *for* $j \leq 4$ *and* $b_j = \frac{1}{3}$ *for* $j \geq 5$. *Then*

$$G(\{(1,2),(3,4),(5,6),(7,8)\},\{1,3,5,7\}) > G(\{(1,2,3,4),(5,6,7,8)\},\{1,5\}).$$

2. Let $n = 8$, $\mathbf{L} = \mathbf{1}$, *and* $b_j = -\frac{1}{12}$ *for* $j \leq 4$ *and* $b_j = \frac{1}{12}$ *for* $j \geq 5$. *Then*

$$G(\{(1,2,3,4),(5,6,7,8)\},\{1,5\}) > G(\{(1,2),(3,4),(5,6),(7,8)\},\{1,3,5,7\}).$$

In the first example, grouping all four agents with identical biases makes the leader possess information of quantity $L = 4$, resulting in an information loss of $\sigma^2(\frac{1}{3}, 4)$ from the group. The communication equilibrium is played at an inefficient point ($b = \frac{L}{12}$). Instead, the principal can divide the four agents into two groups, $\{1, 2\}$ and $\{3, 4\}$, making each leader to possess information of quantity $L = 2$, thereby reducing the overall information loss to $\sigma^2(\frac{1}{3}, 2)+\sigma^2(\frac{1}{3}, 2) < \sigma^2(\frac{1}{3}, 4)$. Note that the agents' biases and the differences between these biases are sufficiently large so that neither retaining authority nor delegation functions effectively. The analogous argument elucidates the second example, but in this case grouping all four agents is preferable to splitting them into two.

The above examples suggest that determining the optimal grouping is highly nuanced as it depends on specific values of b. The following proposition, however, offers a modest general statement, suggesting that if a considerable number of agents share the same bias, it is suboptimal to consolidate them into a single group.

Proposition 11 *Take $K \subset \mathcal{N}$ such that $b_i = b$ for all $i \in K$. Then for almost all b, there is m such that if $|K| > m$, any grouping (P, l) such that $P_j = K$ for some j is not optimal.*

Proposition 11 arises from the observation that, for almost all values of b, when the principal retains authority, the information loss from communication between the principal and a team leader with m members is strictly smaller than $\frac{m|b|}{3}$, leading to an average information loss per agent of less than $\frac{b}{3}$. Because the information loss in communication is approximated by $\frac{m|b|}{3}$ when m is large, each agent's average information loss approaches $\frac{|b|}{3}$ if the principal consolidates all of them into a single group. Instead of it, she can divide the group into subgroups (or retain authority), thereby reducing the overall information loss.

7 Conclusion

In this paper we have analysed communication and delegation decisions in a multi-agent setting that extends the results of Dessein (2002) and Harris and Raviv (2005). We show that the effects identified in those papers extend beyond the one-agent case, but that other effects also arise and make the analysis more complex. The decision to delegate or not as well as to whom to delegate is determined by a trade-off between 1) enhanced communication when the delegated agent's preferences are closely aligned with those of the other agents, and 2) the costs incurred from distorted decisions arising due to biased preferences of the delegated agent.

8 Appendix

Proof of Proposition 2.

Suppose that the principal delegates to Agent i. Denote by $y_i(\theta) : \Pi_{j=1}^{n}[-\frac{L_j}{2}, \frac{L_j}{2}] \to R$ the equilibrium mapping from states to decisions when the principal delegates to agent

i, and denote by Ξ_j for $j \neq i$ the partition of state space $[-\frac{L_j}{2}, \frac{L_j}{2}]$ generated as a result of communication game between agent j (as a sender) and agent i (as a receiver). From Hori (2006), communication between an agent and the principal does not affect the set of equilibrium partitions of the other comunication games between senders $s \neq i$ and agent i, Ξ_j is determined independently from the realizations of $\theta_{s \neq j}$. For each θ_j, denote by $\Xi_j(\theta_j) \in \Xi_j$ the element of Ξ_j that contains θ_j.

From agent i's optimality condition $y_i(\theta) = \sum_{j \neq i} E[\theta_j | \Xi_j(\theta_j)] + \theta_i + b_i$. Using the independence of θ_j, $E_\theta \left[-\left(\sum_j \theta_j - y_i(\theta) \right)^2 \right]$, the principal's expected payoff, is computed as:

$$
\begin{aligned}
& E_\theta \left[-\left(\sum_{j \neq i} \theta_j - \sum_{j \neq i} E[\theta_j | \Xi_j(\theta_j)] + b_i \right)^2 \right] \\
= \; & E_{\theta_{-i}} \left[-\sum_{j \neq i} \left(\theta_j - E[\theta_j | \Xi_j(\theta_j)]\right)^2 + 2b_i \sum_{j \neq i} (\theta_j - E[\theta_j | \Xi_j(\theta_j)]) \right. \\
& \left. - \sum_{k \neq i} \sum_{j \neq k, i} (\theta_k - E[\theta_k | \Xi_k(\theta_k)])(\theta_j - E[\theta_j | \Xi_j(\theta_j)]) \right] - (b_i)^2 \\
= \; & E_{\theta_{-i}} \left[-\sum_{j \neq i} \left(\theta_j - E[\theta_j | \Xi_j(\theta_j)]\right)^2 \right] - (b_i)^2 = -\sum_{j \neq i} E_{\theta_j} \left(\theta_j - E[\theta_j | \Xi_j(\theta_j)]\right)^2 - (b_i)^2 \\
= \; & -\sum_{j \neq i} \sigma^2(b_j - b_i, L_j) - (b_i)^2 .
\end{aligned}
$$

Similar proof applies to the case where the principal retains power.

Proof of Corollary 1.

1. It follows because fixing L, it holds that $\sigma^2(b^-, L) = \sigma^2(b^+, L)$ at the point of discontinuity of $N(b, L)$ in b, ($|b| = L/2i(i-1)$ for $i = 2, 3, ..$ Also, at points where σ^2 is differentiable, $\frac{d\sigma^2}{db} = \frac{2b(N^2-1)}{3} > 0$. The second statement follows analogously.

2. From Proposition 1-2, $|b| \in (\frac{L}{2(N+1)N}, \frac{L}{2N(N-1)})$ for each $N = N(b, L)$. Hence if $b > 0$,

$$\frac{d\sigma^2}{db} = \frac{2b(N^2-1)}{3} \in (\frac{L(N-1)}{3N}, \frac{L(N+1)}{3N}).$$

As $\frac{2b(N^2-1)}{3}$ is increasing in b, $\frac{d\sigma^2}{db}$ increases from $\frac{L(N-1)}{3N}$ to $\frac{L(N+1)}{3N}$. Thus $\inf_b \frac{d\sigma}{db} = \min_N \frac{L(N-1)}{3N} = \frac{L}{6}$ (at $N = 2$) and $\sup_b \frac{d\sigma^2}{db} = \max_N \frac{L(N+1)}{3N} = \frac{L}{2}$ (at $N = 2$). Finally, simple calculations verify that $\sigma^2(b, L) = \frac{bL}{3}$ when $b = \frac{L}{2(N+1)N}$ or $\frac{L}{2N(N-1)}$.

3. Take b' and b'' such that $\frac{L}{4} > b'' > b' > 0$. Because $\sigma^2(b, L)$ is continuous, increasing

and differentiable almost everywhere in b, it holds that:

$$\begin{aligned}\sigma^2(b'', L) - \sigma^2(b', L) &= \int_{b'}^{b''} \left(\lim_{x \uparrow b} \frac{\partial \sigma^2(x, L)}{\partial x} \right) db = \int_{b'}^{b''} \frac{2b(N(b, L)^2 - 1)}{3} db \\ &\geq \int_{b'}^{b''} 2b db = (b'')^2 - (b')^2 = (b'' - b')(b'' + b').\end{aligned}$$

Proof of Proposition 4.

1 and 2 are straightforward and thus ommited. To prove 3, Fix $L_2 > 0$ and $\mathbf{b}$ such that $|b_1| > |b_2|$. From Lemma 1, $W_1(\mathcal{N}, \mathbf{b}, \mathbf{L}) - W_2(\mathcal{N}, \mathbf{b}, \mathbf{L})$ is increasing in L_1. Hence the statement follows from $W_1(\mathcal{N}, \mathbf{b}, \mathbf{L}) - W_2(\mathcal{N}, \mathbf{b}, \mathbf{L}) < 0$ for $L_1 = L_2$.

Proof of Proposition 5.

Suppose that $0 < b_1 < b_2$ and $L_1 = L_2 = L > 0$. That $W_1(\mathcal{N}, \mathbf{b}, \mathbf{L}) > W_2(\mathcal{N}, \mathbf{b}, \mathbf{L}))$ is straightforward. If $b_1 \leq \frac{L}{2\sqrt{3}}$, then;

$$\begin{aligned}W_1(\mathcal{N}, \mathbf{b}, \mathbf{L}) - W_p(\mathcal{N}, \mathbf{b}, \mathbf{L})) &= -(b_1)^2 - \sigma^2(b_2 - b_1, L) + \sigma^2(b_1, L) + \sigma^2(b_2, L) \\ &> -(b_1)^2 + \sigma^2(b_1, L) > 0,\end{aligned}$$

where the last inequality follows from Proposition 1.

If $b_1 > \frac{L}{\sqrt{6}}$, then;

$$\begin{aligned}W_p(\mathcal{N}, \mathbf{b}, \mathbf{L}) - W_1(\mathcal{N}, \mathbf{b}, \mathbf{L})) &= (b_1)^2 + \sigma^2(b_2 - b_1, L) - \sigma^2(b_1, L) - \sigma^2(b_2, L) \\ &> (b_1)^2 - \sigma^2(b_1, L) - \sigma^2(b_2, L) \\ &> (b_1)^2 - \frac{L^2}{12} - \frac{L^2}{12} > 0.\end{aligned}$$

Suppose that $b_1 \in (\frac{L}{2\sqrt{3}}, \frac{L}{\sqrt{6}})$. If the principal retains power, she plays a no-information equilibrium with both agents and her payoff becomes $-\frac{L^2}{6}$, while that from delegating to Agent 1 is $-b_1^2 - \sigma^2(b_2 - b_1)$. Hence she is indifferent between delegation and power retention when:

$$\frac{L^2}{6} = b_1^2 + \frac{L^2}{12\left(N_{p(b_1)-b_1}\right)^2} + \frac{(p(b_1) - b_1)^2\left(\left(N_{p(b_1)-b_1}\right)^2 - 1\right)}{3}.$$

Note that since $b_1 > \frac{L}{2\sqrt{3}}$, it must hold that $N_{p(b_1)-b_1} \geq 2$. Take the total derivative (from Lemma 1, $\sigma^2(b, L)$ is continuous everywhere) and we obtain:

$$2b_1 + \frac{2\left(p(b_1) - b_1\right)\left(\left(N_{p(b_1)-b_1}\right)^2 - 1\right)}{3} \frac{\partial p(b_1)}{\partial b_1} = 0.$$

Thus we have that $\frac{\partial p(b_1)}{\partial b_1} < 0$.

Proof of Proposition 6.

1. Suppose that $b_1 \leq 0 \leq b_2 \leq b_3$. It is straightforward to see that $W_2(\mathcal{N}, \mathbf{b}) > W_3(\mathcal{N}, \mathbf{b})$. If $|b_3 - b_1| \leq \frac{1}{4}$, then;

$$\begin{aligned} W_2(\mathcal{N}, \mathbf{b}) - W_1(\mathcal{N}, \mathbf{b}) &= \sigma(b_3 - b_1)^2 - \sigma(b_3 - b_2)^2 - (b_2)^2 + (b_1)^2 \\ &\geq (b_3 - b_1)^2 - (b_3 - b_2)^2 - (b_2)^2 + (b_1)^2 \\ &= 2(b_3 - b_1 - b_2)(b_2 - b_1) > 0, \end{aligned}$$

where the first inequality follows from Corollary 1-3.

2. Fix $b_1 \leq 0$ and $b_3 \geq 0$. Then; $W_2(\mathcal{N}, \mathbf{b}) - W_p(\mathcal{N}, \mathbf{b}) = 0$ if $b_2 = 0$. Also, at $b_2 = 0$, it holds that;

$$\frac{\partial\{W_2(\mathcal{N}, \mathbf{b}) - W_1(\mathcal{N}, \mathbf{b})\}}{\partial b_2} = -\frac{\partial \sigma(b_2 - b_1)^2}{\partial b_2} - \frac{\partial \sigma(b_3 - b_2)^2}{\partial b_2} - 2b_2 + \frac{\partial \sigma(b_2)^2}{\partial b_2}$$
$$> \quad -\frac{1}{2} + \frac{1}{6} - 2b_2 + \frac{1}{3} = 0.$$

where the inequality follows from Corollary 1-2. Hence the result follows.

Proof of Proposition 7.

Suppose that $b_3 > b_2 > b_1 > 0$ for all i and $b_3 - b_1 < \frac{1}{4}$.

1. Fix $b_1 > \frac{1}{4}$ and b_3. If $b_1 = b_2$, then $W_1(\mathcal{N}, \mathbf{b}) = W_2(\mathcal{N}, \mathbf{b})$. As $-\frac{\partial \sigma^2(b_3 - b_2)}{\partial b_2} < \frac{1}{2}$ from Corollary 1, $\frac{\partial\{W_2(\mathcal{N}, \mathbf{b}) - W_1(\mathcal{N}, \mathbf{b})\}}{\partial b_2} = -\frac{\partial \sigma^2(b_3 - b_2)}{\partial b_2} - 2b_2 < \frac{1}{2} - 2\frac{1}{4} = 0$. Therefore $W_1(\mathcal{N}, \mathbf{b}) > W_2(\mathcal{N}, \mathbf{b})$ if $b_2 > b_1$ and $b_2 < b_1 + k$ for some $k > 0$.

2. Fix $b_1 < \frac{1}{12}$ and b_3. If $b_1 = b_2$, then $W_1(\mathcal{N}, \mathbf{b}) = W_2(\mathcal{N}, \mathbf{b})$. As $-\frac{\partial \sigma^2(b_3 - b_2)}{\partial b_2} > \frac{1}{6}$, we have $\frac{\partial\{W_2(\mathcal{N}, \mathbf{b}) = W_1(\mathcal{N}, \mathbf{b})\}}{\partial b_2} = -\frac{\partial \sigma^2(b_3 - b_2)}{\partial b_2} - 2b_2 > \frac{1}{6} - 2\frac{1}{12} = 0$ and thus the result follows.

Proof of Proposition 9.

The first statement follows from

$$W_{n-1}(\mathcal{N}, \mathbf{b}) - W_n(\mathcal{N}, \mathbf{b}) = \sum_{i \leq n-1} \{\sigma^2(b_n - b_i) - \sigma^2(b_{n-1} - b_i)\} - (b_{n-1})^2 + (b_n)^2 > 0.$$

To prove the second statement, suppose that $|b_n - b_1| < \frac{1}{4}$. Then it follows that; $W_{j+1}(\mathcal{N}, \mathbf{b}) -$

$W_j(\mathcal{N}, \mathbf{b}) =$

$$\begin{aligned}
&\sum_{i \neq j.j+1} \{\sigma^2(b_j - b_i) - \sigma^2(b_{j+1} - b_i)\} - (b_{j+1})^2 + (b_j)^2 \\
> &\sum_{i \neq j.j+1} (2b_i - b_{j+1} - b_j)(b_{j+1} - b_j) - (b_{j+1} + b_j)(b_{j+1} - b_j) \\
= \ &2(b_{j+1} - b_j)\{\sum_{i \neq j.j+1} b_i - \frac{(n-1)(b_{j+1} + b_j)}{2}\},
\end{aligned}$$

where the inequality follows from Lemma 1-3.

Proof of Proposition 10.

Take $(P, l) \in G$. Assume that the strategies employed in communication games within a team adheres to those of a perfectly informative equilibrium, and that the strategies in communication game between the team leader in group P_i and the principal adhere to those of the equilibrium in Proposition 1, with bias b and $L = \sum_{j \in P_i} L_j$. To see that this is supported as an equilibrium, take $k \in P_i$ such that $k \notin l$. Given a realization of $(\theta_1, .., \theta_n)$ and the principal's strategy, l_i sends his optimal message when he is correctly informed of $(\theta_k, \{\theta_j\}_{j \in P_i, j \neq k})$, which is also $k's$ optimal message. By deviating and transmitting a false message to l_i, k can only prompt l_i to believe $(\theta_k', \{\theta_j\}_{j \in P_i, j \neq k})$, which only leads l_i sending a suboptimal message; thus k has no incentive to deviate. Because the same is true for all other agents, supposed strategies form an equilibrium. Then $G(P, l) = -\sum_{i=1}^{g} \sigma^2(b, \sum_{j \in P_i} L_j)$ for all $(P, l) \in G$ and $\sigma^2(b, \sum_{j \in P_i} L_j) > b^2 = -W_i(\mathcal{N}, \mathbf{b})$, readily follow.

Proof of Proposition 11.

Take any $b > 0$ such that $2j(j-1)|b| \neq k$ and $4|b| < k$ for all $j \in N_+$ and some $k \in N_+$; the overall information loss from grouping k number of agents with bias b is $\sigma^2(b, k)$. Fix such k. From Corollary 1-2, it holds that $\sigma^2(b, k) < \frac{bk}{3}$ for almost all b.

Suppose that there are $Jk + s$ number of agents with bias b with $J \in N_+,\ s \in Z$, and $s < k$. The overall information loss from dividing these agents into $J + 1$ groups in which there are J number of groups with k agents and one group with s members, the the overall information loss is $J\sigma^2(b, k) + \sigma^2(b, s)$, while that from grouping all of them in the same group is $\sigma^2(b, Jk + s)$.

Because $\sigma^2(b, k) < \frac{kb}{3}$, it holds that

$$\frac{J\sigma^2(b, k) + s\sigma^2(b, s)}{\sigma^2(b, Jk + s)} < \frac{\frac{Jkb}{3} + k\sigma^2(b, s)}{\sigma^2(b, Jk + s)} = \frac{\frac{Jkb}{(Jk+s)b} + \frac{k\sigma^2(b,s)}{(Jk+s)b}}{\frac{\sigma^2(b,Jk+s)}{(Jk+s)b}}.$$

As $J \to \infty$, it holds that $\frac{Jkb}{(Jk+s)b} \to 1$, $\frac{k\sigma^2(b,s)}{(Jk+s)b} \to 0$, and $\frac{\sigma^2(b,Jk+s)}{(Jk+s)b} \to 1$. Thus the last term converges to 1. Thus we conclude that $\frac{J\sigma^2(b,k)+s\sigma^2(b,s)}{\sigma^2(b,Jk+s)} < 1$ for large enough J, which proves the statement.

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9 Declaration of generative AI and AI-assisted technologies in the writing process

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