The Reserve Army and the Direction of Technical Change in a Dual Labour Market

Takashi OHNO* Kengo KOGURE

Doshisha University Institute of Science Tokyo

September 4, 2026

Working Paper Series No.67

Faculty of Economics, Doshisha University

Abstract

This paper develops a macro-dynamic model of overeducation and induced technical change in a dual labour market. Higher education expansion raises the incidence of overeducation through credential crowding without increasing employment in the short run. Sector-1-biased technical progress reduces total employment and raises the peripheral-sector employment share. With symmetric technology-adoption opportunities, an interior steady state equalizes sectoral labour-cost shares, while endogenous adjustment can raise overeducation and reduce employment peripheralization. Finally, the reserve-army wage elasticity bounds the intensity of local distribution–technology oscillations. Key results remain robust under an alternative endogenous-investment closure.

Key words: dual labour market; overeducation; induced technical change; technology adoption; reserve army; employment peripheralization

JEL Classification: E24; E25; J24; J42; O33

*Faculty of Economics, Doshisha University, Karasuma-higashi-iru, Imadegawa-dori, Kamigyo-ku, Kyoto 602-8580, Japan. E-mail: takaono@mail.doshisha.ac.jp

Kengo Kogure: Institute of Science Tokyo. E-mail: kkogure@ila.isct.ac.jp

This work was supported by JSPS KAKENHI Grant Number JP25K16648.

1 Introduction

Despite the expansion of higher education, a substantial fraction of graduates work in jobs that do not require a university degree. We refer to this phenomenon as *overeducation*. We focus not on temporary mismatches arising from search frictions or imperfect information immediately after graduation, but on a structural allocation problem in which the number of degree-requiring jobs need not adjust to the supply of educated workers.[1]

The literature on overeducation has examined its incidence, wage penalties, measurement, and consistency with human-capital and assignment theories (McGuinness, 2006; Leuven and Oosterbeek, 2011). Its main focus, however, has been the match between individual educational attainment and job requirements. Less attention has been paid to the macroeconomic determination of the number of graduate jobs and, in particular, to why the supply of such jobs need not expand in proportion to the supply of graduates. We address this question within a dual-labour-market framework.

The issue is also closely connected to recent changes in technology and the composition of employment. Advances in automation technologies, including AI and robotics, have drawn renewed attention to the contraction of routine middle-skill jobs, the polarization of employment towards high-skill occupations and low-wage service jobs, and the decline in the labour share (Goos and Manning, 2007; Acemoglu and Restrepo, 2018, 2022; Restrepo, 2024; Acemoglu and Loebbing, 2026). The key questions are how technical change affects employment, what determines its direction, and how that direction reshapes employment composition and distribution. We use the term employment peripheralization to denote an increase in sector 2's share of total employment, where sector 2 is the low-wage, low-bargaining-power segment of the labour market. This is distinct from an increase in the absolute level of sector-2 employment. When we use the term "McJobization," it refers only to this compositional shift.[2]

A leading framework for endogenizing the direction of technical change is Acemoglu's (1998, 2002) theory of directed technical change (DTC). In DTC, the direction of innovation is shaped by a price effect arising from factor prices and a market-size effect arising from factor supplies; an increase in the supply of skilled labour can therefore induce technologies that complement skilled labour. Related work using task-based models analyses the employ-

[1] The overeducation literature uses several measurement strategies, including differences between years of schooling and job requirements, self-assessment, and occupation-specific standard education levels. We do not take a position on these measurement disputes. Instead, we introduce into a macroeconomic model a job-requirement-based concept: graduates working in jobs that do not require a university degree.

[2] "McJobization" is a paper-specific shorthand inspired by the discussion in Ritzer (1993); it is not attributed to Ritzer as his own term. The analytical term used throughout the paper is employment peripheralization, while McJobization is used only as an intuitive label for the same compositional shift.

ment and distributional effects of automation and the creation of new tasks (Acemoglu and Restrepo, 2018, 2019; Acemoglu, Kong and Restrepo, 2025; Acemoglu and Loebbing, 2026; Jacobo, 2025). Our model also endogenizes the direction of technical change, but through a different causal mechanism. Whereas DTC determines the direction of innovation through price and market-size effects, our model focuses on a distributional channel in which sectoral labour-cost shares in an institutionally segmented labour market direct technology adoption. The supply of higher education therefore does not enter the technology-adoption rule directly. Instead, educational expansion first changes employment allocation and distribution within the dual labour market; these changes then affect the direction of technology adoption.[3]

The idea that distributional relations can shape the direction of technical choice has a long tradition in the classical–Marxian literature on induced technical change (ITC). The insight of Kennedy (1964) was subsequently connected to Goodwin-type distributional dynamics and Okishio-type criteria of technical choice (Goodwin, 1967; Okishio, 1961; Shah and Desai, 1981; van der Ploeg, 1987; Foley, 2003; Julius, 2005). Tavani (2012, 2013), in particular, links the labour share generated by wage bargaining to endogenous technical choice, yielding a feedback mechanism in which distribution induces technical change, which in turn feeds back on distribution and employment. Subsequent work has connected this framework to demand, accumulation, and stagnation (Tavani and Zamparelli, 2015, 2017; Zamparelli, 2015; Rada et al., 2023; Stamegna, 2025). Goodwin-type dynamics have also recently been reassessed on both theoretical and empirical grounds (Basu and Martins, 2026).

Existing ITC models, however, generally assume a single homogeneous labour market. A macroeconomic treatment of overeducation requires, at a minimum, a distinction between jobs that require a university degree and those that do not, together with a mechanism that allows educated workers to be allocated to the latter. Important contributions to the dual-labour-market literature include Doeringer and Piore (1971), Taylor (1983, 2004), and Skott (2005, 2006). In particular, Skott (2006) links Marx's (1867/1976) concept of the industrial reserve army to a peripheral labour-market segment and analyses labour-market hierarchy, including overeducation. The direction of technical change, however, is not determined there by endogenous firm-level choice.

We therefore combine a Skott-type dual labour market with a Tavani-type mechanism of distribution-induced technical choice. Access to sector 1 requires a university degree, whereas sector 2 has no such requirement. In sector 1, the employment rate determines wage growth; in sector 2, wages are benchmarked to sector-1 wages and discounted according to

[3] DTC and the task-based approach also incorporate factor prices, institutions, and heterogeneity across firms and workers. Our distinction is not that these approaches ignore distribution, but that our analysis places at its centre an ITC-type causal mechanism in which sector-specific labour-cost shares directly induce the direction of technology adoption.

the pressure exerted by the reserve army. We study how this institutional asymmetry shapes the interaction among distribution, employment, and technology adoption.[4]

We make four main theoretical contributions. First, we derive a sharp credential-crowding result for educational expansion within a dual labour market. Decomposing sector-2 employment by educational status, we show that, in the short run, an expansion of higher education changes neither sector's employment level but replaces non-graduates with graduates in sector 2, thereby increasing the incidence of overeducation among employed graduates. Thus, educational expansion generates credential crowding rather than job creation in the short run. Moreover, even in the symmetric steady state with endogenous technology adoption, educational expansion raises the incidence of overeducation.

Second, we identify a mechanism through which distribution-induced sectoral technology adoption changes employment composition. Differences in sectoral labour-cost shares induce different rates of technology adoption, and these differences cumulate over time into changes in relative productivity. In the short run, under the fixed-coefficient benchmark, sector-1-biased technical progress reduces total employment and raises the sector-2 employment share. Appendix D shows that the short-run employment-level and compositional effects in Proposition 3 carry over to a general constant-returns-to-scale final-good technology with the appropriately generalized surplus closure.

Third, we show that endogenous technology adoption tends to equalize sectoral labour-cost shares. Because the cost-saving gains from adoption are larger in the sector with the higher labour-cost share, symmetric adoption costs imply equal sectoral labour-cost shares at an interior steady state. This equalization concerns the disappearance of sector 1's distributional advantage, not equality of real wages; with asymmetric adoption possibilities, the ranking of sectoral labour-cost shares can even be reversed.

Fourth, we characterize the local oscillatory dynamics of distribution and relative productivity. In the symmetric benchmark, the steady-state elasticity of the reserve-army wage response, ε_ρ^*, provides the upper bound $\sqrt{\varepsilon_\rho^*}$ on the ratio of the imaginary to the real part of the local eigenvalues. Actual oscillation also depends on the relative responsiveness of technology adoption and wage adjustment. This characterization identifies the reserve-army elasticity and the relative response speed as natural empirical targets for evaluating the distribution–technology feedback.

We derive the first three results under a benchmark closure with an exogenous surplus requirement and assess their robustness in Section 5 by endogenizing investment. The short-

[4]We adopt a classical–Marxian closure because the analysis centres on persistent unemployment, institutionally distinct wage-setting mechanisms, and a causal link from distribution to technology adoption. The model is not designed to derive every distributional relation from the optimization problem of a representative agent.

run credential-crowding result, the short-run employment effects of sector-biased technical progress, and equalization of labour-cost shares under symmetric technology adoption survive the alternative closure. By contrast, the zero-sum distributional relation between sectors and local stability without additional parameter restrictions are features of the benchmark closure. The closed-form oscillation bound in the fourth contribution is likewise a characterization of the symmetric benchmark system and need not survive under the alternative closure.

The remainder of the paper is organized as follows. Section 2 develops the model and the short-run equilibrium. Section 3 analyses comparative statics and long-run dynamics. Section 4 endogenizes technology adoption. Section 5 introduces an alternative macroeconomic closure. Section 6 concludes.

2 A Dual Labour Market and the Benchmark Distributional Closure

This section sets out the dual labour market and a benchmark closure based on differences in saving propensities and an exogenous surplus requirement. We then show how employment and output are sequentially determined in the short run.

2.1 Production Technology and the Structure of the Dual Labour Market

The economy comprises a large number of homogeneous, vertically integrated firms, each negligible relative to the economy as a whole. Individual firms therefore take aggregate labour-cost shares as given and cannot alter them through their own decisions. This assumption will be used in Section 4 when we analyse firm-level technology adoption.

Each firm produces two intermediate inputs, X_1 in sector 1 and X_2 in sector 2, and a final good Y that uses both inputs. Intermediate production technologies are

$$X_1 = aN_1, \qquad X_2 = bN_2,$$

and the benchmark final-good technology is fixed-coefficient:

$$Y = \min(X_1, X_2) = \min(aN_1, bN_2).$$

Here N_1 and N_2 denote employment in the two sectors, and a and b are labour productivities. The fixed-coefficient specification provides a transparent benchmark for complementary intermediate inputs at different stages of production; Appendix D extends the analysis to a general final-good production function.[5]

Firms advance wages and means of production, while aggregate profit Π is the residual after wage payments and accrues to capital owners. Following Foley and Michl (1999), capital is treated as a social relation that enables the appropriation of surplus; the capital stock does not enter the benchmark production function explicitly.[6]

Efficient use of labour implies

$$Y = aN_1 = bN_2.$$

Define relative productivity as

$$\kappa \equiv \frac{a}{b}.$$

Then

$$N_2 = \kappa N_1$$

is the technological link between sectoral employment levels.

Let the labour force be N_s, with a fraction $\theta \in (0, 1)$ holding a university degree. Sector-1 jobs require a degree, so the labour pool eligible for sector 1 is

$$N_{s1} \equiv \theta N_s.$$

All workers employed in sector 1, N_1, are therefore graduates.

Sector-2 jobs, by contrast, do not require a degree. The sector-2 labour pool thus includes both non-graduates, $(1-\theta)N_s$, and graduates who have not obtained a sector-1 job, $\theta N_s - N_1$. Hence

$$N_{s2} = (1-\theta)N_s + (\theta N_s - N_1) = N_s - N_1.$$

[5] Vertical integration allows the same firm to choose technology adoption in both sectors in Section 4. An extension with separate sectoral firms and explicit intermediate-goods markets would require an additional determination of intermediate-goods prices. While vertical integration is a standard simplification in macro-dynamic models, it serves as a neutral benchmark that isolates the distributional channel of induced technical change from intermediate-goods price dynamics. Introducing separate sectoral firms with explicit market transactions would require modelling terms of trade and intermediate market power; while adding realism, this would complicate the price-setting mechanism and obscure the core interaction between wage bargaining, reserve-army pressure, and sectoral technology adoption.

[6] The model does not impose zero profits through free entry under perfect competition. It assumes the advance of capital and asymmetric ownership of the means of production, with profit accruing residually to capital owners. Except for the simplified investment closure in Section 5, the dynamics of the capital stock and the rate of profit are not modelled explicitly.

We decompose sector-2 employment by educational status as

$$N_2 = N_2^H + N_2^L,$$

and define graduate employment in sector 2, N_2^H, as overeducated employment. As a neutral benchmark, workers in sector 2 have the same productivity b and receive the same wage w_2 regardless of education; we therefore impose a common employment rate z_2 within the sector-2 labour pool (proportional rationing). It follows that

$$N_2^H = z_2(\theta N_s - N_1), \qquad N_2^L = z_2(1-\theta)N_s,$$

with $N_2 = z_2(N_s - N_1)$. This decomposition does not alter the aggregate short-run equilibrium.[7]

Define sectoral employment rates as

$$z_1 \equiv \frac{N_1}{\theta N_s}, \qquad z_2 \equiv \frac{N_2}{N_s - N_1}.$$

Using $N_2 = \kappa N_1$ gives

$$z_2 = \kappa \frac{\theta z_1}{1-\theta z_1} \tag{1}$$

or, equivalently,

$$\theta z_1 = \frac{z_2}{\kappa + z_2} \tag{2}$$

Throughout, we restrict attention to the following interior region with less than full employment in both sectors:[8]

$$0 < z_1 < 1, \qquad 0 < z_2 < 1.$$

[7]Proportional rationing is the most neutral benchmark. If graduates receive a constant relative hiring weight $\eta > 0$ in sector 2, so that $N_2^H = N_2\eta(\theta N_s - N_1)/[\eta(\theta N_s - N_1) + (1-\theta)N_s]$, then N_2^H is still increasing in θ as long as N_1 and N_2 remain independent of θ in the short run. The qualitative credential-crowding result therefore does not depend on proportional rationing itself. Furthermore, if employers in sector 2 systematically prefer graduates because of credential screening, represented by a relative hiring weight $\eta > 1$, such preferential hiring strengthens the displacement of non-graduates by graduates rather than eliminating overeducation among employed graduates. Therefore, relative to cases in which employers give graduates a hiring advantage $\eta > 1$, proportional rationing $\eta = 1$ provides a conservative baseline for the extent of credential crowding.

[8]Equation (1) implies parameter restrictions consistent with less than full employment in both sectors. Because we focus on interior distributional and technology-adoption dynamics rather than boundary solutions, we restrict attention to parameter values satisfying these restrictions.

2.2 Wage Determination in the Two Sectors

Define the sectoral labour-cost shares as $\pi_w \equiv w_1/a$ in sector 1 and $\Omega \equiv w_2/b$ in sector 2. The institutional asymmetry of the dual labour market is represented by different wage-setting rules across sectors.

Sector 1: bargained wage determination. We assume ongoing wage bargaining supported by firm-specific skills, internal labour markets, and worker organization. The employment rate affects the rate of wage growth rather than the wage level:

$$\hat{w}_1 = \phi(z_1), \qquad \phi' > 0 \tag{3}$$

Thus, defining $\hat{a} \equiv \dot{a}/a$, the growth rate of the sector-1 labour-cost share is

$$\hat{\pi}_w = \phi(z_1) - \hat{a} \tag{4}$$

At any point in time, π_w reflects the cumulative outcome of past wage bargaining and is predetermined for the short-run equilibrium.

Sector 2: relative wages and the reserve-army effect. Sector 2 does not share the bargaining institutions of sector 1. Its wage is set relative to the reference wage w_1 and discounted according to reserve-army pressure:

$$w_2 = \rho(z_2) w_1, \qquad \rho' > 0, \qquad 0 < \rho < 1 \tag{5}$$

A higher employment rate therefore narrows the wage gap. In the model, the reserve-army mechanism is represented by labour-market slack in the residual labour pool facing sector 2, as captured by z_2; sector 2 itself is not identified one-for-one with the reserve army. This specification is consistent with the institutional asymmetry emphasized in the internal- and dual-labour-market literature (Doeringer and Piore, 1971; Skott, 2005, 2006).[9]

Using $w_1 = \pi_w a$ and $\kappa = a/b$ yields

[9] One could alternatively specify sector-2 wages independently as $w_2 = \omega(z_2)$. Doing so would, however, remove the institutional link between the sector-1 reference wage and sector-2 wage formation. We therefore use a relative-wage specification to represent the hierarchy of the dual labour market. The benchmark also assumes that the relative wage does not respond directly to the educational composition of the sector-2 labour pool. Let $h_2 \equiv (\theta N_s - N_1)/(N_s - N_1)$ denote the graduate share of the sector-2 labour pool. More generally, with $\rho = \rho(z_2, h_2)$ and $\partial\rho/\partial h_2 \neq 0$, $\partial z_2/\partial\theta = 0$ would not generally hold, and the employment-invariance part of Proposition 1 would become conditional. We focus on the neutral benchmark that abstracts from the direct effect of educational composition on sector-2 wage formation. While micro-level feedback effects—such as changes in productivity or bargaining power within sector 2—may arise as the number of university graduates increases, we abstract from these channels to isolate the dual labour market's fundamental institutional asymmetry: bargained wage growth in sector 1 and the reserve-army effect in sector 2.

$$\Omega = \kappa\rho(z_2)\pi_w \tag{6}$$

so the sector-2 labour-cost share is determined by the sector-1 labour-cost share, relative productivity, and reserve-army pressure.

2.3 Class-Based Distribution and the Benchmark Surplus Closure

Let s, s_1, and s_2 denote the saving propensities of capitalists, sector-1 workers, and sector-2 workers, respectively, and assume

$$s > s_1 \geq 0, \qquad s > s_2 \geq 0.$$

Because we focus on interior benchmark equilibria, we restrict attention to configurations in which the sector-2 labour-cost share is positive. From the benchmark surplus condition below, this requires

$$(s - \sigma) > (s - s_1)\pi_w > 0,$$

and therefore, in particular,

$$\sigma < s.$$

We maintain this interior-feasibility condition throughout the benchmark analysis. It is part of the economically feasible region formalized below.

In the benchmark, σ denotes an exogenous surplus-absorption requirement: the share of output that must be matched by aggregate saving. It is most naturally interpreted as a reduced-form accumulation requirement rather than as a behavioural investment function. This classical closure determines the distribution consistent with a given surplus requirement; investment is endogenized in Section 5.

Let aggregate profit be Π and sectoral wage bills be W_1 and W_2. Consistency between aggregate saving and the surplus requirement requires

$$s\Pi + s_1 W_1 + s_2 W_2 = \sigma Y \tag{7}$$

Substituting $\Pi = Y - W_1 - W_2$ and dividing by Y yields

$$(s - s_1)\pi_w + (s - s_2)\Omega = s - \sigma \tag{8}$$

Equation (8) is not a general goods-market equilibrium condition. It is a consistency condition between the surplus requirement and distribution under the benchmark closure: a higher σ reduces the labour-cost shares compatible with the required surplus.

2.4 Short-Run Equilibrium

At any point in time, π_w is predetermined. Under the benchmark closure, the remaining short-run variables are determined sequentially from it.

Stage 1: the sector-2 labour-cost share. Solving (8) for Ω gives

$$\Omega = \frac{(s-\sigma)-(s-s_1)\pi_w}{s-s_2} \equiv \Omega^e(\pi_w;\sigma) \tag{9}$$

with

$$\frac{\partial \Omega^e}{\partial \pi_w} = -\Lambda < 0, \qquad \frac{\partial \Omega^e}{\partial \sigma} = -\frac{1}{s-s_2} < 0,$$

where

$$\Lambda \equiv \frac{s-s_1}{s-s_2} > 0.$$

The negative slope captures a zero-sum distributional relation specific to the exogenous-surplus benchmark.

Stage 2: the sector-2 employment rate. Combining (6) and (9),

$$\rho(z_2) = \frac{(s-\sigma)-(s-s_1)\pi_w}{(s-s_2)\kappa\pi_w} \tag{10}$$

Since $\rho' > 0$, an interior solution for z_2 is unique whenever it exists, and

$$\frac{\partial z_2}{\partial \pi_w} < 0, \qquad \frac{\partial z_2}{\partial \kappa} < 0, \qquad \frac{\partial z_2}{\partial \sigma} < 0, \qquad \frac{\partial z_2}{\partial \theta} = 0$$

(Appendix A). The independence of z_2 from θ is central to the overeducation result.

Stage 3: employment, output, and the educational composition of sector-2 employment. Equation (2) gives

$$z_1 = \frac{1}{\theta}\frac{z_2}{\kappa+z_2} \equiv Z(\pi_w,\kappa;\sigma,\theta) \tag{11}$$

and therefore

$$N_1 = \theta N_s z_1 = N_s \frac{z_2}{\kappa+z_2}, \qquad N_2 = \kappa N_1, \qquad Y = aN_1.$$

Once N_1 and z_2 are determined,

$$N_2^H = z_2(\theta N_s - N_1), \qquad N_2^L = z_2(1-\theta)N_s$$

are determined uniquely as well. Because educational composition does not feed back into sector-2 productivity b or wage w_2, this decomposition leaves the aggregate short-run equilibrium unchanged.

Two features of this sequential structure are important. First, when π_w is predetermined, the benchmark surplus condition determines Ω, which in turn determines z_2 through (5). Second, once z_2 is fixed,

$$N_1 = N_s \frac{z_2}{\kappa + z_2},$$

so the supply of graduates, θ, does not enter the short-run determination of the number of sector-1 jobs. Education supply changes the size of the pool eligible for a given number of jobs; it does not directly determine how many such jobs exist.

Moreover, because $\partial\Omega^e/\partial\pi_w < 0$ in the benchmark, a higher sector-1 labour-cost share implies a lower sector-2 labour-cost share and, through $\rho' > 0$, greater labour-market slack in sector 2. Section 5 examines the extent to which this zero-sum distributional mechanism depends on the macroeconomic closure.

Define the economically meaningful region as

$$\mathcal{F} = \left\{(\pi_w, \kappa, \Omega, z_1, z_2) : \ \pi_w > 0,\ \kappa > 0,\ \Omega > 0,\ 1 - \pi_w - \Omega > 0,\ 0 < z_1, z_2 < 1\right\}.$$

In the dynamic analysis we restrict attention to state values whose implied economic configuration belongs to $\mathcal{F}$.

Total employment is

$$N = N_1 + N_2 = (1 + \kappa)N_1.$$

3 Comparative Statics and Long-Run Equilibrium

This section uses the sequential structure in Section 2.4 to analyse the effects of exogenous parameters on employment and distribution. In Sections 3.1–3.3, π_w is held fixed.

3.1 Expansion of Higher Education and Overeducation

Define the incidence of overeducation among employed graduates as

$$q \equiv \frac{N_2^H}{N_1 + N_2^H}.$$

Proposition 1. Higher education expansion raises overeducation without increasing sector-1 employment.

An increase in the graduate share of the labour force, θ, lowers the sector-1 employment rate z_1 but leaves N_1, N_2, N, and Y unchanged. Graduate employment in sector 2, N_2^H, increases, while non-graduate employment in sector 2, N_2^L, falls by the same amount. The incidence of overeducation among employed graduates, q, rises. In particular, z_1 declines proportionately with θ, so its elasticity with respect to θ is -1.

Proof. Equation (10) does not contain θ, so $\partial z_2/\partial\theta = 0$. Moreover,

$$N_1 = N_s \frac{z_2}{\kappa + z_2}$$

implies $\partial N_1/\partial\theta = 0$. Since $N_2 = \kappa N_1$, $N = (1+\kappa)N_1$, and $Y = aN_1$,

$$\frac{\partial N_2}{\partial\theta} = \frac{\partial N}{\partial\theta} = \frac{\partial Y}{\partial\theta} = 0.$$

Because $z_1 = N_1/(\theta N_s)$,

$$\frac{\partial z_1}{\partial\theta} = -\frac{N_1}{\theta^2 N_s} = -\frac{z_1}{\theta}, \qquad \epsilon_\theta \equiv \frac{\theta}{z_1}\frac{\partial z_1}{\partial\theta} = -1.$$

For the educational composition of sector-2 employment, z_2 and N_1 are independent of θ, hence

$$\frac{\partial N_2^H}{\partial\theta} = z_2 N_s > 0, \qquad \frac{\partial N_2^L}{\partial\theta} = -z_2 N_s < 0.$$

Finally, since $N_2^H = z_2(\theta N_s - N_1)$,

$$\frac{\partial q}{\partial\theta} = \frac{z_2 N_s N_1}{(N_1 + N_2^H)^2} > 0. \qquad \blacksquare$$

Implication. In the short run, an expansion of higher education does not create jobs; it changes who occupies the existing jobs in sector 2. A larger graduate share neither raises the number of sector-1 jobs nor changes total employment in sector 2. Graduates unable to obtain a sector-1 job enter the sector-2 labour pool. The share employed in sector 2 is z_2, while the remaining share, $1 - z_2$, is unemployed. Since aggregate sector-2 employment is unchanged, the increase in graduate employment in sector 2 is matched one-for-one by a fall in non-graduate employment. This is credential crowding: the diffusion of educational credentials changes the educational composition of employment without changing the number of jobs.

The unit elasticity in absolute value is mechanical and should not be interpreted as a measure of the severity of search frictions. Because education supply does not enter the equation determining short-run sector-1 employment, an increase in education supply shows up in employment rates and educational composition rather than in employment levels. The invariance of aggregate employment in Proposition 1 relies on the benchmark assumption that educational composition does not directly affect sector-2 wage formation. By contrast, the credential-crowding mechanism itself is more robust to alternative hiring weights across educational groups, provided the aggregate short-run employment levels remain unchanged.

3.2 A Higher Surplus Requirement: A Benchmark-Closure Result

Proposition 2. A higher exogenous surplus requirement reduces employment.

Under the benchmark closure, an increase in the surplus requirement σ lowers employment rates, employment levels in both sectors, and total employment.

Proof. Equation (10) implies $\partial z_2/\partial\sigma < 0$. Since $N_1 = N_s z_2/(\kappa + z_2)$ is increasing in z_2, $\partial N_1/\partial\sigma < 0$, and $z_1 = N_1/(\theta N_s)$ implies $\partial z_1/\partial\sigma < 0$. With κ fixed, $N_2 = \kappa N_1$ and $N = (1+\kappa)N_1$, so $\partial N_2/\partial\sigma < 0$ and $\partial N/\partial\sigma < 0$. ■

This result depends on the benchmark closure, in which the exogenous σ imposes a trade-off between distributional shares. Because π_w is predetermined, an increase in σ lowers Ω, increases labour-market slack in sector 2, and contracts employment. The model does not include a channel through which capital accumulation expands labour demand. Proposition 2 should therefore not be read as claiming that faster accumulation generally raises unemployment. Section 5 examines this closure dependence.

3.3 Asymmetric Technical Progress and Employment Peripheralization

The comparative static with respect to κ is conditional on the current distributional state: it compares short-run configurations holding π_w fixed. It should not be interpreted as the instantaneous response to an unanticipated productivity shock at a fixed nominal wage, since such a shock would itself change $\pi_w = w_1/a$.

Proposition 3. An increase in sector 1's relative productivity reduces employment and raises the sector-2 employment share.

Under the fixed-coefficient benchmark, an increase in relative productivity, $\kappa = a/b$, reduces sector-1 employment N_1 and total employment N, while increasing the sector-2 share of total employment, N_2/N.

Proof. Using $\partial z_2/\partial\kappa < 0$, total differentiation of $N_1 = N_s z_2/(\kappa + z_2)$ with respect to κ gives

$$\frac{dN_1}{d\kappa} = \frac{N_s}{(\kappa + z_2)^2}\left[\kappa\frac{\partial z_2}{\partial \kappa} - z_2\right] < 0.$$

Moreover,

$$N = N_s\frac{z_2(1+\kappa)}{\kappa + z_2}$$

implies

$$\frac{dN}{d\kappa} = \frac{N_s}{(\kappa + z_2)^2}\left[(\kappa^2 + \kappa)\frac{\partial z_2}{\partial \kappa} + z_2(z_2 - 1)\right] < 0.$$

Finally,

$$\frac{N_2}{N} = \frac{\kappa}{1+\kappa},$$

so

$$\frac{\partial}{\partial \kappa}\left(\frac{N_2}{N}\right) = \frac{1}{(1+\kappa)^2} > 0. \qquad \blacksquare$$

Two distinct mechanisms underlie the proposition. First, the decline in N_1 and N depends on a distributional and reserve-army mechanism: an increase in κ lowers the relative wage ρ implied by (6), and, since $\rho' > 0$, reduces z_2. Second, under the fixed-coefficient benchmark, the rise in the sector-2 employment share follows transparently from the complementary production structure $N_2/N_1 = \kappa$. Appendix D shows that the short-run employment-level and compositional effects in Proposition 3 carry over to a general constant-returns-to-scale final-good technology with the appropriately generalized surplus closure.

Employment peripheralization refers to an increase in N_2/N. The sign of the absolute change in sector-2 employment, $N_2 = \kappa N_1$, is generally indeterminate because the change in the required labour ratio and the decline in N_1 work in opposite directions. Thus "McJobization" here does not mean an absolute expansion of sector-2 employment; it means a greater weight of the peripheral sector in total employment.

Proposition 4. An increase in sector 2's relative productivity raises employment and reduces the sector-2 employment share.

Under the fixed-coefficient benchmark, an increase in sector 2's productivity relative to sector 1—that is, a decline in κ—has the opposite effects to those in Proposition 3. Sector-1 employment and total employment rise, while the sector-2 employment share falls.

Proof. A rise in sector-2 productivity b, holding a fixed, lowers $\kappa = a/b$. The result therefore follows from Proposition 3 with the sign of the comparative-static change in κ

reversed. ■

The contrast between Propositions 3 and 4 shows that the employment effects of a given productivity improvement depend on the sector in which it occurs. This asymmetry motivates the endogenous technology-adoption analysis in Section 4.

3.4 Long-Run Dynamics under Exogenous Technical Progress

Throughout Sections 3.4 and 4, a "steady state" means a stationary configuration of employment rates, distributional shares, and relative productivity. The productivity levels a and b may continue to grow at a common positive rate; in that sense, the steady state is a balanced-growth configuration in productivity levels.

So far, π_w has been held fixed. We now assume that both sectors grow at the same exogenous productivity growth rate, $\hat{a} = \hat{b} = \gamma_a$, so that κ remains constant, and consider the adjustment of π_w. From (4) and (11),

$$\dot{\pi}_w = \pi_w \left[\phi\big(Z(\pi_w, \kappa; \sigma, \theta)\big) - \gamma_a\right] \tag{12}$$

Proposition 5. Characterization and local stability of an interior steady state. *Suppose that $\gamma_a \in \phi((0,1))$ and that the corresponding interior steady-state configuration belongs to the economically feasible region $\mathcal{F}$. Then the steady-state sector-1 employment rate z_1^* is uniquely characterized by*

$$\phi(z_1^*) = \gamma_a,$$

and the steady state is locally asymptotically stable.

Proof. At an interior steady state with $\pi_w^* > 0$, $\dot{\pi}_w = 0$ requires $\phi(z_1^*) = \gamma_a$, and $\phi' > 0$ ensures uniqueness of z_1^*. Linearization gives

$$\left.\frac{\partial \dot{\pi}_w}{\partial \pi_w}\right|_* = \pi_w^* \phi' \frac{\partial z_1}{\partial \pi_w} < 0,$$

where $\partial z_1 / \partial \pi_w < 0$ follows from Appendix A. Hence, conditional on the existence of an interior steady state, local asymptotic stability requires no additional parameter restriction. ■

At the steady state, z_1^* is independent of θ, κ, and σ. The long-run comparative statics are summarized in Table 1.

The $\kappa \uparrow$ row compares steady states with different constant values of relative productivity, holding the common exogenous growth rate $\hat{a} = \hat{b} = \gamma_a$ fixed. It therefore does not describe a dynamic change in κ itself. When θ rises, N_2^* increases together with N_1^* and N^*; when

Table 1. Long-run comparative statics under exogenous technical progress.

Exogenous variable	z_1^*	N_1^*	z_2^*	Ω^*	π_w^*	N^*	N_2^*/N^*	q^*
Graduate share $\theta \uparrow$	unchanged	increases	increases	increases	decreases	increases	unchanged	increases
Surplus requirement $\sigma \uparrow$	unchanged	unchanged	unchanged	decreases	decreases	unchanged	unchanged	unchanged
Relative productivity $\kappa \uparrow$	unchanged	unchanged	increases	increases	decreases	increases	increases	increases

σ rises, N_2^* remains unchanged. When κ rises, N_1^* is unchanged while $N_2^* = \kappa N_1^*$ and $N^* = (1+\kappa)N_1^*$ both increase. Endogenous changes in relative productivity are analysed in Section 4.

For higher education, the constancy of z_1^* implies

$$N_1^* = \theta N_s z_1^*,$$

which increases with θ. From (1),

$$z_2^* = \frac{\kappa\theta z_1^*}{1-\theta z_1^*},$$

so z_2^* also rises. Hence

$$q^* = \frac{z_2^*(1-z_1^*)}{z_1^* + z_2^*(1-z_1^*)}$$

increases as well. Thus, although sector-1 employment expands in the long run, the share of employed graduates who are overeducated in sector 2 also rises. An expansion of education can therefore increase sector-1 employment through long-run wage and distributional adjustment without eliminating overeducation.

The adjustment to a higher σ differs between the short run and the long run. In the short run, Ω and z_2 fall and employment contracts. In the long-run steady state, z_1^*, z_2^*, N_1^*, and N_2^* are unchanged, while both π_w^* and Ω^* fall (Appendix A.6). Thus the burden of adjustment appears as employment contraction in the short run and as lower labour-cost shares in both sectors in the long run. This implication is specific to the benchmark closure.

The increase in κ also illustrates the distinction between the short-run comparative statics of Proposition 3 and long-run steady-state comparisons. In the short run, predetermined π_w implies declines in N_1 and total employment N. In the long run, distributional adjustment returns z_1 to its steady-state level z_1^*. The difference reflects the contrast between a short-run response before distributional adjustment and a comparison of steady states after that adjustment is complete.

4 Induced Technical Change and Uneven Productivity Growth

The previous section treated productivity growth as exogenous. We now introduce a firm-level rule under which firms adopt technology in response to current sectoral labour-cost shares, thereby endogenizing relative productivity, κ.

4.1 A Firm-Level Rule for Technology Adoption

As assumed in Section 2.1, the economy contains a large number of homogeneous, vertically integrated firms, each of which takes the aggregate values of π_w and Ω as given. Under the fixed-coefficient benchmark, unit labour cost per unit of final output is

$$c = \frac{w_1}{a} + \frac{w_2}{b} = \pi_w + \Omega.$$

What we endogenize is not invention through R&D, but the direction in which available technologies are adopted. In the spirit of Kennedy (1964) and Tavani (2012, 2013), we associate sectoral productivity-growth rates $\hat{a}$ and $\hat{b}$ with separable technology-adoption cost functions that have increasing marginal costs,

$$f_a(\hat{a}), \qquad f_b(\hat{b}), \qquad f_a', f_b' > 0, \quad f_a'', f_b'' > 0.$$

These functions capture adjustment and adoption costs under the current factor-price structure. Firms are myopic in the sense that they evaluate current benefits and do not internalize future wages.[10]

Holding current wages fixed, the instantaneous reduction in unit labour cost generated by productivity growth is, to a first-order approximation,

$$-\dot{c}\big|_{\text{tech}} = \pi_w \hat{a} + \Omega \hat{b}.$$

The firm's technology-adoption problem can therefore be written as

[10] In this sense, f_a and f_b are not intended to represent an innovation possibility frontier. They provide a reduced-form representation of the Kennedy–Tavani idea that factor prices induce the direction of technical choice, here expressed through sector-specific convex technology-adoption costs. The same qualitative adoption rule can be derived from a simple present-value formulation. Suppose that a marginal productivity improvement is permanent, firms discount future cost savings at a constant rate $r > 0$, and they extrapolate current sectoral labour-cost shares when evaluating those savings. The present values of the marginal cost reductions are then proportional to π_w/r and Ω/r, so the first-order conditions become $f_a'(\hat{a}) = \pi_w/r$ and $f_b'(\hat{b}) = \Omega/r$. Because r is common to both sectors, the constant factor $1/r$ can be absorbed into the definitions of g_a and g_b. Hence $g_a' > 0$, $g_b' > 0$, the symmetry condition $g_a = g_b$, and the subsequent steady-state results are unchanged. We therefore normalize this constant scaling factor in the adoption problem below.

$$\max_{\hat{a},\hat{b}} \ \Psi = \left(\pi_w \hat{a} + \Omega \hat{b}\right) - \left(f_a(\hat{a}) + f_b(\hat{b})\right) \tag{13}$$

The first-order conditions for an interior solution are

$$f'_a(\hat{a}) = \pi_w, \qquad f'_b(\hat{b}) = \Omega,$$

Inverting these conditions yields

$$\hat{a} = g_a(\pi_w), \qquad \hat{b} = g_b(\Omega), \qquad g'_a, g'_b > 0 \tag{14}$$

A higher labour-cost share increases the cost-saving return to adopting productivity-enhancing technologies and therefore induces a higher rate of labour-saving technology adoption in that sector. We do not model the innovation process itself; rather, we specify a reduced-form adoption mechanism that determines the direction of technical change. With homogeneous firms making symmetric choices, the firm-level adoption rates coincide with the aggregate growth rates of a and b.[11]

4.2 Uneven Productivity Growth and Employment Composition

Individual firms choose their adoption rates taking π_w and Ω as given and do not internalize the aggregate distributional constraint (8). Hence, in general,

$$\hat{a} \neq \hat{b}.$$

The growth rate of relative productivity, $\kappa = a/b$, is

$$\hat{\kappa} = \hat{a} - \hat{b} = g_a(\pi_w) - g_b(\Omega) \tag{15}$$

Differences in sectoral labour-cost shares therefore induce different rates of technology adoption, and these differences cumulate over time into changes in relative productivity. Under the fixed-coefficient benchmark, Proposition 3 implies that an increase in κ reduces total employment and raises N_2/N. Firm-level cost-saving behaviour can thus generate employment peripheralization as an unintended macroeconomic outcome. For a given path of relative productivity and the cost-minimizing input mix, Appendix D shows that, under a general production function, employment composition shifts towards sector 2 whenever

[11] Because homogeneous firms choose the same technology-adoption rates in a symmetric equilibrium, the firm-level adoption rules coincide with the aggregate growth rates of a and b. The model abstracts from firm heterogeneity and lags in technology diffusion.

$\hat{\kappa} + \hat{m} > 0$.

4.3 Two-Dimensional Dynamics and Symmetric Technology-Adoption Costs

Under the benchmark closure, $\Omega = \Omega^e(\pi_w)$ and Ω does not depend directly on κ. Equations (4) and (15) therefore give the two-dimensional system in the state variables (π_w, κ):

$$\begin{cases} \dot{\pi}_w = \pi_w \left[\phi\big(Z(\pi_w, \kappa; \sigma, \theta)\big) - g_a(\pi_w) \right], \\ \dot{\kappa} = \kappa \left[g_a(\pi_w) - g_b\big(\Omega^e(\pi_w)\big) \right] \end{cases} \tag{16}$$

Proposition 6. Equalization of sectoral labour-cost shares and local stability. *Suppose the technology-adoption functions are symmetric across sectors, $g_a = g_b$, and that system* (16) *has an interior steady state (π_w^*, κ^*) whose implied economic configuration belongs to the economically feasible region $\mathcal{F}$. Then*

$$\pi_w^* = \Omega^* = \frac{s - \sigma}{2s - s_1 - s_2},$$

and the steady state is locally asymptotically stable.

Proof: steady state. Since $\dot{\kappa} = 0$ and $\kappa^* > 0$,

$$g_a(\pi_w^*) = g_b(\Omega^*).$$

Because $g_a = g_b$ and the common function is strictly increasing,

$$\pi_w^* = \Omega^*.$$

Substitution into (8) gives

$$\pi_w^*[(s - s_1) + (s - s_2)] = s - \sigma,$$

and hence

$$\pi_w^* = \Omega^* = \frac{s - \sigma}{2s - s_1 - s_2}.$$

For this closed-form distributional solution, the necessary and sufficient condition for $\pi_w^* = \Omega^* > 0$ and a positive profit share $1 - \pi_w^* - \Omega^* > 0$ is

$$\frac{s_1 + s_2}{2} < \sigma < s.$$

This condition concerns only positivity of the distributional variables and does not by itself guarantee that the entire steady state is interior. In addition, we assume that the values of z_1^*, z_2^*, and κ^* implied by $\phi(z_1^*) = g_a(\pi_w^*)$, (2), and (6) satisfy $0 < z_i^* < 1$. The proposition is therefore conditional on the implied steady state lying in the interior of the feasible region.

Proof: local stability. Linearizing (16) at the steady state gives the Jacobian matrix (Appendix B)

$$J = \begin{pmatrix} \pi_w^* \left[\phi' \dfrac{\partial z_1}{\partial \pi_w} - g_a'\right] & \pi_w^* \phi' \dfrac{dz_1}{d\kappa} \\ \kappa^* \left[g_a' + g_b' \Lambda\right] & 0 \end{pmatrix}.$$

Appendix A establishes $\partial z_1/\partial \pi_w < 0$ and $dz_1/d\kappa < 0$. Since $\phi', g_a', g_b', \Lambda > 0$,

$$J_{11} < 0, \qquad J_{12} < 0, \qquad J_{21} > 0, \qquad J_{22} = 0.$$

Therefore,

$$\operatorname{tr} J < 0, \qquad \det J > 0,$$

and the interior steady state is locally asymptotically stable. Conditional on existence and feasibility of the steady state, local stability under the benchmark closure requires no additional parameter restrictions. ■

Corollary 1. Higher education expansion with endogenous technology adoption.

Suppose an interior steady-state branch exists in the symmetric case of Proposition 6 for the values of θ under consideration. Holding all other primitives fixed, including $s, s_1, s_2, \sigma, \phi, g_a = g_b, \rho$, and N_s, an increase in the graduate share θ leaves z_1^*, π_w^*, and Ω^* unchanged, raises z_2^*, and lowers relative productivity κ^*. Sector-1 employment N_1^*, total employment N^*, and the incidence of overeducation q^* increase, while the sector-2 share of total employment, N_2^*/N^*, falls. The change in the absolute level of sector-2 employment N_2^* is generally indeterminate.

Proof. Proposition 6 implies that $\pi_w^* = \Omega^*$ is independent of θ, and

$$\phi(z_1^*) = g_a(\pi_w^*)$$

implies that z_1^* is also independent of θ. Equation (6) gives

$$\kappa^* \rho(z_2^*) = 1.$$

From (2),

$$\theta z_1^* = \frac{z_2^*}{\kappa^* + z_2^*}.$$

Substituting $\kappa^* = 1/\rho(z_2^*)$ yields

$$z_2^* \rho(z_2^*) = \frac{\theta z_1^*}{1 - \theta z_1^*}.$$

Let $H(z_2) \equiv z_2 \rho(z_2)$. Then

$$H'(z_2) = \rho(z_2) + z_2 \rho'(z_2) > 0,$$

while the right-hand side is increasing in θ. Hence

$$\frac{dz_2^*}{d\theta} > 0.$$

Since $\kappa^* = 1/\rho(z_2^*)$,

$$\frac{d\kappa^*}{d\theta} < 0.$$

Moreover, $N_1^* = \theta N_s z_1^*$ implies $dN_1^*/d\theta > 0$. Because

$$\frac{N_2^*}{N^*} = \frac{\kappa^*}{1 + \kappa^*} = \frac{1}{1 + \rho(z_2^*)},$$

we have

$$\frac{d}{d\theta}\left(\frac{N_2^*}{N^*}\right) < 0.$$

The incidence of overeducation is

$$q^* = \frac{z_2^*(1 - z_1^*)}{z_1^* + z_2^*(1 - z_1^*)},$$

which is increasing in z_2^* for fixed z_1^*, so $dq^*/d\theta > 0$. For total employment, using $\kappa^* = 1/\rho(z_2^*)$ and the steady-state relation above gives

$$\frac{N^*}{N_s} = \frac{z_2^*[1 + \rho(z_2^*)]}{1 + z_2^* \rho(z_2^*)}.$$

Differentiating with respect to z_2^*,

$$\frac{d(N^*/N_s)}{dz_2^*} = \frac{1 + \rho(z_2^*) + z_2^*(1 - z_2^*)\rho'(z_2^*)}{[1 + z_2^* \rho(z_2^*)]^2} > 0,$$

so $dN^*/d\theta > 0$. By contrast,

$$\frac{N_2^*}{N_s} = \frac{z_2^*}{1 + z_2^* \rho(z_2^*)}$$

has derivative $[1 - (z_2^*)^2 \rho'(z_2^*)]/[1 + z_2^* \rho(z_2^*)]^2$, whose sign is generally indeterminate. ■

This result highlights the difference from the exogenous-technology case in Section 3.4. There, κ is fixed, so an expansion of higher education leaves $N_2^*/N^* = \kappa/(1+\kappa)$ unchanged. When technology adoption is endogenous, however, the increase in z_2^* raises the relative wage $\rho(z_2^*)$, lowers $\kappa^* = 1/\rho(z_2^*)$, and therefore reduces the sector-2 employment share. Endogenizing technology adoption thus allows changes in education supply to affect not only the educational composition of employment but also the sectoral composition of employment. At the same time, the incidence of overeducation q^* rises in both cases. Overeducation and employment peripheralization are therefore distinct phenomena.

4.4 Asymmetric Technology-Adoption Costs

The equalization result in Proposition 6 depends on symmetry in technology-adoption opportunities across sectors. In reality, automation possibilities may differ across cognitive tasks, interpersonal services, and production processes.

Corollary 2. Steady-state distribution under asymmetric technology-adoption costs.

If $g_a(x) > g_b(x)$ for every x, then an interior steady state satisfies $\pi_w^* < \Omega^*$. Conversely, if $g_a(x) < g_b(x)$ for every x, then $\pi_w^* > \Omega^*$.

Proof. At the steady state,

$$g_a(\pi_w^*) = g_b(\Omega^*).$$

If $g_a(x) > g_b(x)$ for every x, then

$$g_a(\Omega^*) > g_b(\Omega^*) = g_a(\pi_w^*),$$

and monotonicity of g_a implies $\Omega^* > \pi_w^*$. The reverse case follows analogously. ■

More generally, the long-run ranking of sectoral labour-cost shares is determined endogenously by relative technology-adoption opportunities. If automation is easier in sector 1, its initial distributional advantage can disappear and even be reversed.

4.5 Reserve-Army Elasticity, Damped Oscillations, and the Interpretation of the Steady State

The local stability result in Proposition 6 can be sharpened in the symmetric case. Write the common technology-adoption function as $g_a = g_b = g$, and define the steady-state elasticity of the sector-2 relative-wage function as

$$\varepsilon_\rho^* \equiv \frac{z_2^* \rho'(z_2^*)}{\rho(z_2^*)} > 0.$$

Also define

$$A^* \equiv - \left. \frac{\partial z_1}{\partial \pi_w} \right|_* > 0, \qquad \tau^* \equiv \frac{g'(\pi_w^*)}{\phi'(z_1^*) A^*} > 0.$$

The parameter τ^* measures the responsiveness of technology adoption relative to the wage-adjustment channel at the steady state.

Proposition 7. Reserve-army elasticity and local oscillatory dynamics. *Under the symmetric benchmark of Proposition 6, the Jacobian satisfies*

$$\frac{4 \det J}{(\operatorname{tr} J)^2} = (1 + \varepsilon_\rho^*) \frac{4\tau^*}{(1 + \tau^*)^2}.$$

Hence the linearized local convergence is oscillatory if and only if

$$\left(\sqrt{1 + \varepsilon_\rho^*} - \sqrt{\varepsilon_\rho^*} \right)^2 < \tau^* < \left(\sqrt{1 + \varepsilon_\rho^*} + \sqrt{\varepsilon_\rho^*} \right)^2.$$

Whenever the eigenvalues are complex,

$$\left| \frac{\operatorname{Im} \lambda}{\operatorname{Re} \lambda} \right| = \sqrt{(1 + \varepsilon_\rho^*) \frac{4\tau^*}{(1 + \tau^*)^2} - 1} \le \sqrt{\varepsilon_\rho^*},$$

with equality at $\tau^* = 1$. Thus $\sqrt{\varepsilon_\rho^*}$ is the maximum local oscillatory intensity attainable, for a given reserve-army elasticity, as the relative response parameter τ^* varies.

Proof. See Appendix B. ■

Proposition 7 sharpens the generic discriminant condition $\Delta = (\operatorname{tr} J)^2 - 4 \det J < 0$. The reserve-army elasticity does not by itself determine whether a given economy oscillates: actual local dynamics depend jointly on ε_ρ^* and τ^*. It does, however, provide a sufficient statistic for the upper envelope of oscillatory intensity. In light of Basu and Martins' (2026) reassessment of Goodwin-type dynamics, empirical evaluation of the present mechanism would therefore naturally focus on both the elasticity of the sector-2 wage discount with respect to the reserve-army employment rate and the relative responsiveness of technology adoption and wage adjustment.

Figure 1 summarizes the local geometry and the education comparative static. In panel (a), the $\dot{\kappa} = 0$ locus is vertical in the symmetric case, while the $\dot{\pi}_w = 0$ locus is downward sloping. The directional field rotates counter-clockwise around the steady state; when $\Delta < 0$, nearby trajectories spiral inward. Panel (b) illustrates Corollary 1: a higher graduate share shifts the $\dot{\pi}_w = 0$ locus downward while leaving the vertical $\dot{\kappa} = 0$ locus unchanged, so the steady state moves to a lower value of κ.

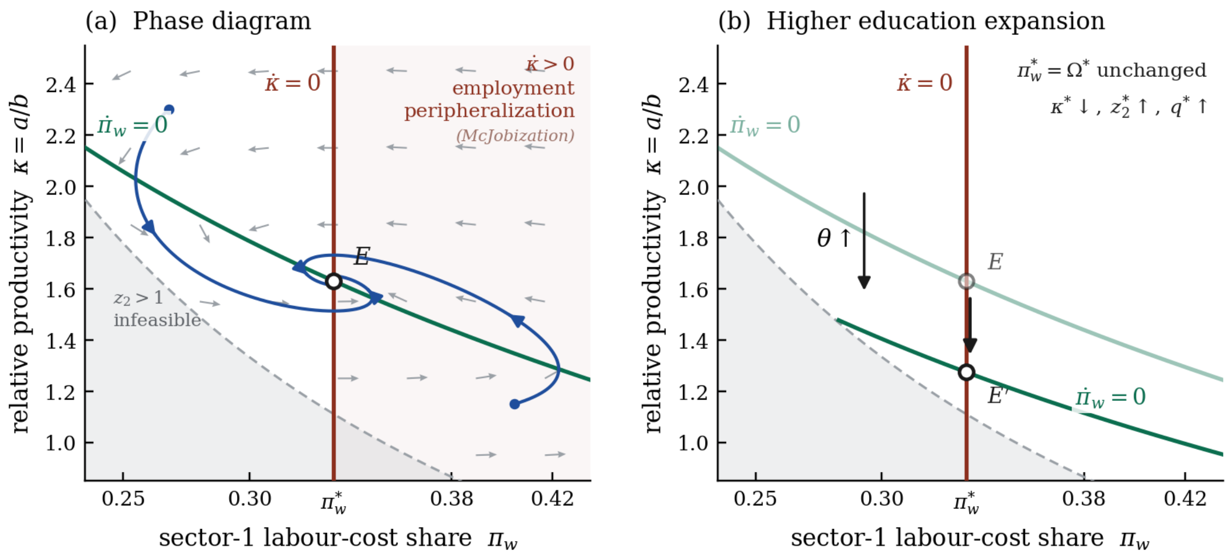

Figure 1. **Distribution–technology dynamics and the effects of higher education expansion.** In the symmetric case $g_a = g_b$, panel (a) shows the local phase diagram. The $\dot{\kappa} = 0$ locus is vertical at $\pi_w^* = \Omega^*$, while the $\dot{\pi}_w = 0$ locus slopes downward. The blue trajectory illustrates damped counter-clockwise convergence for the displayed parameterization, for which $\Delta < 0$. The shaded region satisfies $\pi_w > \Omega$ and hence $\dot{\kappa} > 0$, corresponding to employment peripheralization (the McJobization phase) under fixed coefficients. The dashed curve is the $z_2 = 1$ boundary of the feasible region projected onto the (π_w, κ) plane. Panel (b) shows the effect of increasing θ: the vertical locus and $\pi_w^* = \Omega^*$ remain unchanged, while the $\dot{\pi}_w = 0$ locus shifts downward and the steady state moves from E to E'. Consequently, κ^* falls, z_2^* and q^* rise, and N_2^*/N^* falls (Corollary 1). Illustrative parameters are $s = 0.8$, $s_1 = 0.2$, $s_2 = 0.05$, $\sigma = 0.35$, $\phi(z_1) = -0.0713 + 0.15z_1$, $g(x) = 0.1366x$, $\rho(z_2) = 0.9z_2^3$, and $\theta = 0.45$ in the baseline and 0.55 after the education expansion.

The equality $\pi_w^* = \Omega^*$ in Proposition 6 does not imply equality of real wages. Equation (5) gives

$$w_2^* = \rho(z_2^*)w_1^* < w_1^*,$$

so the sector-2 real wage remains below that in sector 1. Equalization applies to sectoral labour-cost shares, not to real wages. Proposition 6 therefore describes the erosion of sector 1's distributional advantage. At the steady state, $\hat{a}^* = \hat{b}^*$ and κ is constant. Under the symmetric adoption benchmark of Proposition 6, if $\pi_w > \Omega$ during the transition, then κ rises and, under the fixed-coefficient benchmark, employment composition shifts towards sector 2. We refer to this transitional phase, in the fixed-coefficient benchmark, as McJobization. Under a general production function, the additional condition identified in Appendix D is required.

5 Robustness: An Alternative Macroeconomic Closure with Endogenous Investment

Sections 2–4 used an exogenous surplus requirement σ to keep the causal structure transparent. This section endogenizes investment and examines which results survive under an alternative macroeconomic closure.

5.1 The Endogenous-Investment Closure

Let the profit share be

$$\pi = 1 - \pi_w - \Omega,$$

and let the investment-output ratio be $i(\pi)$, with

$$i' > 0.$$

Define the aggregate saving rate as

$$\bar{s}(\pi_w, \Omega) = s - (s - s_1)\pi_w - (s - s_2)\Omega.$$

The stationary goods-market consistency condition can then be written as

$$i(1 - \pi_w - \Omega) = s - (s - s_1)\pi_w - (s - s_2)\Omega.$$

Suppose the goods-market condition defines a locally unique interior branch for Ω as a function of π_w, with $i' \neq s - s_2$, and write it as

$$\Omega = \widetilde{\Omega}(\pi_w).$$

Crucially, neither θ nor κ enters this equation directly. Thus, in short-run comparative statics holding π_w fixed,

$$\frac{\partial\widetilde{\Omega}}{\partial\theta} = 0, \qquad \frac{\partial\widetilde{\Omega}}{\partial\kappa} = 0.$$

The slope of $\widetilde{\Omega}$ with respect to π_w, however, differs from the benchmark. Implicit differentiation gives

$$\frac{d\widetilde{\Omega}}{d\pi_w} = \frac{(s - s_1) - i'}{i' - (s - s_2)}. \tag{17}$$

so the benchmark relation $d\Omega/d\pi_w = -\Lambda < 0$ is not a general law of distribution; it is a consequence of the exogenous-surplus closure.

5.2 Robustness of the Main Short-Run Results to the Closure

Under the alternative closure, equation (6) becomes

$$\rho(z_2) = \frac{\widetilde{\Omega}(\pi_w)}{\kappa\pi_w}.$$

Since $\widetilde{\Omega}$ is independent of θ,

$$\frac{\partial z_2}{\partial\theta} = 0$$

still holds. The short-run results of Proposition 1 therefore remain valid:

$$\frac{\partial N_1}{\partial\theta} = \frac{\partial N_2}{\partial\theta} = 0, \qquad \frac{\partial N_2^H}{\partial\theta} > 0, \qquad \frac{\partial N_2^L}{\partial\theta} < 0, \qquad \frac{\partial q}{\partial\theta} > 0.$$

These results therefore do not depend on the exogeneity of σ.

Likewise, because $\widetilde{\Omega}$ does not depend directly on κ, holding π_w fixed implies

$$\frac{\partial z_2}{\partial\kappa} < 0.$$

Thus the short-run employment effects in Propositions 3 and 4 under the fixed-coefficient benchmark also survive the introduction of endogenous investment.

Taken together, our two central short-run results—that educational expansion generates credential crowding and that sector-biased technical progress changes both employment levels and employment composition—do not depend on the exogeneity of the surplus requirement.

5.3 Local Stability of Goods-Market Adjustment

Suppose output adjusts to excess demand according to

$$\dot{Y} = \beta Y \left[i(\pi) - \bar{s}(\pi_w, \Omega) \right], \qquad \beta > 0.$$

Holding π_w fixed and evaluating the adjustment locally around equilibrium, Appendix C shows that the corresponding local stability condition—the Keynesian stability condition in the present model—is

$$i' > s - s_2. \tag{18}$$

This condition arises because the sector-2 wage responds to the employment rate, so changes in output affect the saving rate through their distributional effects. Higher output raises z_2, ρ, and Ω, shifting income towards workers with lower saving propensities and reducing the saving rate. Relative to an adjustment process with fixed distribution, stability therefore requires investment to be sufficiently sensitive to the profit share.[12]

Condition (18) alone does not determine the sign of $d\widetilde{\Omega}/d\pi_w$. Under (18), the denominator in (17) is positive, while the numerator has the sign of $(s - s_1) - i'$. Hence, if

$$i' > \max\{s - s_1, s - s_2\},$$

then

$$\frac{d\widetilde{\Omega}}{d\pi_w} < 0.$$

By contrast, if

$$s - s_2 < i' < s - s_1,$$

—a non-empty interval only when $s_2 > s_1$—then $d\widetilde{\Omega}/d\pi_w > 0$, and the two sectoral labour-cost shares move in the same direction. Thus the sign of the intersectoral distributional relation is closure- and parameter-dependent rather than a general zero-sum property.

[12]For debates on stability conditions in Kaleckian growth models—particularly the distinction between Keynesian and Harrodian instability and the specification of the investment function—see Hein, Lavoie and van Treeck (2011) and Skott (2012). Equation (18) should not be read as the standard stability condition from those models. It is a local stability condition generated by the specific structure of our model, in which sector-2 employment and distribution respond to output adjustment.

5.4 Long-Run Equalization and Stability

For the analysis in this subsection, we use the reduced two-dimensional distribution–technology system obtained by imposing the local goods-market relation $\Omega = \widetilde{\Omega}(\pi_w)$. This corresponds to assuming that goods-market adjustment is sufficiently fast relative to the dynamics of distribution and technology for the economy to remain locally close to that relation. The stability conditions below therefore apply to this reduced system; they are not a claim about the local stability of the full three-dimensional system that would treat output Y as an additional state variable.

Endogenizing investment does not change the steady-state condition for technology adoption. At an interior steady state, $\dot{\kappa} = 0$ implies

$$g_a(\pi_w^*) = g_b(\Omega^*).$$

Thus, if $g_a = g_b$,

$$\pi_w^* = \Omega^*$$

continues to hold. The common value is no longer determined in closed form by (8); instead, it is jointly determined by the investment–saving equilibrium condition. Hence the qualitative equalization of sectoral labour-cost shares is robust to the closure, whereas the closed-form value of the common labour-cost share is benchmark-specific. The ranking result in Corollary 2 under asymmetric g_a and g_b survives for the same reason.

Local stability of the two-dimensional system, by contrast, depends on the closure. Under the alternative closure,

$$J_{21} = \kappa^* \left[g_a' - g_b' \frac{d\widetilde{\Omega}}{d\pi_w} \right],$$

so when $d\widetilde{\Omega}/d\pi_w > 0$, its sign is not determined automatically. Likewise, the response of z_2 to π_w satisfies

$$\rho'(z_2)\frac{\partial z_2}{\partial \pi_w} = \frac{\pi_w \widetilde{\Omega}' - \widetilde{\Omega}}{\kappa \pi_w^2},$$

which need not be negative as in the benchmark.

A convenient set of sufficient conditions for retaining the benchmark sign configuration is

$$\frac{d \ln \widetilde{\Omega}}{d \ln \pi_w} < 1,$$

which implies $\partial z_2/\partial \pi_w < 0$ and hence $\partial Z/\partial \pi_w < 0$, together with

$$g_a' > g_b' \frac{d\widetilde{\Omega}}{d\pi_w},$$

which implies $J_{21} > 0$. These conditions show that responses that are automatically stabilizing under the benchmark can become conditional once investment is endogenized.

Thus, the short-run credential-crowding result, the short-run employment effects of sector-biased technical progress, and equalization of labour-cost shares under symmetric technology adoption survive the alternative closure, whereas zero-sum intersectoral distribution, the closed-form steady-state distribution, and local stability without additional restrictions are benchmark-specific. The closed-form oscillation characterization in Proposition 7 is also benchmark-specific because it uses the linear benchmark relation $d\Omega^e/d\pi_w = -\Lambda$.

6 Conclusion

We develop a macro-dynamic model combining a degree-segmented dual labour market, reserve-army wage determination, and distribution-induced sectoral technology adoption. The model treats education supply, employment composition, distribution, and technical change as mutually interacting processes within an institutionally segmented labour market.

First, an expansion of higher education does not automatically create more graduate jobs. In the short run, sector-1 employment N_1 and total sector-2 employment N_2 are independent of the graduate share θ. Educational expansion therefore changes the educational composition of existing sector-2 employment rather than creating new sector-2 jobs: graduate employment in sector 2, N_2^H, and the incidence of overeducation among employed graduates, q, both rise. This credential-crowding mechanism treats overeducation not as a temporary search friction, but as a structural outcome of the disconnect between the number of degree-requiring jobs and the supply of educated labour. In the long run, sector-1 employment rises with educational expansion, yet the incidence of overeducation also increases. Under symmetric endogenous technology adoption, an increase in education similarly raises z_2^* and q^* while lowering κ^* and the sector-2 employment share. Thus, a lower degree of employment peripheralization can coexist with a higher incidence of overeducation. Expanding education alone does not eliminate the mismatch between educational credentials and job requirements.

Second, the direction of technical change has asymmetric effects on both the level and composition of employment. In the short run, sector-1-biased technical progress lowers z_2, N_1, and N and, under the fixed-coefficient benchmark, raises N_2/N. Employment peripheralization, or McJobization, refers to this rise in the sector-2 employment share and does not imply an absolute increase in N_2. Appendix D shows that these short-run employment-level and compositional effects carry over to a general constant-returns-to-scale final-good technology with the appropriately generalized surplus closure.

Third, distribution induces technology adoption, which in turn erodes sectoral distributional advantages. A higher labour-cost share induces stronger labour-saving technology adoption, so under symmetric adoption costs an interior steady state satisfies $\pi_w^* = \Omega^*$. This is the disappearance of sector 1's distributional advantage rather than equality of real wages; with asymmetric adoption possibilities, the ranking of sectoral labour-cost shares can be reversed.

Fourth, the local distribution–technology dynamics admit a structural characterization. In the symmetric benchmark, oscillatory convergence depends jointly on the steady-state reserve-army elasticity ε_ρ^* and the relative response parameter τ^*. The reserve-army elasticity alone determines the maximum attainable ratio $|\operatorname{Im}\lambda/\operatorname{Re}\lambda|$, which is $\sqrt{\varepsilon_\rho^*}$. This result connects the reserve-army wage mechanism directly to the strength of local distribution–technology cycles and identifies measurable objects for empirical evaluation.

The alternative closure in Section 5 clarifies the extent to which the first three results depend on the benchmark closure. Once investment is endogenized, the zero-sum relationship between π_w and Ω and local stability without additional parameter restrictions need not survive. Credential crowding, the short-run effects of sector-biased technical progress, and equalization of labour-cost shares under symmetric technology adoption, however, remain. The closed-form oscillation bound in Proposition 7 is a benchmark result and is not claimed to survive the alternative closure.

Our analysis has three limitations. First, the benchmark does not explicitly model the capital stock or capacity utilization. Section 5 introduces endogenous investment only as a robustness exercise rather than as a fully endogenous accumulation model. A natural extension would connect the framework to the Sraffian Supermultiplier (Serrano, 1995; Freitas and Serrano, 2015; Lavoie, 2016) or to a Kaleckian growth model so that the labour-demand effects of accumulation and the labour-saving effects of technical change can be analysed jointly.

Second, within sector 2 we impose the same employment rate across educational groups and specify the relative-wage function as $\rho = \rho(z_2)$, independently of the educational composition of the labour pool. This provides a neutral benchmark for educational allocation and wage formation within sector 2, but abstracts from employer preferences for credentials, credential screening, and the possibility that an influx of graduates changes the bargaining environment in sector 2 itself. More generally, with $\rho = \rho(z_2, h_2)$, educational expansion could affect z_2 directly, making the employment-invariance part of Proposition 1 conditional. Jointly endogenizing education-specific hiring probabilities and wage formation that depends on educational composition would permit a more general analysis of the strength of credential crowding and the displacement of non-graduates. We likewise hold the sector-1 wage-bargaining function ϕ independent of the graduate share θ. Allowing $\phi = \phi(z_1, \theta)$ would make the long-run invariance of z_1^* with respect to educational expansion conditional on the specification of this bargaining channel.

Third, firms are homogeneous and vertically integrated, and the model abstracts from intermediate-goods markets and firm heterogeneity. This simplification makes the causal link from distribution to technology adoption transparent. Allowing for sector-specific firms, intermediate-goods prices, and heterogeneity in technology adoption would permit a richer analysis of the interaction between the direction of technical change and labour-market segmentation.

The central message is simple: educational expansion, technical progress, and the expansion of higher-wage, degree-requiring employment are distinct processes. The number of jobs, access to them, and the direction of technology adoption are shaped by institutional segmentation and the distributional structure of the labour market. In a dual labour market, technical change does more than raise productivity: it can reshape both employment composition and the distributional structure.

A Benchmark Short-Run Equilibrium and Long-Run Comparative Statics

A.1 The Sector-2 Labour-Cost Share

From (9),

$$\frac{\partial \Omega^e}{\partial \pi_w} = -\Lambda < 0, \qquad \frac{\partial \Omega^e}{\partial \sigma} = -\frac{1}{s - s_2} < 0, \qquad \Lambda = \frac{s - s_1}{s - s_2} > 0.$$

Ω^e does not depend directly on θ or κ.

A.2 The Sector-2 Employment Rate

Let the right-hand side of (10) be

$$R(\pi_w, \kappa, \sigma) = \frac{(s-\sigma) - (s-s_1)\pi_w}{(s-s_2)\kappa\pi_w}.$$

At an interior solution,

$$R = \rho(z_2) > 0.$$

Since $s - s_2 > 0$, $\kappa > 0$, and $\pi_w > 0$, this implies

$$(s-\sigma) > (s-s_1)\pi_w > 0,$$

and hence $s - \sigma > 0$. The partial derivatives are therefore

$$\frac{\partial R}{\partial \pi_w} = -\frac{s-\sigma}{(s-s_2)\kappa\pi_w^2} < 0,$$

$$\frac{\partial R}{\partial \kappa} = -\frac{R}{\kappa} < 0, \qquad \frac{\partial R}{\partial \sigma} = -\frac{1}{(s-s_2)\kappa\pi_w} < 0,$$

and $\partial R/\partial\theta = 0$. Since $\rho' > 0$, the inverse-function theorem implies

$$\frac{\partial z_2}{\partial \pi_w} < 0, \qquad \frac{\partial z_2}{\partial \kappa} < 0, \qquad \frac{\partial z_2}{\partial \sigma} < 0, \qquad \frac{\partial z_2}{\partial \theta} = 0.$$

A.3 Employment Levels

From

$$N_1 = N_s \frac{z_2}{\kappa + z_2},$$

we obtain

$$\frac{\partial N_1}{\partial z_2} = N_s \frac{\kappa}{(\kappa + z_2)^2} > 0, \qquad \frac{\partial N_1}{\partial \theta} = 0.$$

Accounting for the response of z_2 to κ,

$$\frac{dN_1}{d\kappa} = \frac{N_s}{(\kappa+z_2)^2}\left[\kappa\frac{\partial z_2}{\partial \kappa} - z_2\right] < 0.$$

Also,

$$N = N_s \frac{z_2(1+\kappa)}{\kappa + z_2},$$

so

$$\frac{dN}{d\kappa} = \frac{N_s}{(\kappa + z_2)^2}\left[(\kappa^2 + \kappa)\frac{\partial z_2}{\partial \kappa} + z_2(z_2 - 1)\right] < 0.$$

A.4 Overeducation

Under proportional rationing,

$$N_2^H = z_2(\theta N_s - N_1), \qquad N_2^L = z_2(1 - \theta)N_s.$$

Since $\partial z_2/\partial\theta = \partial N_1/\partial\theta = 0$ in the short run,

$$\frac{\partial N_2^H}{\partial\theta} = z_2 N_s > 0, \qquad \frac{\partial N_2^L}{\partial\theta} = -z_2 N_s < 0.$$

For

$$q = \frac{N_2^H}{N_1 + N_2^H},$$

we have

$$\frac{\partial q}{\partial\theta} = \frac{z_2 N_s N_1}{(N_1 + N_2^H)^2} > 0.$$

A.5 The Sector-1 Employment Rate

Since $z_1 = N_1/(\theta N_s)$,

$$\frac{\partial z_1}{\partial \pi_w} = \frac{1}{\theta N_s}\frac{\partial N_1}{\partial z_2}\frac{\partial z_2}{\partial \pi_w} < 0,$$

and

$$\frac{dz_1}{d\kappa} = \frac{1}{\theta N_s}\frac{dN_1}{d\kappa} < 0, \qquad \frac{\partial z_1}{\partial\theta} = -\frac{z_1}{\theta} < 0.$$

A.6 Long-Run Comparative Statics under Exogenous Technical Progress

At the steady state, $\phi(z_1^*) = \gamma_a$, so z_1^* is fixed. Therefore,

$$N_1^* = \theta N_s z_1^*.$$

Solving (1) for z_2^* gives

$$z_2^* = \frac{\kappa\theta z_1^*}{1-\theta z_1^*}.$$

Define

$$R^* \equiv \kappa\rho(z_2^*) > 0, \qquad \Omega^* = R^*\pi_w^*.$$

From (8),

$$\pi_w^* = \frac{s-\sigma}{(s-s_1)+(s-s_2)R^*},$$

and

$$\Omega^* = \frac{(s-\sigma)R^*}{(s-s_1)+(s-s_2)R^*}.$$

Thus an increase in either θ or κ raises z_2^* and R^*, lowers π_w^*, and raises Ω^*. By contrast, an increase in σ leaves z_1^*, z_2^*, and R^* unchanged while lowering both π_w^* and Ω^*.

Moreover,

$$q^* = \frac{z_2^*(1-z_1^*)}{z_1^*+z_2^*(1-z_1^*)},$$

which is increasing in z_2^* for fixed z_1^*.

B Local Stability of the Two-Dimensional Dynamic System

Let the right-hand sides of (16) be

$$F(\pi_w,\kappa) = \pi_w[\phi(Z(\pi_w,\kappa;\sigma,\theta)) - g_a(\pi_w)],$$

and

$$G(\pi_w,\kappa) = \kappa[g_a(\pi_w) - g_b(\Omega^e(\pi_w))].$$

Because both bracketed terms vanish at the steady state, the Jacobian is

$$J_{11} = \pi_w^*\left[\phi'\frac{\partial z_1}{\partial \pi_w} - g_a'\right],$$

$$J_{12} = \pi_w^* \phi' \frac{dz_1}{d\kappa},$$

$$J_{21} = \kappa^* \left[g_a' - g_b' \frac{\partial \Omega^e}{\partial \pi_w} \right] = \kappa^* [g_a' + g_b' \Lambda],$$

and

$$J_{22} = 0.$$

Appendix A gives

$$\frac{\partial z_1}{\partial \pi_w} < 0, \qquad \frac{dz_1}{d\kappa} < 0.$$

Since $\phi', g_a', g_b', \Lambda > 0$,

$$J_{11} < 0, \qquad J_{12} < 0, \qquad J_{21} > 0, \qquad J_{22} = 0.$$

Hence

$$\operatorname{tr} J = J_{11} < 0,$$

and

$$\det J = -J_{12} J_{21} > 0.$$

Conditional on the existence of an interior steady state, the necessary and sufficient conditions for local asymptotic stability are therefore satisfied. If, in addition,

$$\Delta = (\operatorname{tr} J)^2 - 4 \det J < 0,$$

then the linearized local dynamics exhibit damped oscillations.

B.1 Reserve-Army Elasticity and the Oscillation Bound

We now derive Proposition 7. Under symmetry, write $g_a = g_b = g$. At the steady state, $\pi_w^* = \Omega^*$, so $g_a' = g_b' = g'$, and define

$$A^* \equiv - \left. \frac{\partial z_1}{\partial \pi_w} \right|_* > 0, \qquad B^* \equiv - \left. \frac{dz_1}{d\kappa} \right|_* > 0.$$

The trace and determinant become

$$\operatorname{tr} J = -\pi_w^*\big(\phi' A^* + g'\big), \qquad \det J = \pi_w^* \phi' B^* \kappa^* g'(1+\Lambda),$$

where ϕ', g', and ρ' below are evaluated at the steady state.

To express A^* and B^* in primitives, use

$$R(\pi_w, \kappa) = \frac{\Omega^e(\pi_w)}{\kappa \pi_w} = \rho(z_2).$$

At the symmetric steady state, $\Omega^* = \pi_w^*$ and $\kappa^* \rho(z_2^*) = 1$. Hence

$$\left.\frac{\partial R}{\partial \pi_w}\right|_* = -\frac{1+\Lambda}{\kappa^* \pi_w^*}, \qquad \left.\frac{\partial R}{\partial \kappa}\right|_* = -\frac{1}{(\kappa^*)^2}.$$

Implicit differentiation of $R = \rho(z_2)$ gives

$$\left.\frac{\partial z_2}{\partial \pi_w}\right|_* = -\frac{1+\Lambda}{\kappa^* \pi_w^* \rho'}, \qquad \left.\frac{\partial z_2}{\partial \kappa}\right|_* = -\frac{1}{(\kappa^*)^2 \rho'}.$$

Since

$$z_1 = \frac{z_2}{\theta(\kappa + z_2)},$$

let

$$D^* \equiv \frac{1}{\theta(\kappa^* + z_2^*)^2} > 0.$$

Then

$$A^* = D^* \frac{1+\Lambda}{\pi_w^* \rho'}, \qquad B^* = D^* \left(\frac{1}{\kappa^* \rho'} + z_2^*\right).$$

Therefore,

$$\frac{\kappa^* B^* (1+\Lambda)}{\pi_w^* A^*} = 1 + \kappa^* z_2^* \rho' = 1 + \frac{z_2^* \rho'}{\rho(z_2^*)} = 1 + \varepsilon_\rho^*,$$

where the second equality uses $\kappa^* \rho(z_2^*) = 1$. Define

$$\tau^* \equiv \frac{g'}{\phi' A^*} > 0.$$

Substitution into the trace–determinant ratio yields

$$\frac{4 \det J}{(\operatorname{tr} J)^2} = (1 + \varepsilon_\rho^*) \frac{4\tau^*}{(1+\tau^*)^2}.$$

The eigenvalues are complex if and only if this expression exceeds one, or equivalently

$$(\tau^*)^2 - 2(1+2\varepsilon_\rho^*)\tau^* + 1 < 0.$$

The two roots are

$$\left(\sqrt{1+\varepsilon_\rho^*} - \sqrt{\varepsilon_\rho^*}\right)^2, \qquad \left(\sqrt{1+\varepsilon_\rho^*} + \sqrt{\varepsilon_\rho^*}\right)^2,$$

which gives the oscillation interval stated in Proposition 7. For complex eigenvalues,

$$\left|\frac{\operatorname{Im}\lambda}{\operatorname{Re}\lambda}\right| = \sqrt{\frac{4\det J}{(\operatorname{tr} J)^2} - 1}.$$

Finally, $4\tau/(1+\tau)^2 \leq 1$ for every $\tau > 0$, with equality at $\tau = 1$. Hence

$$\left|\frac{\operatorname{Im}\lambda}{\operatorname{Re}\lambda}\right| \leq \sqrt{\varepsilon_\rho^*},$$

and the upper bound is attained when $\tau^* = 1$.

C Endogenous-Investment Closure

C.1 Distributional Relation

Under endogenous investment,

$$i(1-\pi_w-\Omega) = s - (s-s_1)\pi_w - (s-s_2)\Omega.$$

Differentiating both sides with respect to π_w gives

$$i'\left(-1 - \frac{d\Omega}{d\pi_w}\right) = -(s-s_1) - (s-s_2)\frac{d\Omega}{d\pi_w}.$$

Rearranging,

$$\frac{d\Omega}{d\pi_w} = \frac{(s-s_1)-i'}{i'-(s-s_2)}.$$

C.2 Local Stability of Goods-Market Adjustment

Let output adjust according to

$$\dot{Y} = \beta Y\left[i(\pi) - \bar{s}(\pi_w,\Omega)\right], \qquad \beta > 0.$$

At equilibrium the bracketed term is zero, so

$$\left.\frac{\partial \dot{Y}}{\partial Y}\right|_{*} = \beta Y^{*}\left[\frac{\partial i}{\partial Y} - \frac{\partial \bar{s}}{\partial Y}\right].$$

Holding π_w fixed, an increase in output raises z_2 and Ω, so $d\Omega/dY > 0$. Moreover,

$$\frac{\partial \pi}{\partial Y} = -\frac{d\Omega}{dY},$$

$$\frac{\partial i}{\partial Y} = -i'\frac{d\Omega}{dY}, \qquad \frac{\partial \bar{s}}{\partial Y} = -(s - s_2)\frac{d\Omega}{dY}.$$

Therefore,

$$\left.\frac{\partial \dot{Y}}{\partial Y}\right|_{*} = \beta Y^{*}\frac{d\Omega}{dY}[(s - s_2) - i'].$$

The local stability condition is

$$i' > s - s_2.$$

C.3 Stability of the Reduced Two-Dimensional Technology Dynamics

Writing the alternative closure as $\Omega = \widetilde{\Omega}(\pi_w)$, the Jacobian of the technology dynamics contains

$$J_{21} = \kappa^{*}\left[g_a' - g_b'\widetilde{\Omega}'\right].$$

Also,

$$\rho'(z_2)\frac{\partial z_2}{\partial \pi_w} = \frac{\pi_w\widetilde{\Omega}' - \widetilde{\Omega}}{\kappa \pi_w^2}.$$

Thus, for example, if

$$\frac{d\ln\widetilde{\Omega}}{d\ln \pi_w} < 1$$

and

$$g_a' > g_b'\widetilde{\Omega}',$$

then the benchmark sign configuration $J_{11} < 0, J_{12} < 0, J_{21} > 0, J_{22} = 0$ is preserved and the interior steady state is locally asymptotically stable. These are sufficient conditions, illustrating that stability under the alternative closure is generally conditional.

D Robustness under a General Final-Good Production Technology

To examine whether the compositional effect in Proposition 3 is mechanically driven by the fixed-coefficient final-good technology, replace the final-good production function with a general constant-returns-to-scale function,

$$Y = F(X_1, X_2),$$

where F is increasing in each input and the standard cost-minimization problem has an interior solution. For a vertically integrated firm, the labour cost of one unit of each intermediate input is

$$c_1 = \frac{w_1}{a} = \pi_w, \qquad c_2 = \frac{w_2}{b} = \Omega.$$

Under the fixed-coefficient benchmark, $X_1 = X_2 = Y$, so π_w and Ω are also the two sectors' labour-cost shares in final output. With a general F, this equivalence no longer holds: π_w and Ω should instead be interpreted as the unit labour costs of the two intermediate inputs.

Because F is constant returns to scale, let

$$x_1(\pi_w, \Omega) \equiv \frac{X_1}{Y}, \qquad x_2(\pi_w, \Omega) \equiv \frac{X_2}{Y}$$

denote the cost-minimizing quantities of the two intermediate inputs per unit of final output. Their ratio is

$$m(\pi_w, \Omega) \equiv \frac{X_2}{X_1} = \frac{x_2(\pi_w, \Omega)}{x_1(\pi_w, \Omega)} > 0.$$

The wage bills per unit of final output are therefore

$$\frac{W_1}{Y} = \pi_w x_1(\pi_w, \Omega), \qquad \frac{W_2}{Y} = \Omega x_2(\pi_w, \Omega).$$

Accordingly, the benchmark surplus-absorption condition generalizes to

$$(s - s_1)\pi_w x_1(\pi_w, \Omega) + (s - s_2)\Omega x_2(\pi_w, \Omega) = s - \sigma.$$

Suppose this equation has a locally unique interior solution for Ω conditional on π_w and σ, and write it as

$$\Omega = \Omega^F(\pi_w; \sigma).$$

Crucially, the generalized closure contains no direct dependence on relative productivity $\kappa = a/b$. Hence, holding π_w and σ fixed,

$$\frac{\partial \Omega^F}{\partial \kappa} = 0.$$

The relative-wage relation (6) remains an identity under the general final-good technology,

$$\Omega^F = \kappa \rho(z_2) \pi_w.$$

Differentiating with respect to κ while holding π_w fixed therefore gives

$$\frac{\partial z_2}{\partial \kappa} = -\frac{\rho(z_2)}{\kappa \rho'(z_2)} < 0.$$

Moreover, because $m = m(\pi_w, \Omega^F)$, the cost-minimizing input ratio is constant with respect to κ in this short-run comparative static.

Since $N_1 = X_1/a$ and $N_2 = X_2/b$,

$$\frac{N_2}{N_1} = \frac{X_2/b}{X_1/a} = \kappa m. \tag{19}$$

Hence

$$\frac{N_2}{N} = \frac{\kappa m}{1 + \kappa m}. \tag{20}$$

The fixed-coefficient benchmark is the special case $m = 1$. Since m is constant with respect to κ in the short-run comparison,

$$\frac{\partial}{\partial \kappa}\left(\frac{N_2}{N}\right) = \frac{m}{(1 + \kappa m)^2} > 0. \tag{21}$$

Thus the short-run peripheralization result does not hinge on the Leontief specification.

Using $z_2 = N_2/(N_s - N_1)$ together with (19),

$$N_1 = N_s \frac{z_2}{\kappa m + z_2}. \tag{22}$$

Since $\partial z_2/\partial \kappa < 0$ and $m > 0$,

$$\frac{dN_1}{d\kappa} = \frac{N_s m}{(\kappa m + z_2)^2}\left[\kappa \frac{\partial z_2}{\partial \kappa} - z_2\right] < 0.$$

Total employment is

$$N = N_s \frac{z_2(1+\kappa m)}{\kappa m + z_2},$$

and

$$\frac{dN}{d\kappa} = \frac{N_s}{(\kappa m + z_2)^2}\left[\kappa m(1+\kappa m)\frac{\partial z_2}{\partial \kappa} + m z_2(z_2 - 1)\right] < 0. \tag{23}$$

The short-run result in Proposition 3 therefore carries over to a general constant-returns-to-scale final-good technology with an interior cost-minimizing input mix.

In the dynamics, however, m itself changes with π_w and Ω. Since

$$\frac{N_2}{N_1} = \kappa m,$$

its growth rate is

$$\widehat{\left(\frac{N_2}{N_1}\right)} = \hat{\kappa} + \hat{m}.$$

A sufficient condition for employment composition to shift dynamically towards sector 2 under a general production function is therefore

$$\hat{\kappa} + \hat{m} > 0.$$

Under the fixed-coefficient benchmark, $\hat{m} = 0$, so $\hat{\kappa} > 0$ immediately implies peripheralization. With a general technology, the change in the intermediate-input mix must not be large enough to offset the change in relative productivity.

The extension in this appendix is deliberately limited to the mapping from relative productivity to employment levels and employment composition, together with the associated surplus closure. It does not claim that the firm-level technology-adoption rule in Section 4 is invariant to the final-good production technology. Under a general F, the cost-minimizing input coefficients x_1 and x_2 would also weight the marginal unit-cost gains from productivity growth; a full extension of the technology-adoption problem would therefore require rederiving the adoption first-order conditions. The benchmark adoption mechanism in Section 4 is retained to keep the main model transparent.

References

Acemoglu, D. (1998). "Why Do New Technologies Complement Skills? Directed Technical Change and Wage Inequality." *Quarterly Journal of Economics*, 113(4), 1055–1089.

Acemoglu, D. (2002). "Directed Technical Change." *Review of Economic Studies*, 69(4), 781–809.

Acemoglu, D., Kong, F., & Restrepo, P. (2025). "Tasks at Work: Comparative Advantage, Technology and Labor Demand." In C. Dustmann & T. Lemieux (eds.), *Handbook of Labor Economics*, Vol. 6, 1–114. Elsevier. https://doi.org/10.1016/bs.heslab.2025.08.003.

Acemoglu, D., & Loebbing, J. (2026). "Automation and Polarization." *Journal of Political Economy*, 134(3), 1017–1072. https://doi.org/10.1086/739330.

Acemoglu, D., & Restrepo, P. (2018). "The Race between Man and Machine: Implications of Technology for Growth, Factor Shares, and Employment." *American Economic Review*, 108(6), 1488–1542.

Acemoglu, D., & Restrepo, P. (2019). "Automation and New Tasks: How Technology Displaces and Reinstates Labor." *Journal of Economic Perspectives*, 33(2), 3–30.

Acemoglu, D., & Restrepo, P. (2022). "Tasks, Automation, and the Rise in U.S. Wage Inequality." *Econometrica*, 90(5), 1973–2016.

Basu, D., & Martins, D. (2026). "Theory and Empirics of the Goodwin Model." *Metroeconomica*, 1–16. https://doi.org/10.1111/meca.70014.

Doeringer, P. B., & Piore, M. J. (1971). *Internal Labor Markets and Manpower Analysis*. Lexington, MA: D. C. Heath.

Foley, D. K. (2003). "Endogenous Technical Change with Externalities in a Classical Growth Model." *Journal of Economic Behavior & Organization*, 52(2), 167–189.

Foley, D. K., & Michl, T. R. (1999). *Growth and Distribution*. Cambridge, MA: Harvard University Press.

Freitas, F., & Serrano, F. (2015). "Growth Rate and Level Effects, the Stability of the Adjustment of Capacity to Demand and the Sraffian Supermultiplier." *Review of Political Economy*, 27(3), 258–281.

Goodwin, R. M. (1967). "A Growth Cycle." In C. H. Feinstein (ed.), *Socialism, Capitalism and Economic Growth*, 54–58. Cambridge: Cambridge University Press.

Goos, M., & Manning, A. (2007). "Lousy and Lovely Jobs: The Rising Polarization of

Work in Britain." *Review of Economics and Statistics*, 89(1), 118–133.

Hein, E., Lavoie, M., & van Treeck, T. (2011). "Some Instability Puzzles in Kaleckian Models of Growth and Distribution: A Critical Survey." *Cambridge Journal of Economics*, 35(3), 587–612.

Jacobo, J. (2025). "Automation, Power, and Profit: A Critique and Generalization of the Task-Based Approach." *Metroeconomica*, 1–19. `https://doi.org/10.1111/meca.70006`.

Julius, A. J. (2005). "Steady-State Growth and Distribution with an Endogenous Direction of Technical Change." *Metroeconomica*, 56(1), 101–125.

Kennedy, C. (1964). "Induced Bias in Innovation and the Theory of Distribution." *Economic Journal*, 74(295), 541–547.

Lavoie, M. (2016). "Convergence Towards the Normal Rate of Capacity Utilization in Neo-Kaleckian Models: The Role of Non-Capacity Creating Autonomous Expenditures." *Metroeconomica*, 67(1), 172–201.

Leuven, E., & Oosterbeek, H. (2011). "Overeducation and Mismatch in the Labor Market." In E. A. Hanushek, S. Machin, & L. Woessmann (eds.), *Handbook of the Economics of Education*, Vol. 4, 283–326. Elsevier.

Marx, K. (1867/1976). *Capital: A Critique of Political Economy, Volume I.* Trans. B. Fowkes. Penguin Books.

McGuinness, S. (2006). "Overeducation in the Labour Market." *Journal of Economic Surveys*, 20(3), 387–418.

Okishio, N. (1961). "Technical Change and the Rate of Profit." *Kobe University Economic Review*, 7, 85–99.

Rada, C., Tavani, D., von Arnim, R., & Zamparelli, L. (2023). "Classical and Keynesian Models of Inequality and Stagnation." *Journal of Economic Behavior & Organization*, 211, 442–461.

Restrepo, P. (2024). "Automation: Theory, Evidence, and Outlook." *Annual Review of Economics*, 16, 1–25.

Ritzer, G. (1993). *The McDonaldization of Society.* Thousand Oaks, CA: Pine Forge Press.

Serrano, F. (1995). "Long Period Effective Demand and the Sraffian Supermultiplier." *Contributions to Political Economy*, 14(1), 67–90.

Shah, A., & Desai, M. (1981). "Growth Cycles with Induced Technical Change." *Economic Journal*, 91(364), 1006–1010.

Skott, P. (2005). "Fairness as a Source of Hysteresis in Employment and Relative Wages." *Journal of Economic Behavior & Organization*, 57(3), 305–331.

Skott, P. (2006). "Wage Inequality and Overeducation in a Marxian/Goodwin Model of the Growth Cycle." *Structural Change and Economic Dynamics*, 17(1), 35–53.

Skott, P. (2012). "Theoretical and Empirical Shortcomings of the Kaleckian Investment Function." *Metroeconomica*, 63(1), 109–138.

Stamegna, M. (2025). "A Kaleckian Growth Model of Secular Stagnation with Induced Innovation." *Metroeconomica*, 76(2), 340–371. https://doi.org/10.1111/meca.12486.

Tavani, D. (2012). "Wage Bargaining and Induced Technical Change in a Linear Economy: Model and Application to the US (1963–2003)." *Structural Change and Economic Dynamics*, 23(2), 117–126.

Tavani, D. (2013). "Bargaining over Productivity and Wages when Technical Change is Induced: Implications for Growth, Distribution, and Employment." *Journal of Economics*, 109(3), 207–244.

Tavani, D., & Zamparelli, L. (2015). "Endogenous Technical Change, Employment and Distribution in the Goodwin Model of the Growth Cycle." *Studies in Nonlinear Dynamics & Econometrics*, 19(2), 209–216.

Tavani, D., & Zamparelli, L. (2017). "Endogenous Technical Change in Alternative Theories of Growth and Distribution." *Journal of Economic Surveys*, 31(5), 1272–1303.

Taylor, L. (1983). *Structuralist Macroeconomics: Applicable Models for the Third World*. New York: Basic Books.

Taylor, L. (2004). *Reconstructing Macroeconomics: Structuralist Proposals and Critiques of the Mainstream*. Cambridge, MA: Harvard University Press.

van der Ploeg, F. (1987). "Growth Cycles, Induced Technical Change, and Perpetual Conflict over the Distribution of Income." *Journal of Macroeconomics*, 9(1), 1–12.

Zamparelli, L. (2015). "Induced Innovation, Endogenous Technical Change and Income Distribution in a Labor-Constrained Model of Classical Growth." *Metroeconomica*, 66(2), 243–262.